

Free surface models of partially molten rock with visco-elasto–plastic rheology: numerical solutions using a staggered-grid finite-difference discretisation

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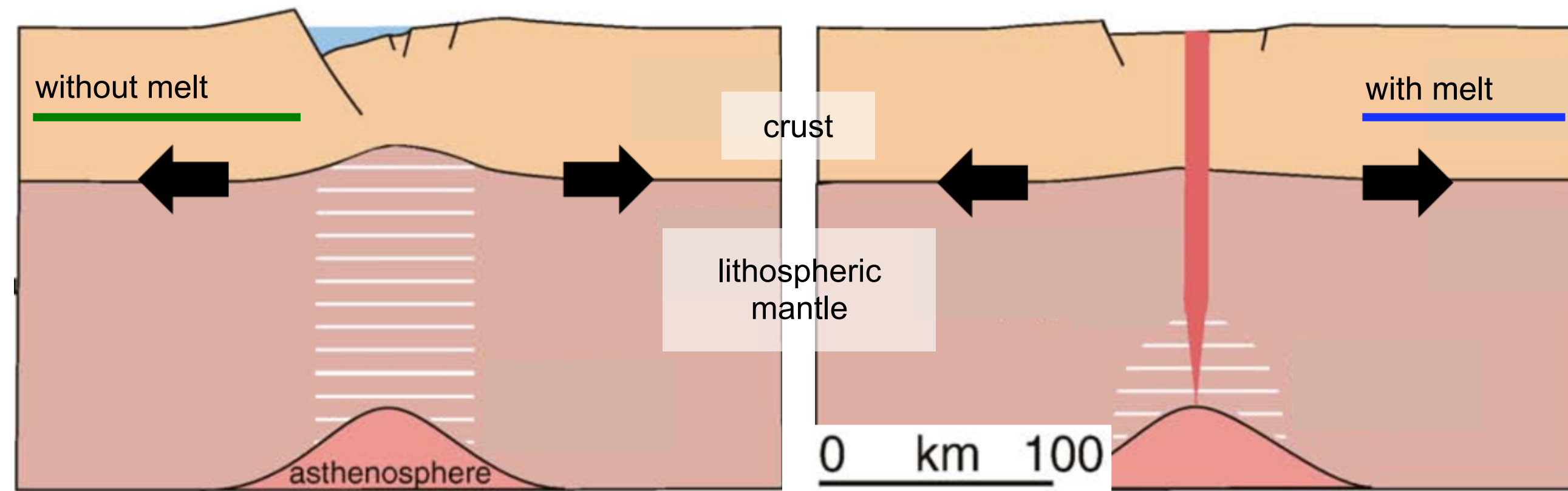
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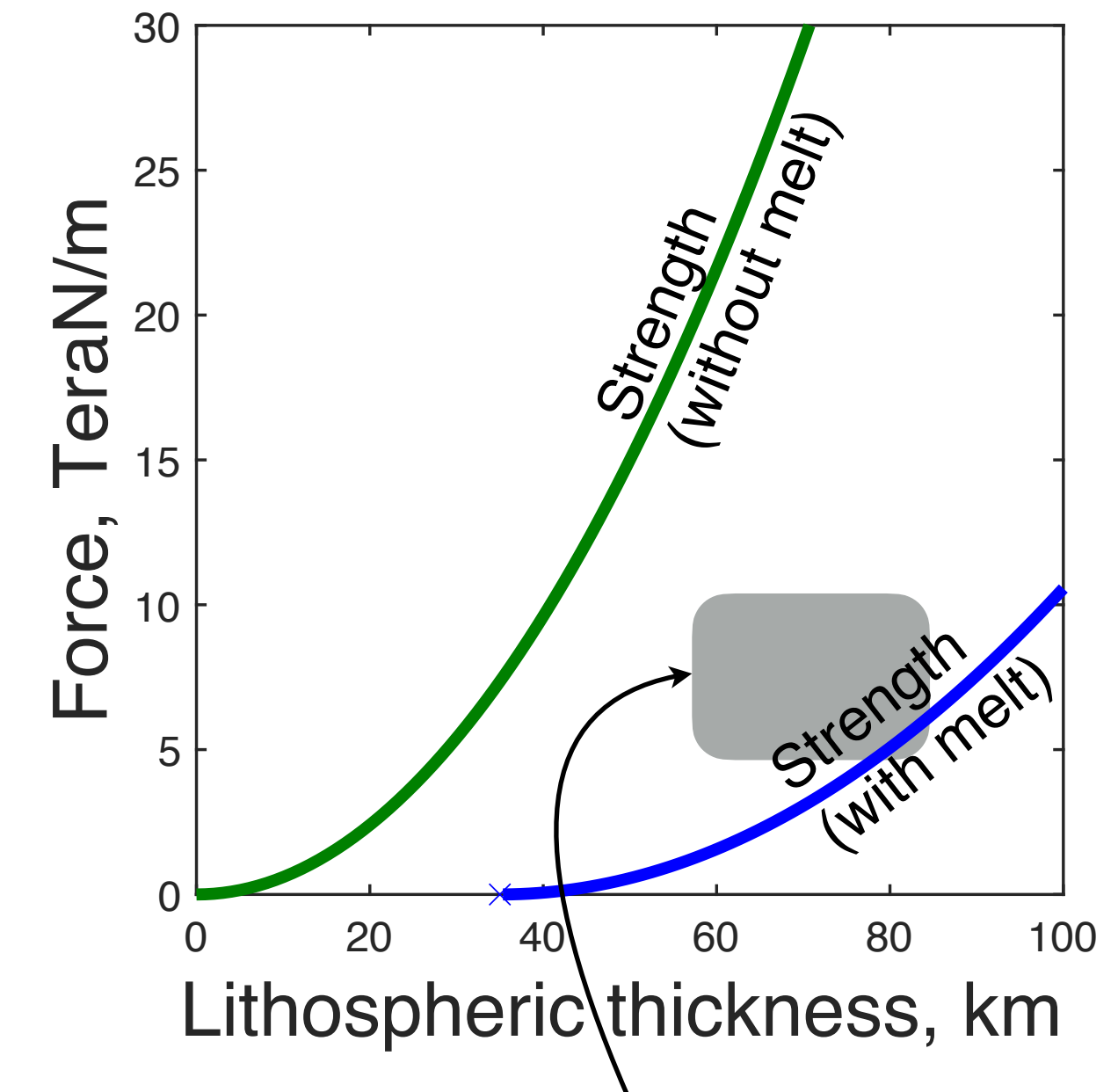
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Project RIFT-O-MAT: to quantify the role of melt in rifting



