

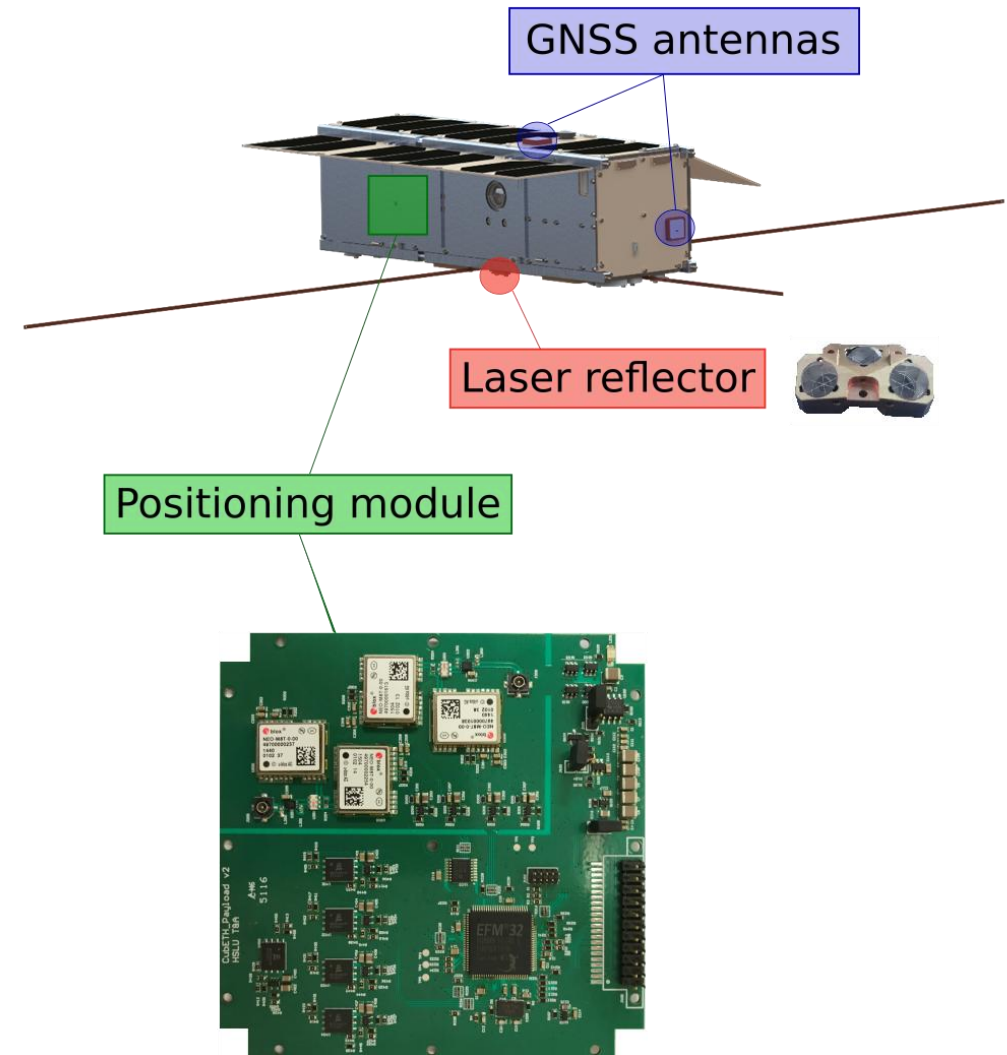
Systematic biases in the on-board navigation solution of a CubeSat GNSS payload board

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Introduction

- Two 3-unit cube satellites, Astrocast-1 and -2, launched in December 2018 and April 2019
- Sun-synchronous orbit with inclination of 97°
- GNSS payload board providing continuous on-board receiver solutions (NAVSOL)
 - Position from GNSS code observations
 - Velocity from Doppler measurements
- Analysis of the quality of the on-board solutions by fitting a dynamic orbit model
- Standard approach based on positions only
- Idea: Additional use of velocity information for orbit estimation



Dynamic orbit fitting

Three different approaches depending on the observations used:

1. Position-only
2. Velocity-only
3. Position and velocity

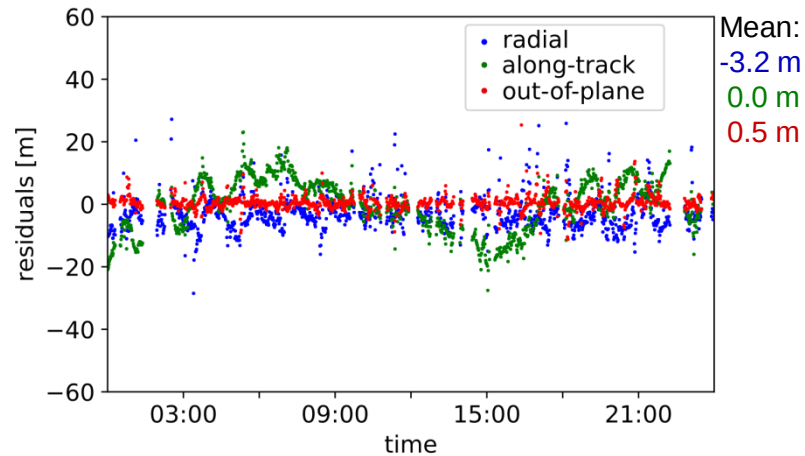
Estimation of 6 Keplerian elements + constant along-track acceleration

Expectations:

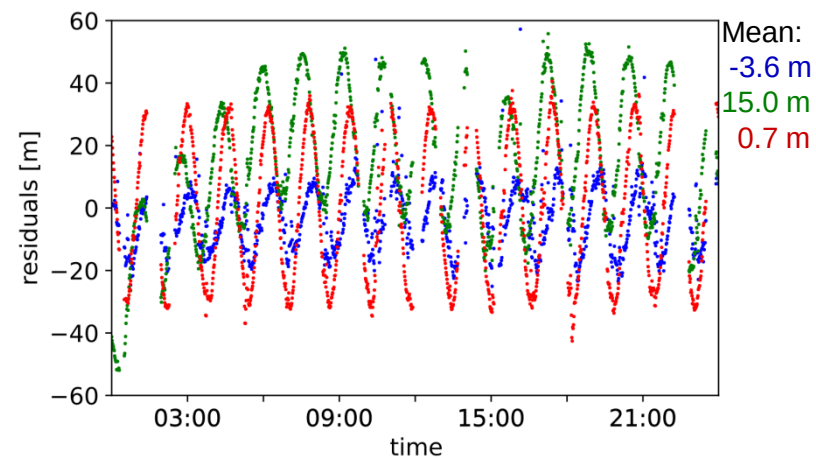
- Systematic effects in the observations → Discrepancies between orbits from positions and from velocities
- Differences in how well certain orbit components or parameters can be estimated
- Improvement of the orbit solutions if the independent observation types are combined

Residuals of one-day orbital arc (Astrocast-1, 18/10/2019)

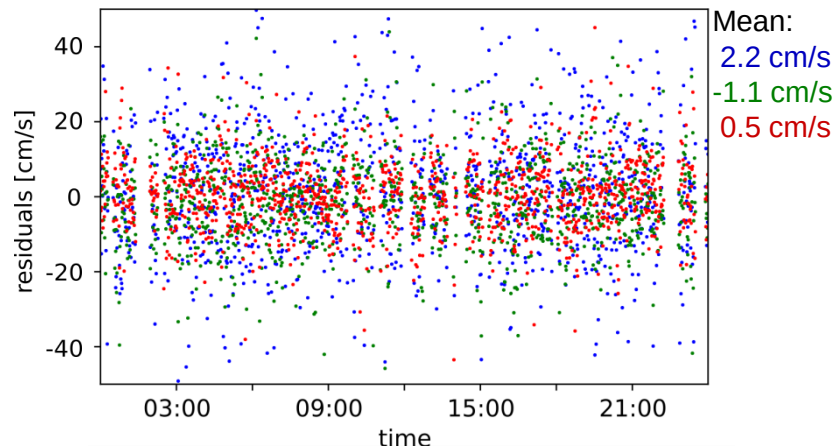
Position-only approach, position residuals



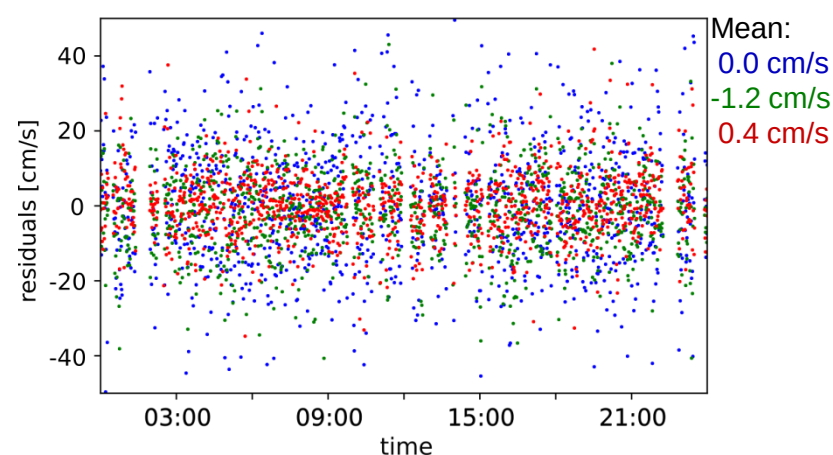
Velocity-only approach, position residuals



Position-only app., velocity residuals



Velocity-only app., velocity residuals

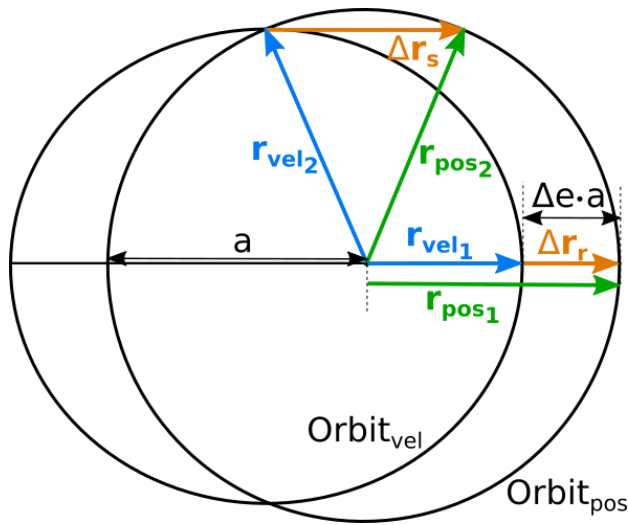


- Once-per-revolution periodicity (96 min) in position residuals
 - Amplitude of about 30 m in along-track and out-of-plane
 - Amplitude of about 15 m in radial direction
- Offsets:
 - Negative radial offset of about 3 m
 - Positive along-track offset of 15 m
 - Positive radial offset of about 2 cm/s in velocities
 - Negative along-track offset of 1 cm/s in velocities

Periodicity – Effect of differences in orbital elements (Astrocast-1, 18/10/2019)

Parameter differences between velocity-only and position-only approach

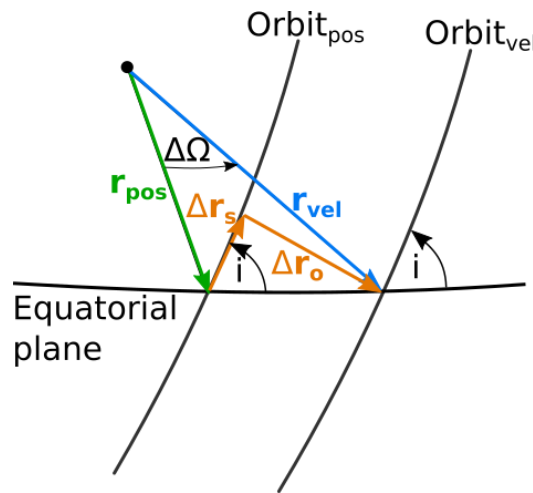
Δa [m]	Δe [10^{-6}]	Δi [10^{-6} rad]	$\Delta \Omega$ [10^{-6} rad]	Δu_0 [10^{-6} rad]	$\Delta \omega$ [10^{-6} rad]
1.0	-1.6	0.2	4.5	3.5	-685.0



Maximum differences in radial

$$\Delta r_r = \Delta e \cdot a = 11.4 \text{ m}$$

Amplitude in along-track about twice as large as in radial: $\Delta r_s \approx 23 \text{ m}$



Out-of-plane difference:

$$\Delta r_o = \Delta \Omega \cdot \|r\| \cdot \sin(i) = 30.8 \text{ m}$$

Along-track difference:

$$\Delta r_s = \Delta \Omega \cdot \|r\| \cdot \cos(i) = -3.8 \text{ m}$$

Argument of perigee poorly defined

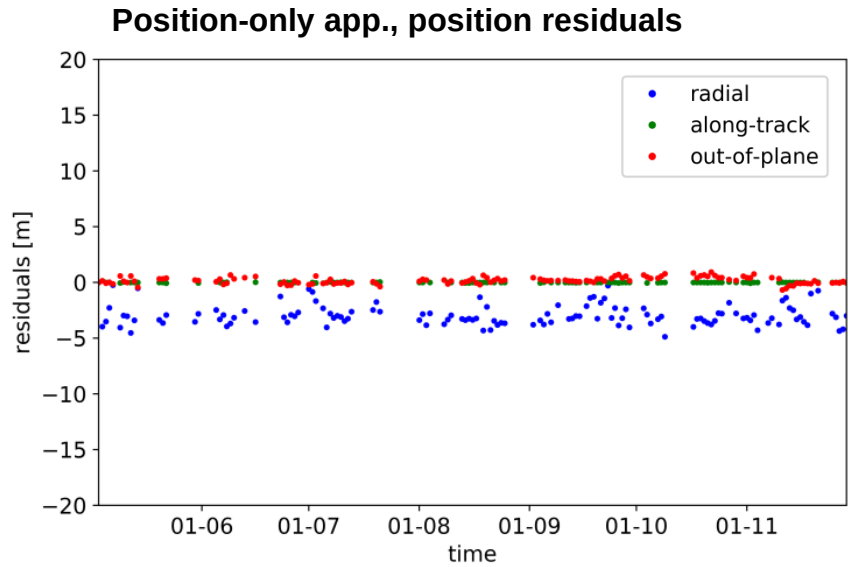
- Due to nearly circular orbit
- Relatively small effect on position residuals

With $e = 0.001$:

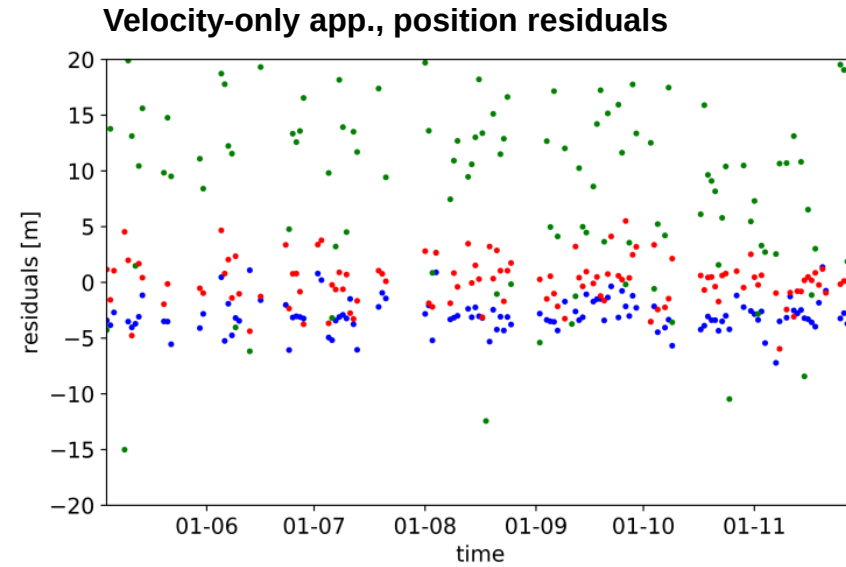
$$\Delta r_r = e \cdot a \cdot \sin(\Delta \omega) = 4.7 \text{ m}$$