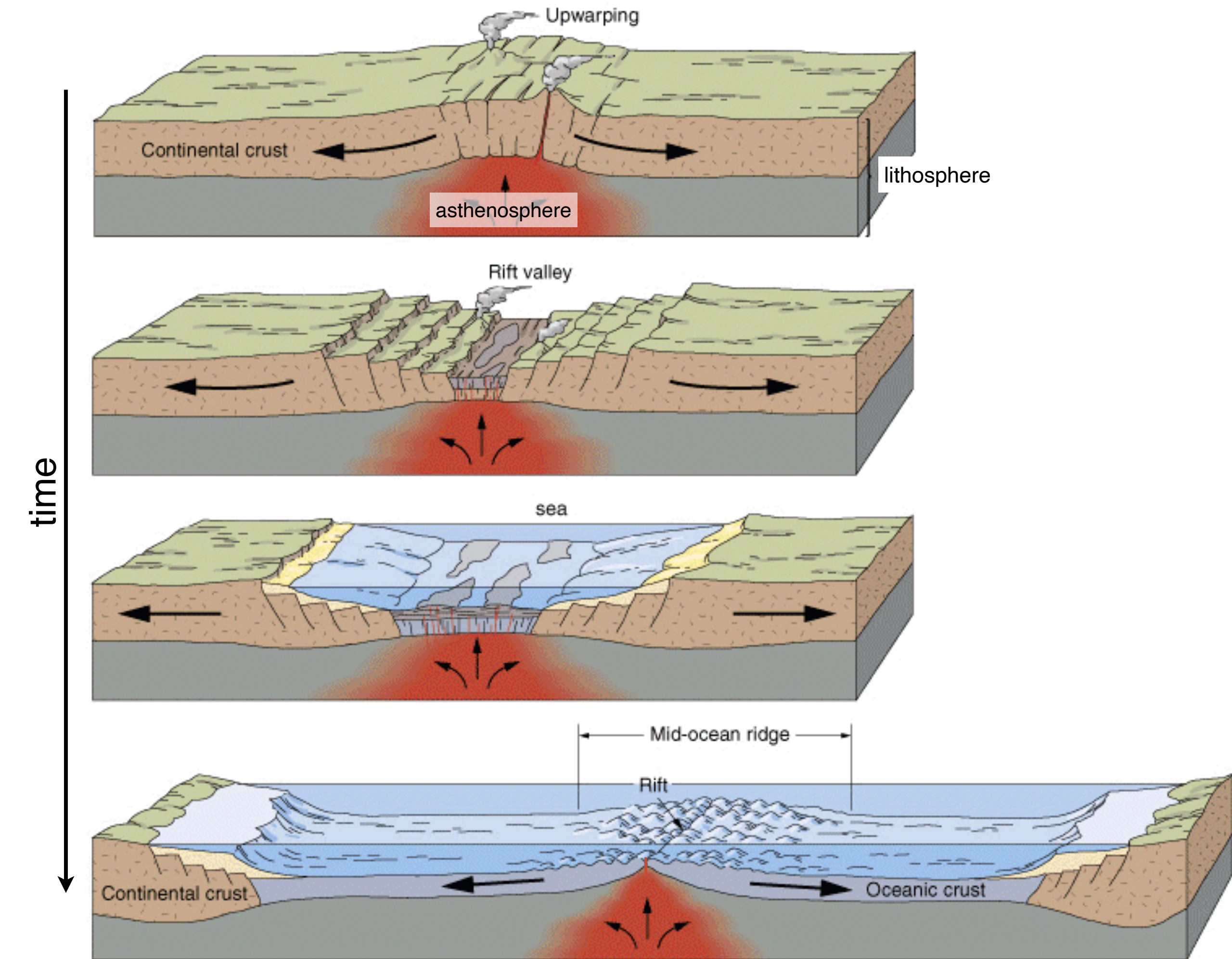
Corti 2009, Buck 2006



Typical conditions for continental rifts

A well accepted hypothesis is that melt intrusion reduces lithospheric strength and promote rifting, but we need to test it in a consistent model and understand it quantitatively.

Conceptual model and key physics



- Consistent modelling of magma/rock interaction (McKenzie 1984)
- Ductile deformation and brittle fracture of rocks (Keller et al. 2013)
- Topographic change on the surface: a key pathway for validation

Numerical framework: FD-PDE

PDE Solver for two-phase flow with magma dynamics
(Pusok et al. 2020)

Magma dynamics using FD-PDE:
a new, PETSc-based, finite-difference staggered-grid framework for
solving partial differential equations

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Visco-elasto-plastic rheology:
a Maxwell body

$$\dot{\epsilon}_{ij} = \underbrace{\frac{1}{2\eta_v} \tau_{ij}}_{\text{Viscosity}} + \underbrace{\frac{1}{2\eta_p} \tau_{ij}}_{\text{Plasticity}} + \underbrace{\frac{1}{2G} \frac{D\tau_{ij}}{Dt}}_{\text{Elasticity}}$$

+

Free surface + the 'sticky' air:
the phase field method
(Mirjalili and Mani 2020).