Amplitude in along-track twice as large: $\Delta r_s \approx 10 \text{ m}$

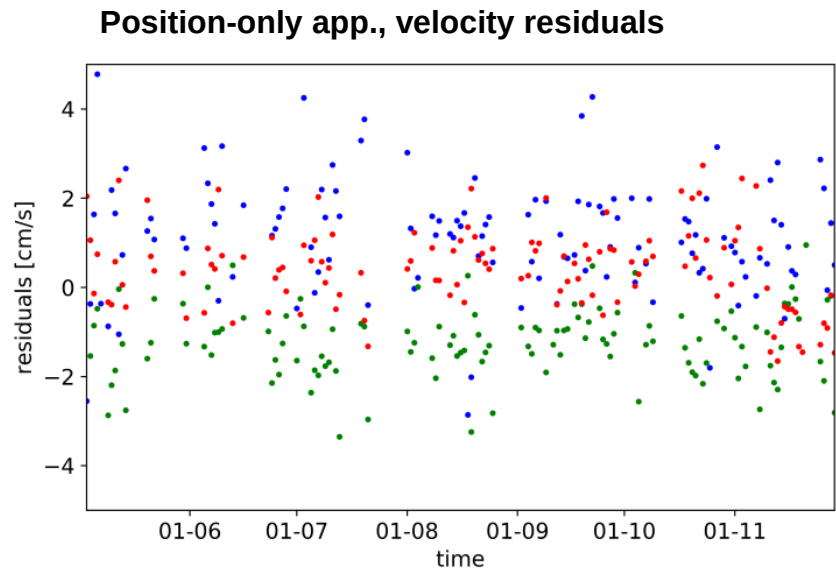
Astrocast-1: Daily mean residuals from May to November 2019



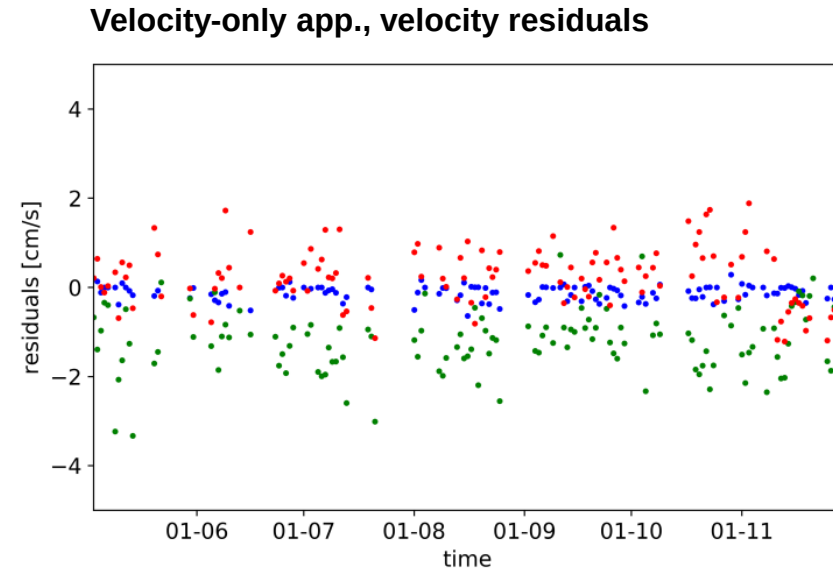
Mean:
-2.9 m
0.0 m
0.2 m



Mean:
-2.9 m
13.3 m
-0.3 m

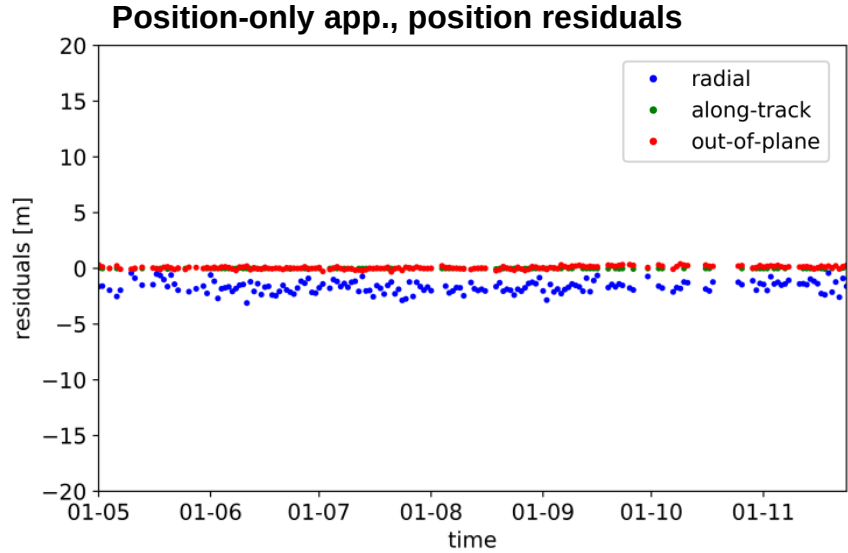


Mean:
1.3 cm/s
-1.2 cm/s
0.6 cm/s

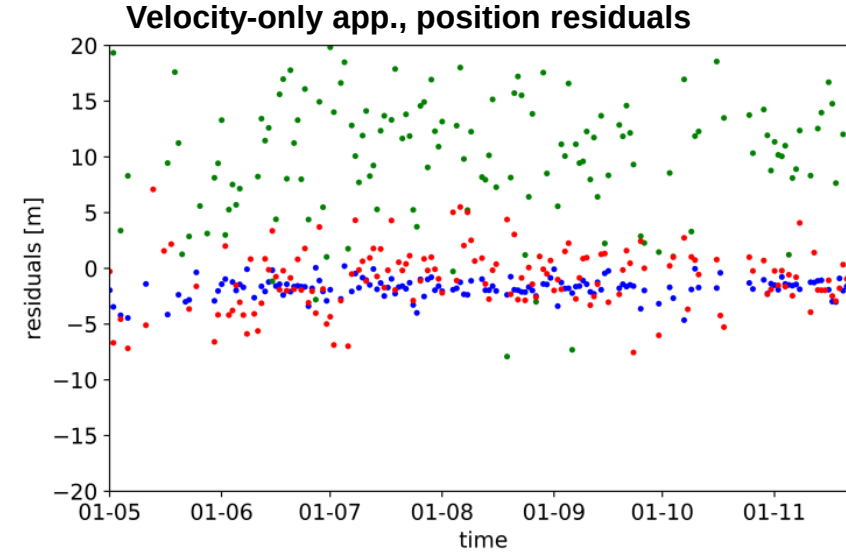


Mean:
-0.4 cm/s
-1.2 cm/s
0.3 cm/s

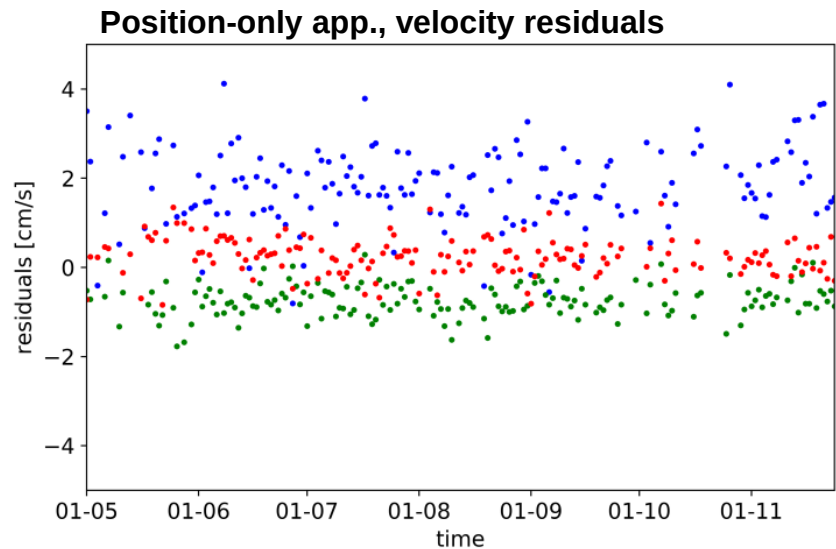
Astrocast-2: Daily mean residuals from May to November 2019