Phase field representation:
a kernel function +
an advection-diffusion equation.

$$\phi(x_n) = \frac{1}{2} \left[1 + \tanh \left(\frac{x_n}{2\epsilon} \right) \right]$$

$$\frac{\partial \phi}{\partial t} + \nabla \cdot (\phi \mathbf{u}) = \nabla \cdot \left[\gamma \left(\epsilon \nabla \phi - \phi(1 - \phi) \frac{\nabla \phi}{|\nabla \phi|} \right) \right]$$

$$\eta_p = \frac{Y}{2\dot{\epsilon}_{II}} \quad Y = C \cos \alpha + p \sin \alpha$$

Boundedness conditions to avoid overshoot
(Mirjalili and Mani 2020):

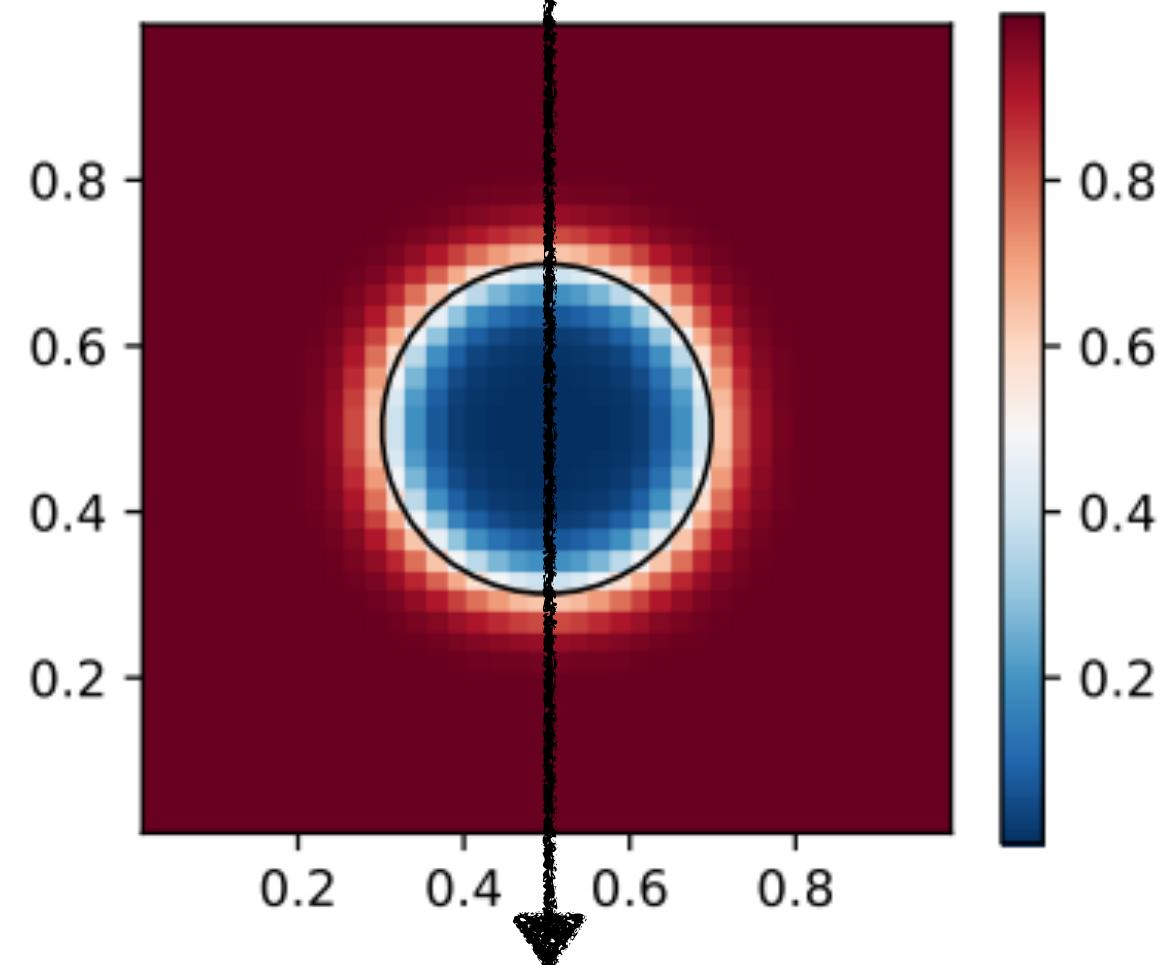
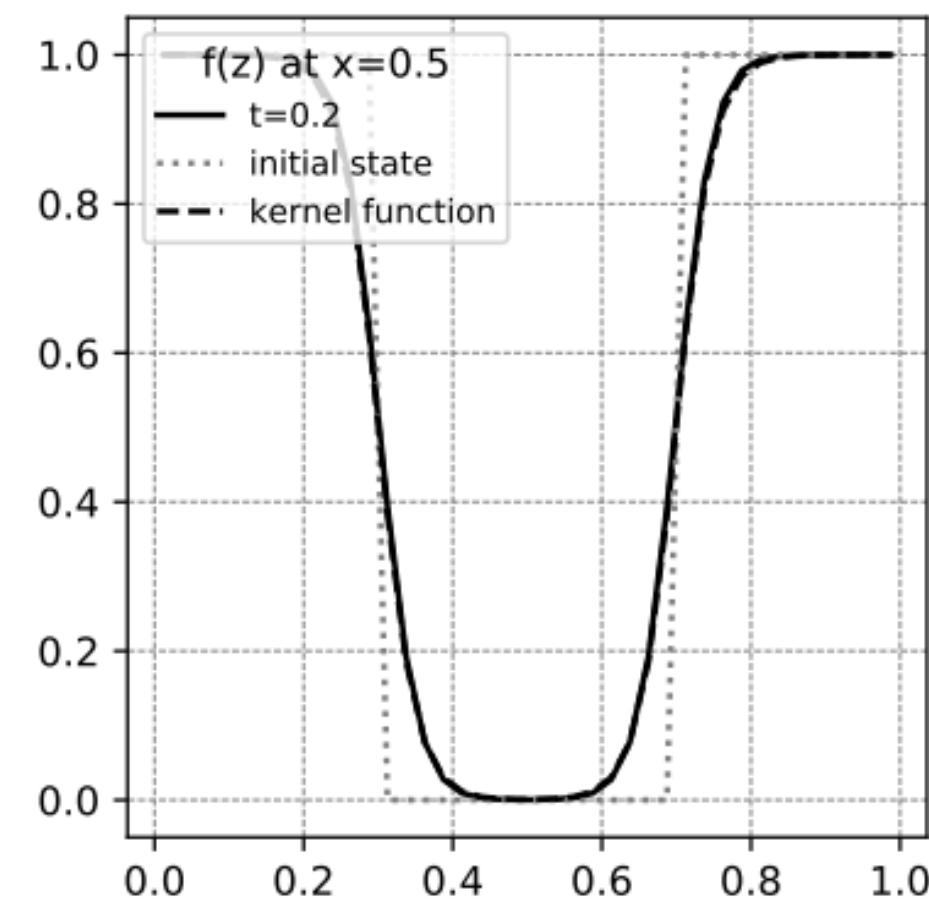
$$\frac{\epsilon}{\Delta x} \geq \frac{\gamma/|\mathbf{u}|_{max} + 1}{2\gamma/|\mathbf{u}|_{max}} \quad \gamma = |\mathbf{u}|_{max}$$