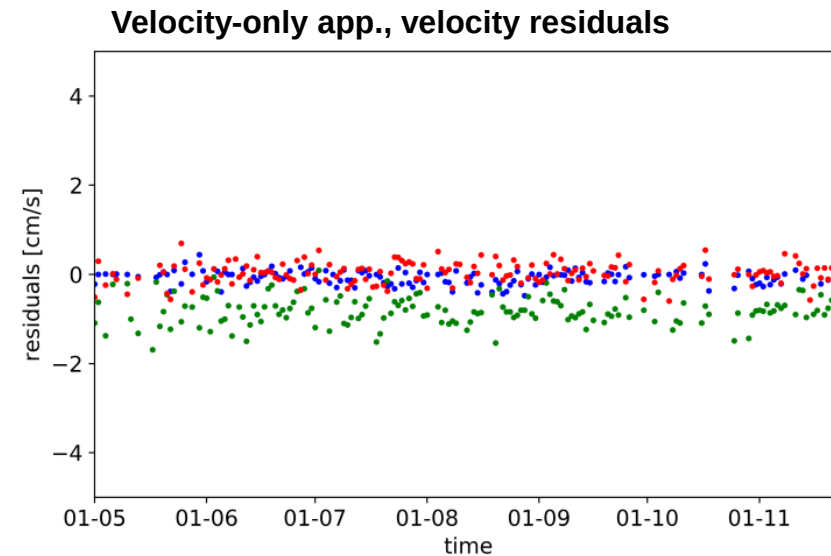
Mean:
-1.6 m
0.0 m
0.1 m



Mean:
-1.8 m
11.4 m
-0.9 m

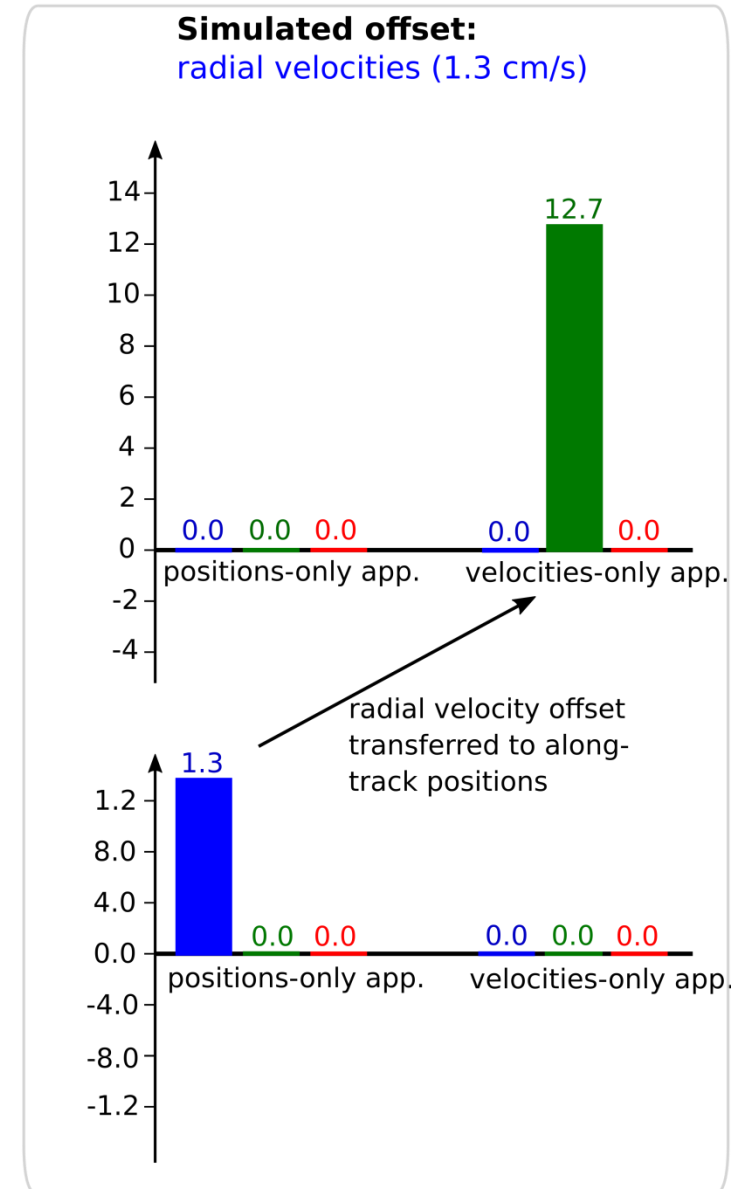
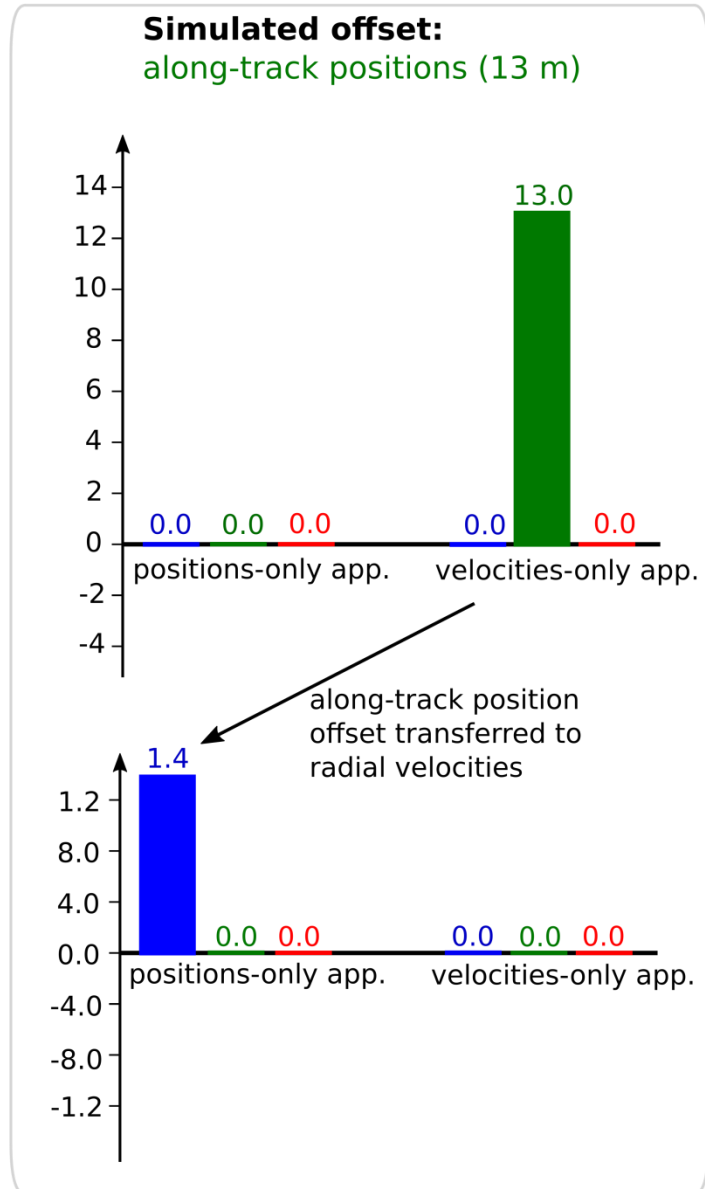
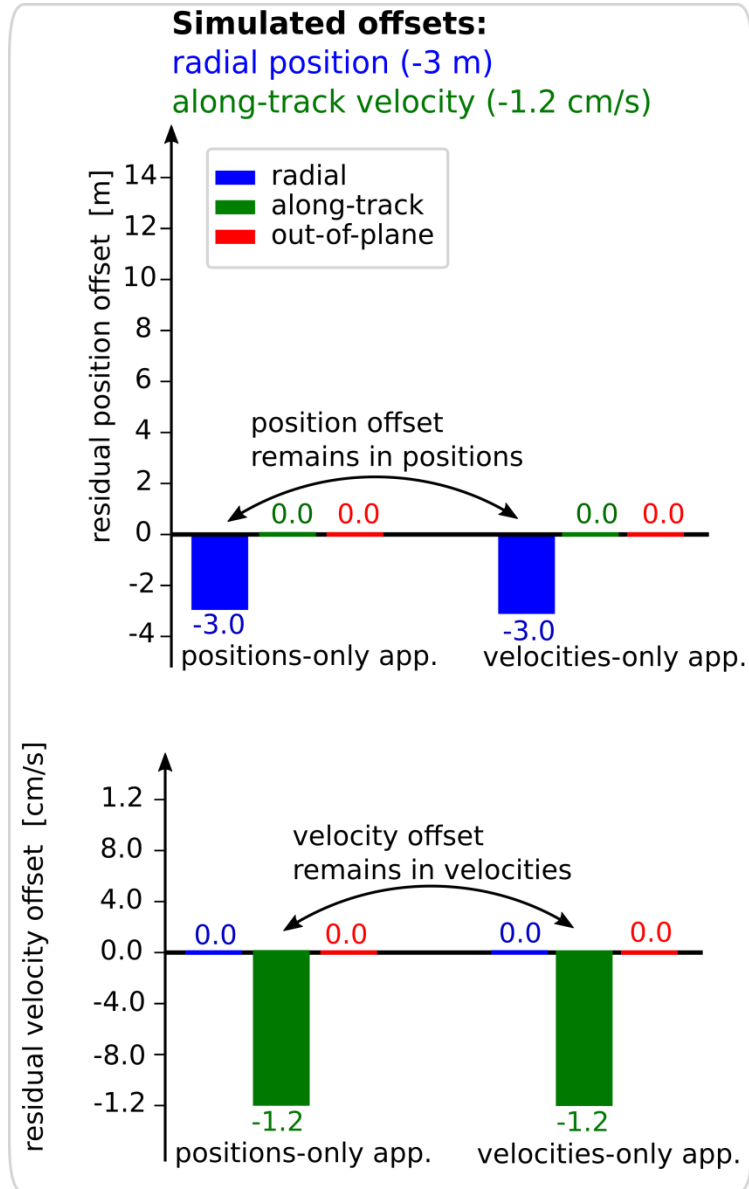