CFL condition: $\Delta t \leq \frac{\Delta x^2}{2\gamma\epsilon}$

Benchmark of the phase field method

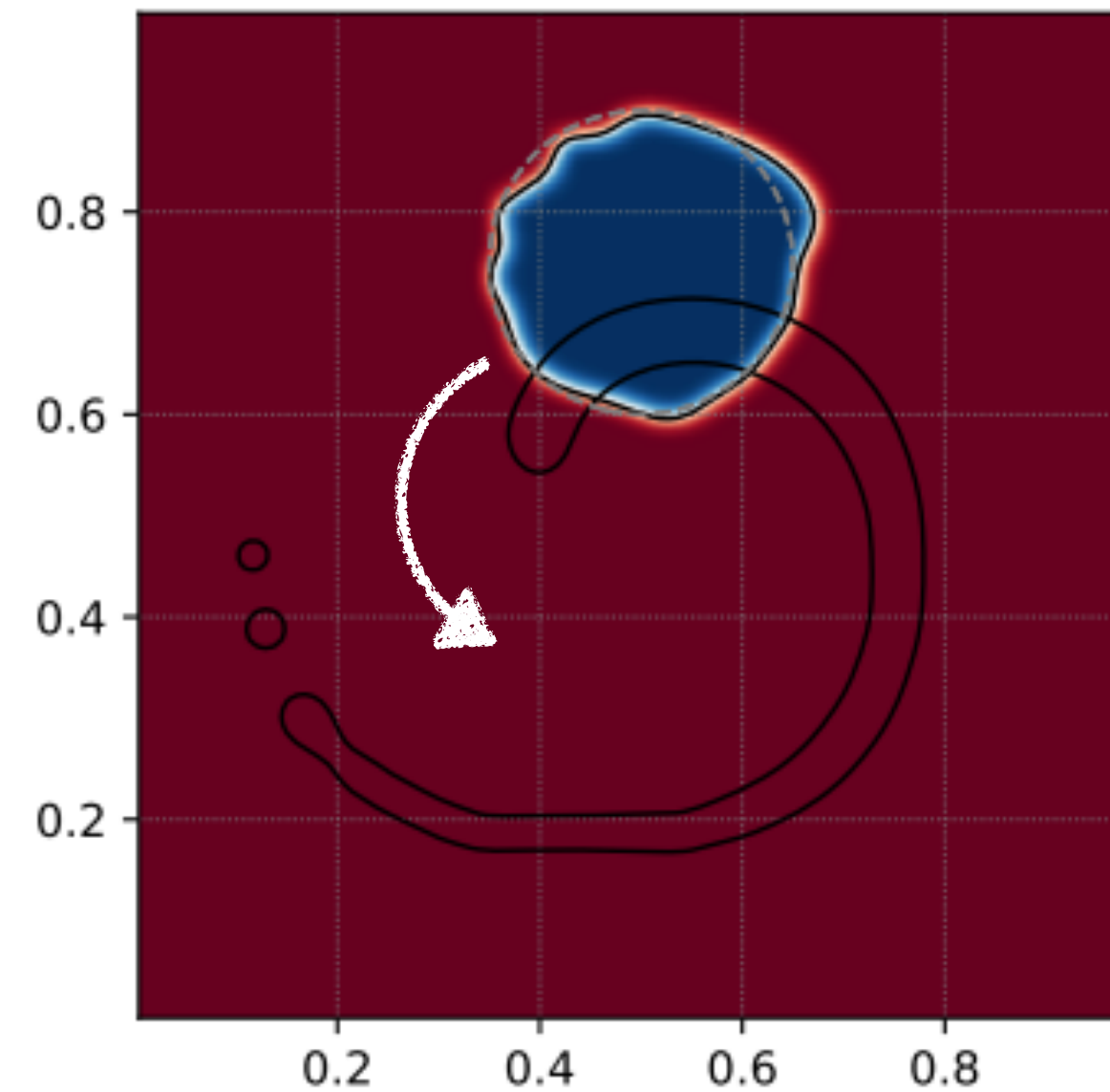
Example 1: Representation of a sharp interface using a diffuse phase field.