Mean:
1.8 cm/s
-0.7 cm/s
0.2 cm/s



Mean:
0.0 cm/s
-0.8 cm/s
0.0 cm/s

Offsets - Simulation of measurement biases



Offsets - Relation between along-track offset and radial velocity

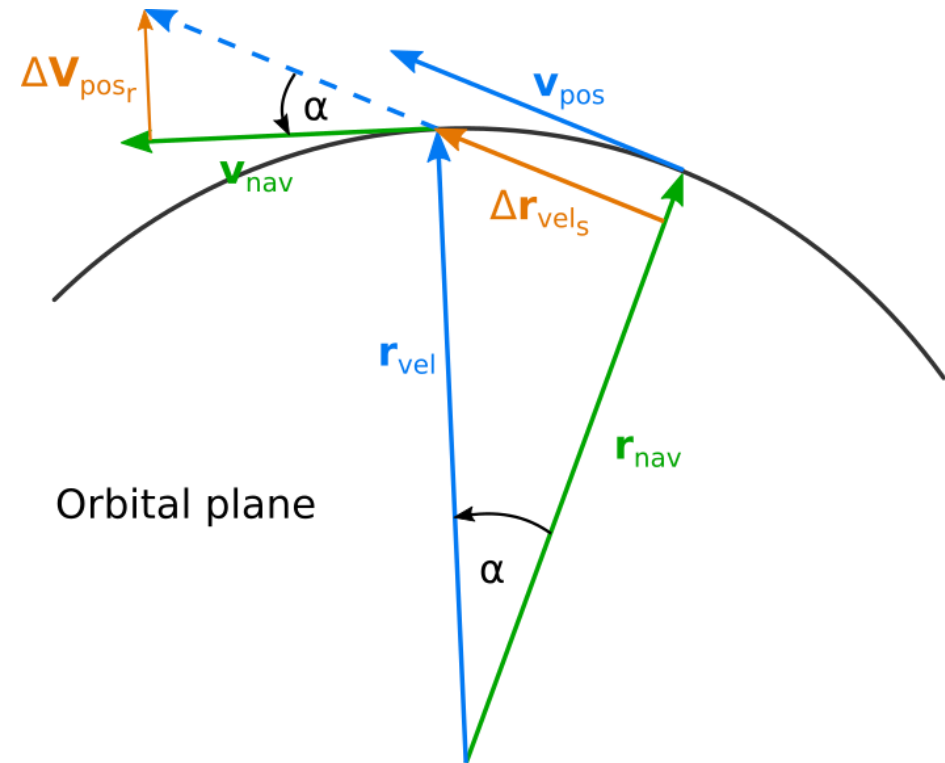
Simple approximation in a circular orbit

$$\frac{\Delta v_{pos_r}}{\|v\|} = \frac{\Delta r_{vel_s}}{\|r\|}$$

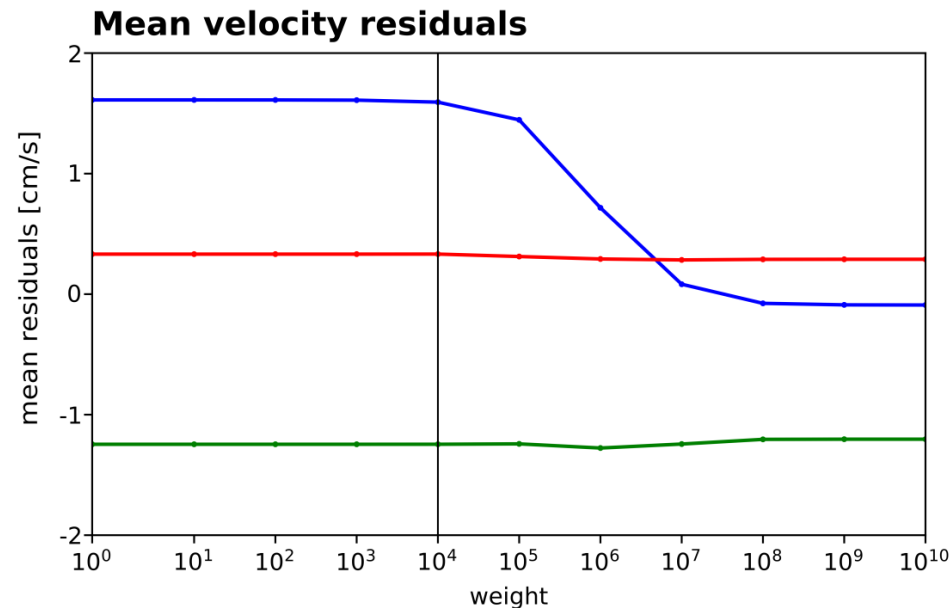
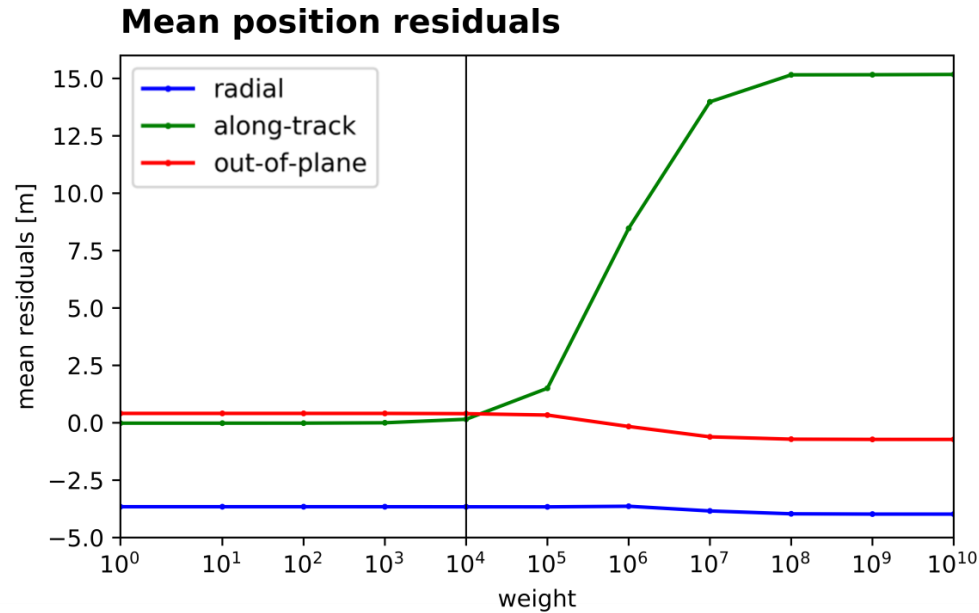
$$\Delta r_{vel_s} \approx 13.3 \text{ m}$$

$$\begin{aligned}\Delta v_{pos_r} &= 13.3 \text{ m} \cdot \frac{7.8 \cdot 10^3 \frac{\text{m}}{\text{s}}}{6.9 \cdot 10^6 \text{ m}} \\ &= 1.5 \text{ cm/s}\end{aligned}$$