A stationary circular drop

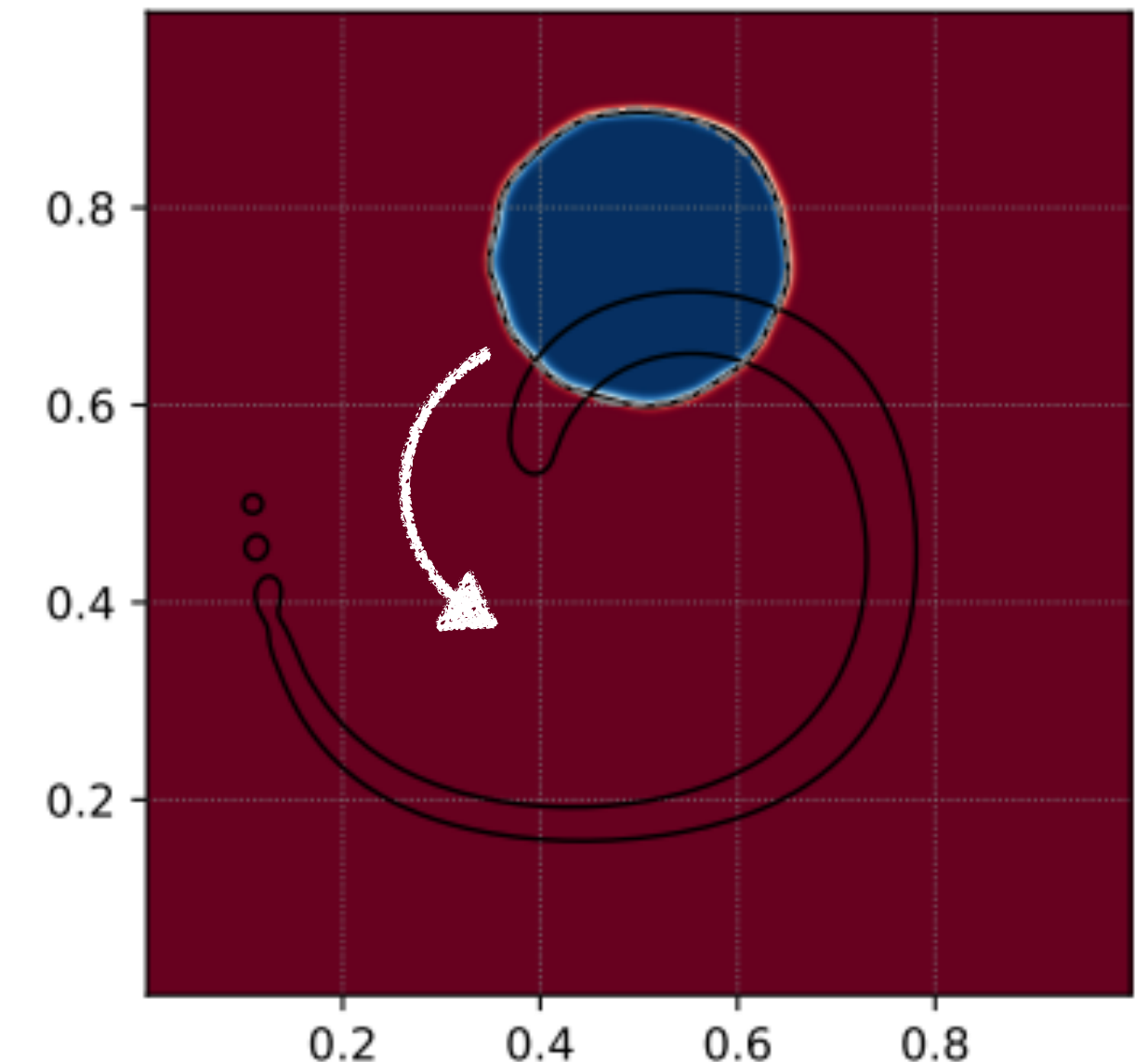


Example 2: Deformation of a circular drop by a single vortex

Streamfunction of the periodic vortex: $\Psi = \frac{1}{\pi} \sin^2(\pi x) \sin^2(\pi z) \cos\left(\frac{\pi t}{T}\right)$