⇒ Inconsistency between position and velocity vector causes shift in along-track



Relative weighting of velocities in combined approach (Astrocast-1, 18/10/2019)



- Need for an appropriate relative weighting factor
- Standard deviation of positions about 2-3 m
- Standard deviation of velocities about 2-3 cm/s

⇒ Expected relative weight of 10^4

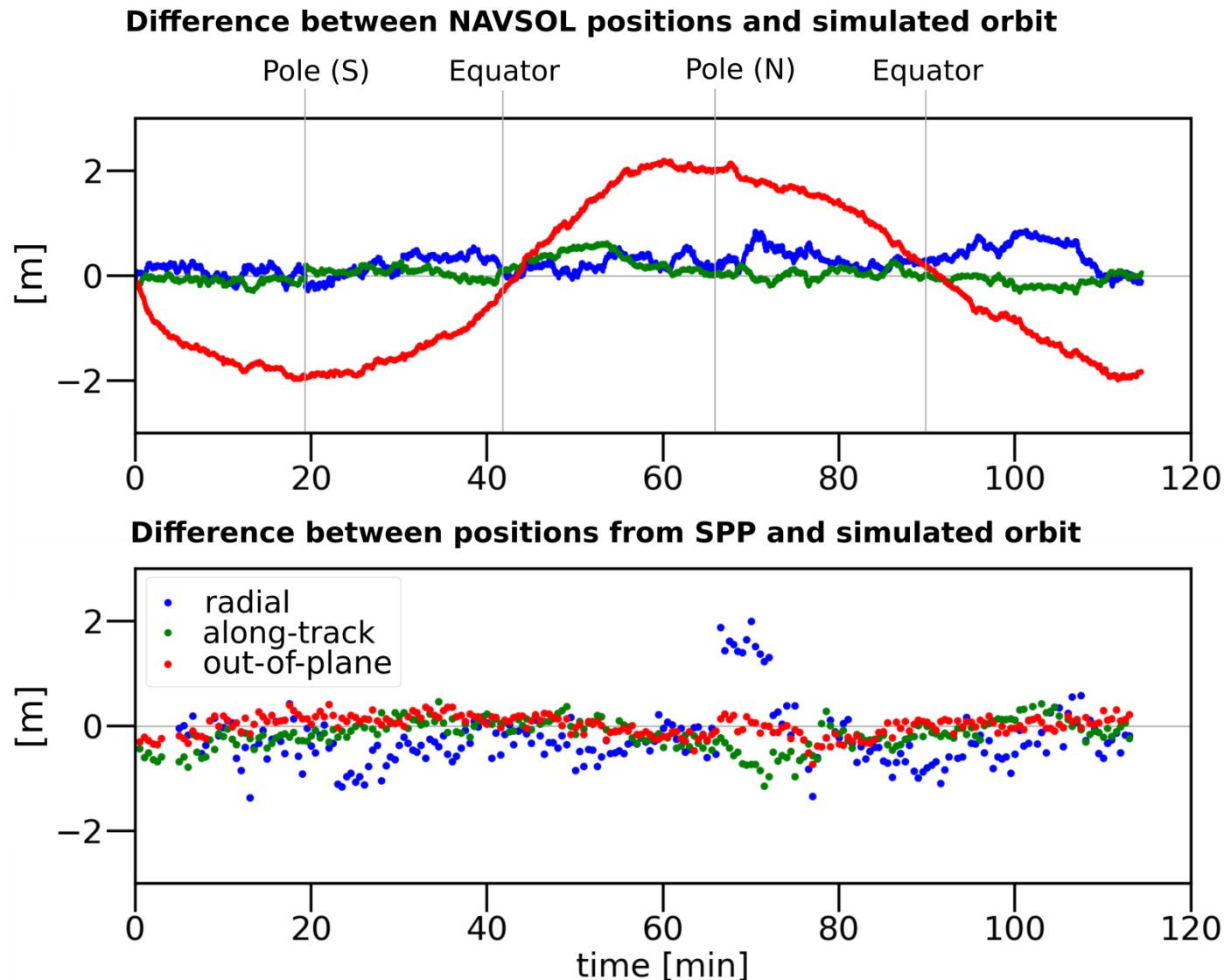
⇒ Measurement bias in velocities rather than in positions

Simulation with the IFEN GNSS signal generator

3 simulations, each covering at least one satellite revolution (~ 96 min)

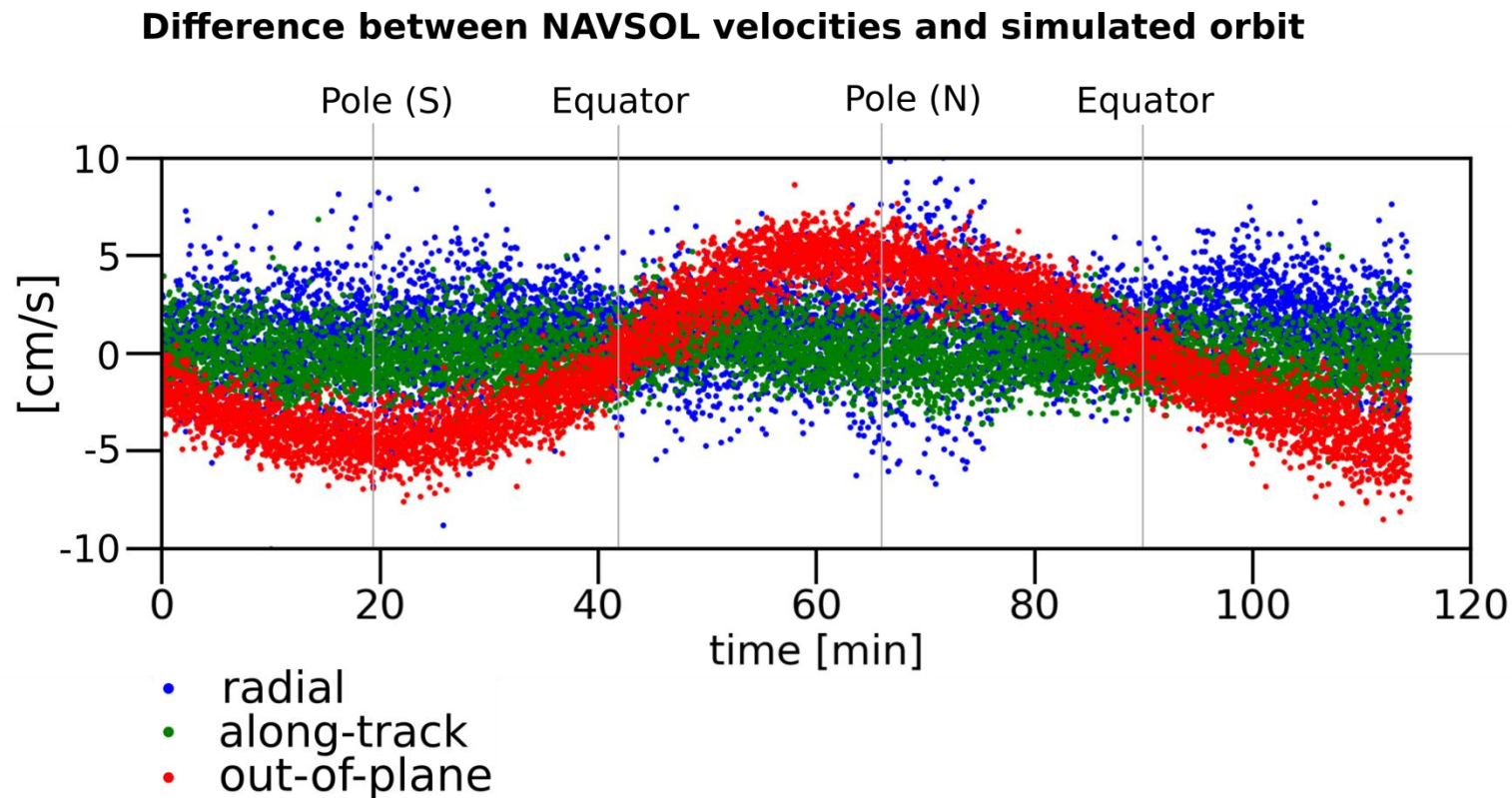
	Satellite	Altitude	Ionosphere model
Simulation 1	Astrocast-2	505 km	None
Simulation 2	Astrocast-2	505 km	NeQuick
Simulation 3	Astrocast-1	575 km	NeQuick