Grid: 128 x 128



Grid: 256 x 256

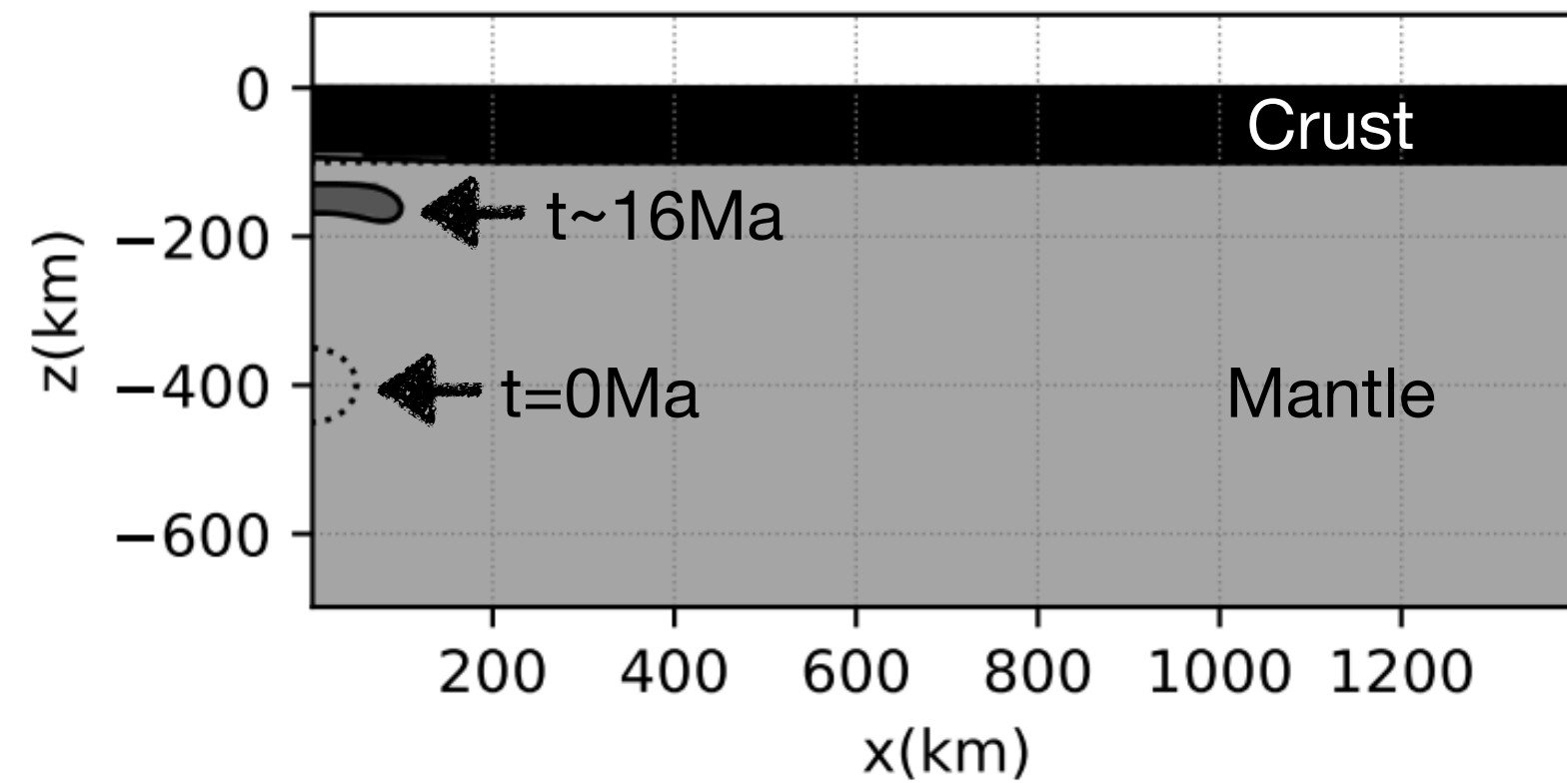
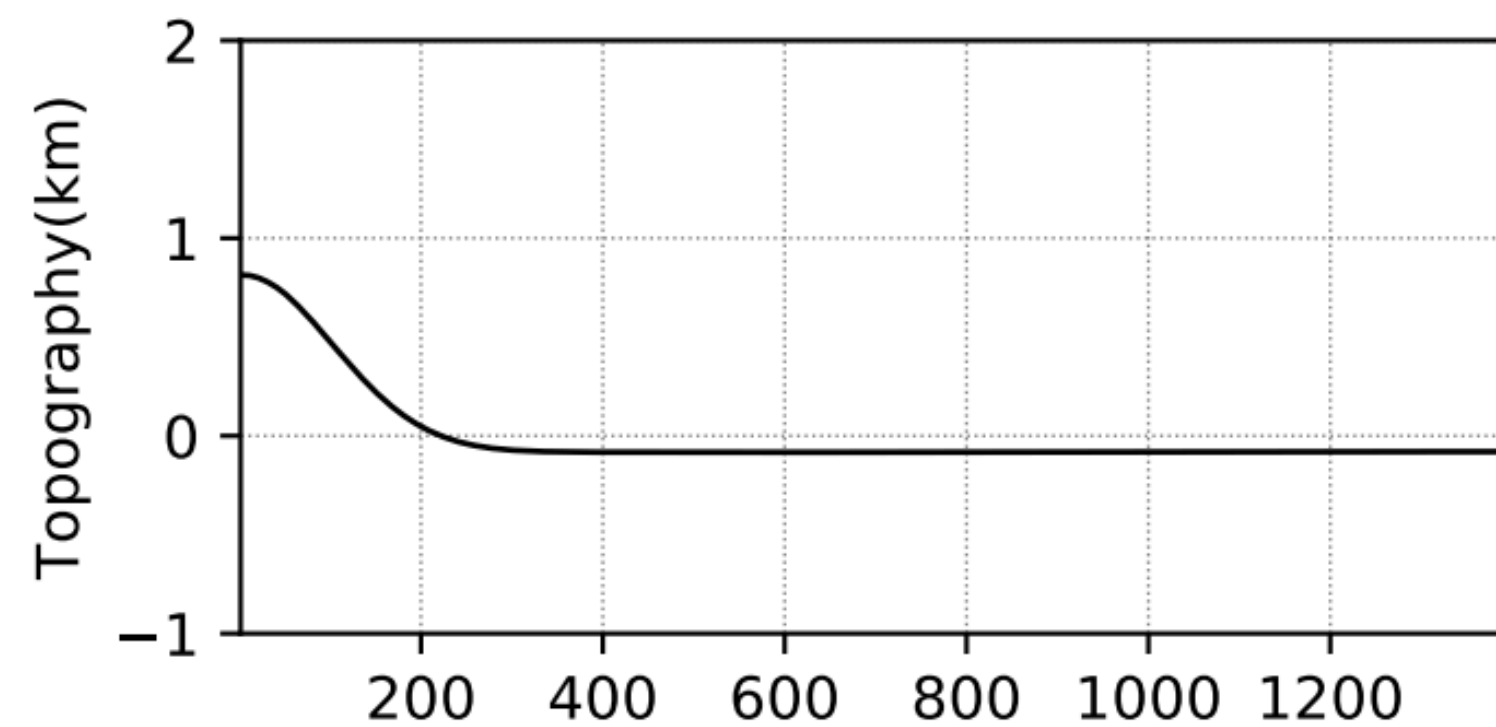
With the resolution of 256 x 256, this computation is accurate enough for the circular drop returning to its initial location and shape after having been deformed by this periodic vortex for one period.