Simulation 1: Astrocast-2, without Ionosphere model



- Once-per-revolution periodicity of 2 m in out-of-plane direction
 - Maximum difference at the poles
 - Orbit fitting absorbs this effects by adjusting the orbit inclination
- Wrong inclination estimated from the position-only orbit fitting, but not detectable in the residuals
- Periodicity might be caused by dynamic effects in the real-time positioning algorithm
 - No such periodicity in raw data solution

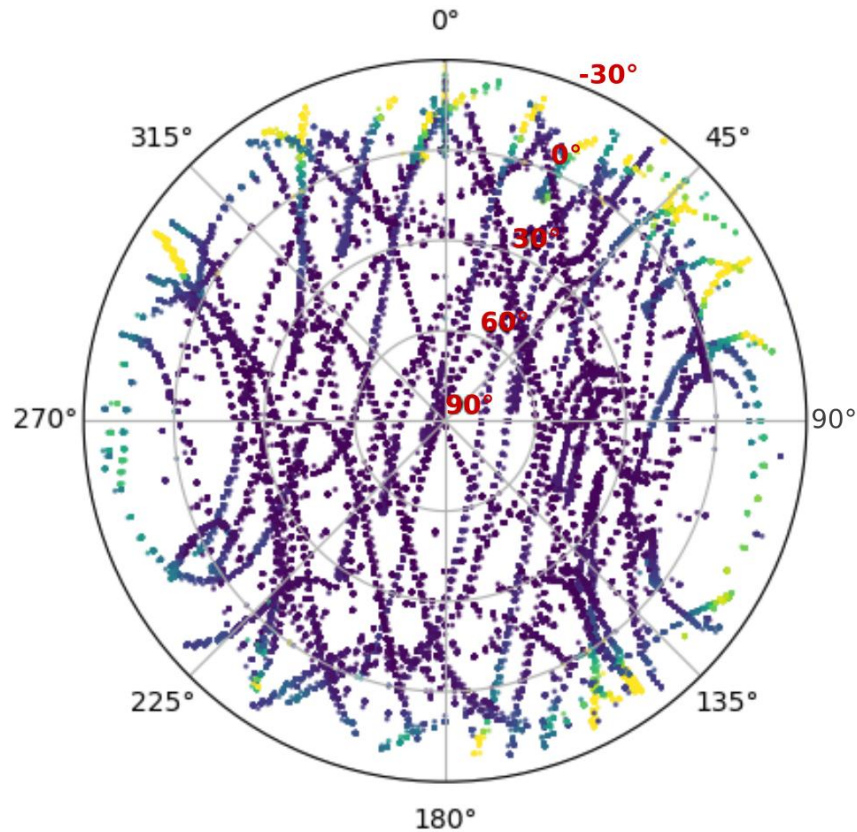
Simulation 1: Astrocat-2, without Ionosphere model



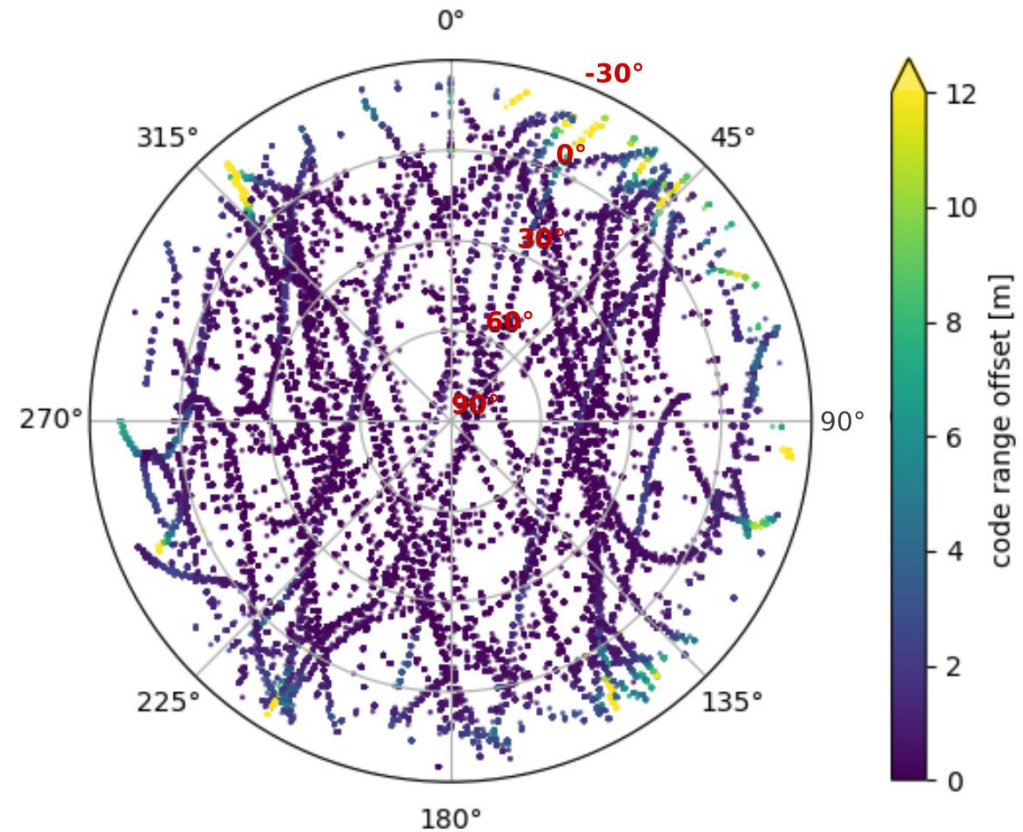
- Similar periodicity in the velocities as in the positions
→ Probably caused by similar dynamic effect
- Corresponds to difference in the right ascension of the ascending node of about $6 \cdot 10^{-6}$ rad
- Similar to the difference in the right ascension of the ascending node of about $5 \cdot 10^{-6}$ rad found when comparing position-only and velocity-only orbit fits