Colormap: the phase field at $t = T$

Solid line: the sharp interface at $t = T/2$ and T

Gray dashed line: the interface at $t = 0$

Example 1: a rising plume

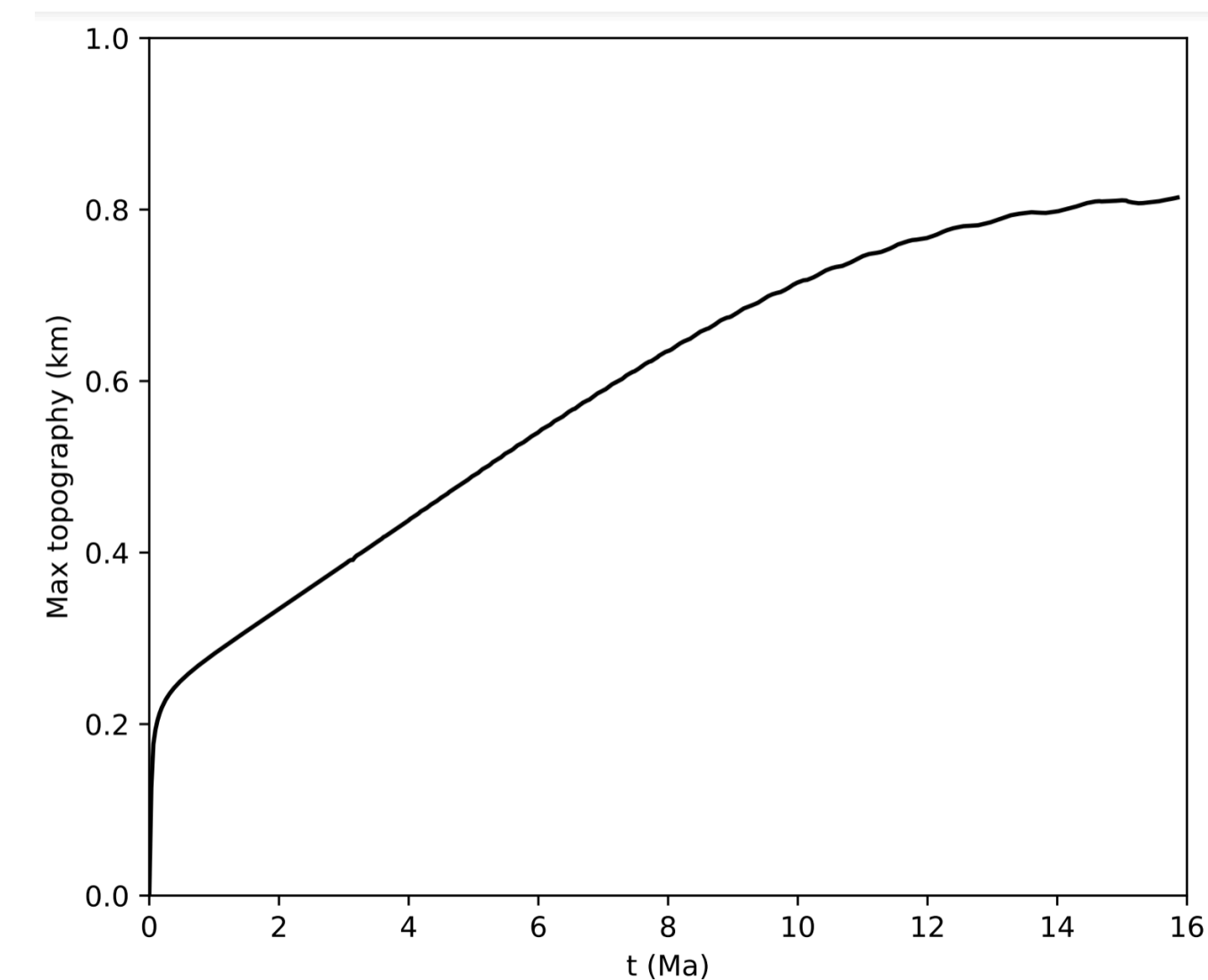


A rising plume in the mantle after $\sim 16\text{Ma}$ from an initially circular shape at a diameter of 100km at 300km below the crust.

Grid size: 4km X 4km

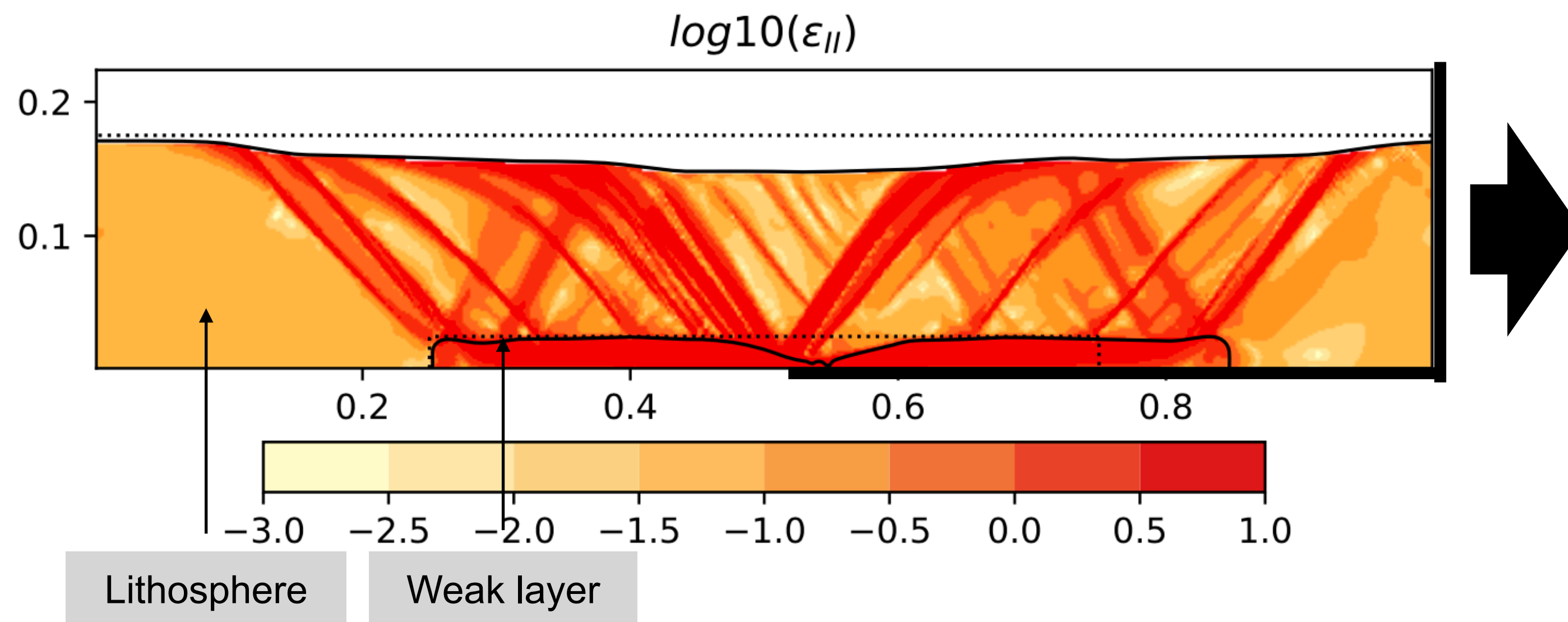
Subgrid-scale displacement of the interface:

From the value of phase field, we can estimate the location of interface up to a scale far below the size of the grid.



Maximum elevation of the surface in time

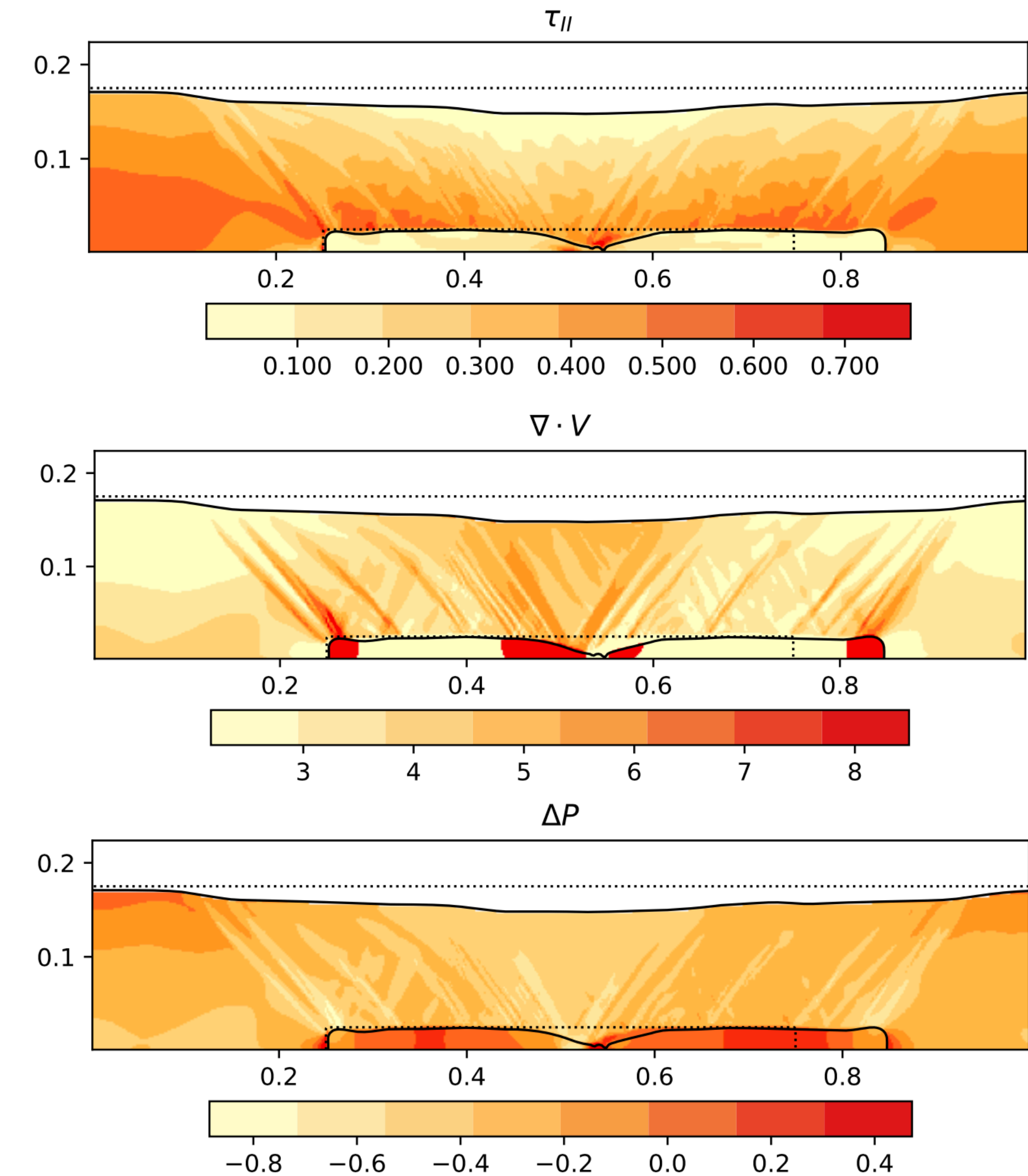
Example 2: extension of a two-phase lithosphere



Localisation of shear bands and topographic changes after a certain time of extension from the right part.

Notes:

1. All values here are non-dimensional ones.
2. Porosity (1%) is assumed as a constant in space and time for this example.



The shear stresses, volumetric strain rates and the compaction pressure of the solid phase.

Conclusions

The FD-PDE, PETSc based finite difference staggered-grid framework, now can:

1. Capturing the free surface, or interface between two fluids, in the sub-grid scale by using the phase field method;
2. Simulating the plastic failure with a visco-elasto-plastic rheology for a two-phase flow with magma dynamics.

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