Simulated pseudorange offsets based on the NeQuick model

Simulation 2: Astrocast-2, height 505 km

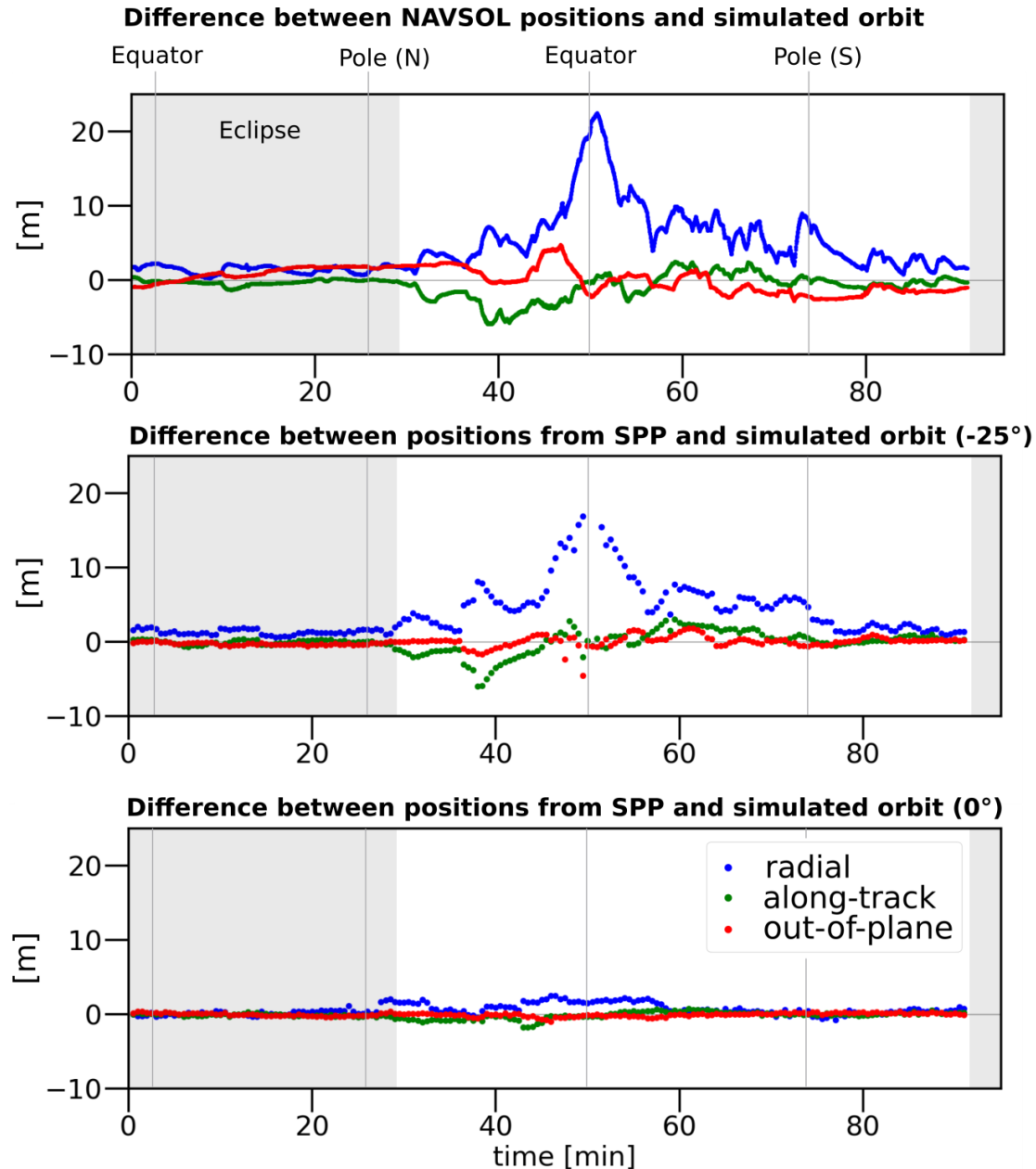


Simulation 3: Astrocast-1, height 575 km



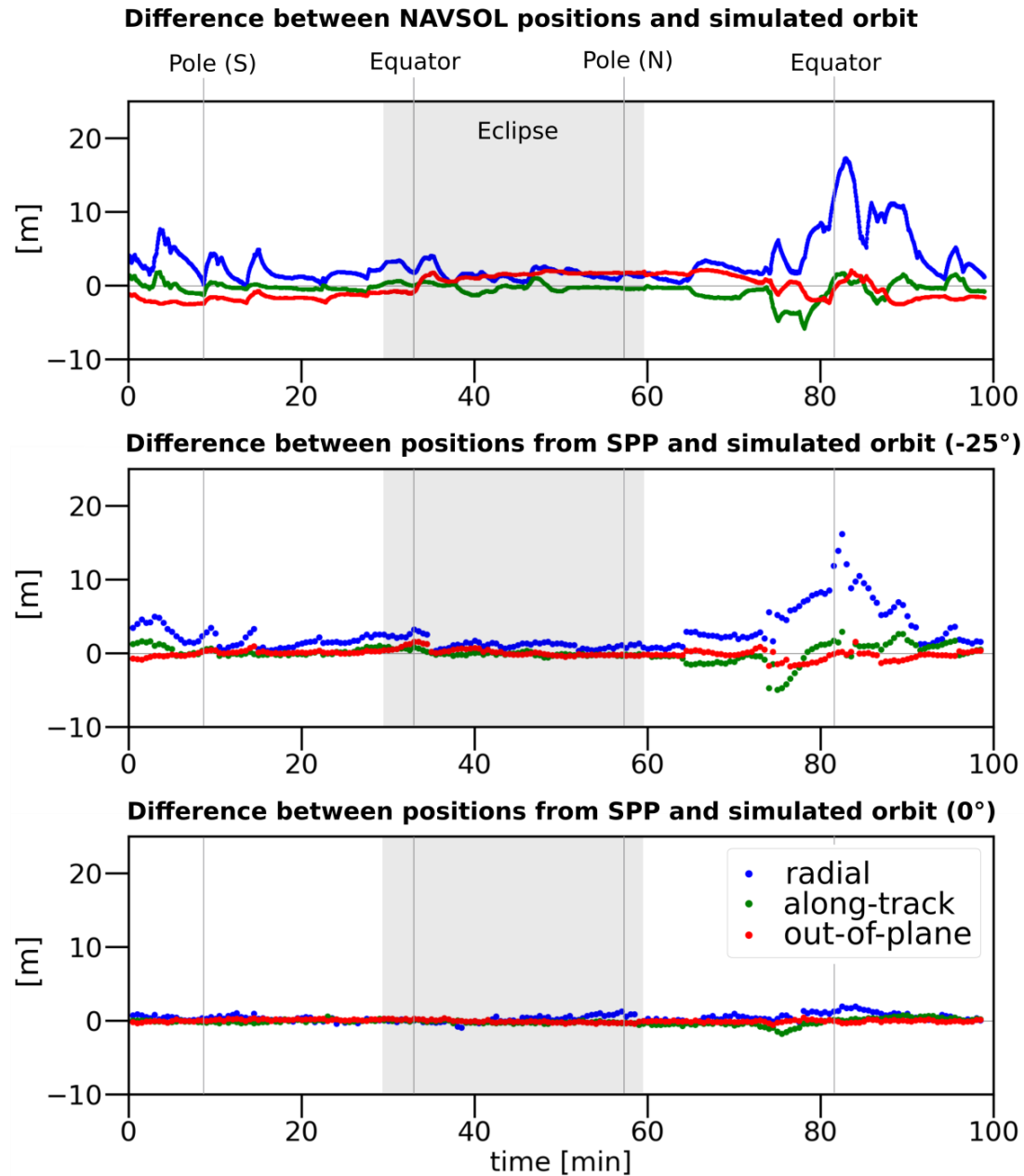
- Largest simulated offsets at negative elevations
- Overall larger offsets for the lower flying satellite (Astrocast-2)

Simulation 2: Astrocast-2, with Ionosphere model



- Positive radial offset over entire time span
 - Significant peak at the equator
 - Similar variations when processing the raw observations with an elevation cut-off angle of -25°
 - But: much smaller effects with an elevation cut-off of 0°
- Ionospheric refraction causes upward shift of the orbit
- Effect of the ionosphere in the NAVSOL positions is dominated by signals coming from elevations below 0°

Simulation 3: Astrocast-1, with Ionosphere model



- Smaller effect of the ionosphere on the NAVSOL positions than for Astrocast-2
- This contrasts with the residuals from the orbit fitting, which shows a higher radial offset for Astrocast-1.
- Changing the elevation cut-off angle has similar effect as for Astrocast-2

Conclusions for the once-per-revolution periodicity

- Periodicity in radial and along-track of about 15 m and 30 m, respectively
 - Related to the orbit eccentricity
 - Probably due to a general deficiency, when using velocities to estimate geometric orbit parameters
- Periodicity in out-of-plane with amplitudes of about 30 m at the equator
 - Corresponds to a difference in the right ascension of the ascending node of about $5 \cdot 10^{-6}$ rad
 - Caused by a periodic velocity bias in the NAVSOL with maxima of about 5 cm/s at the poles
 - Probably due to dynamic effects in the real-time navigation algorithm of the receiver
- 2-m-amplitude in out-of-plane
 - Only present in the differences between the NAVSOL and the true orbit, but not in the residuals from a dynamic orbit fit
 - Possibly similar dynamic effects as the 5-cm/s-periodicity in the velocities

Conclusions for the position and velocity offsets

- Radial offsets in positions of about -3 m for Astrocast-1 and -1.5 m for Astrocast-2
 - Dependent on the satellite altitude → probably caused by ionospheric refraction, but ...
 - Opposite sign as in simulation
 - Despite its higher altitude, Astrocast-1 shows larger radial offset
 - Possible causes
 - Overcorrection of the ionospheric effect due to the use of a correction intended for ground stations
 - Highly variable ionospheric effects cause the fitted orbit to be on average above the NAVSOL positions
 - Along-track offset in velocities shows similar dependency on the satellite altitude and might also be related to the ionospheric refraction
- Along-track offset in positions (about 15 m) and radial offset in velocities (-1 to -2 cm/s)
 - Mostly independent of the satellite altitude
 - Rather caused by dynamic effects than by ionospheric refraction