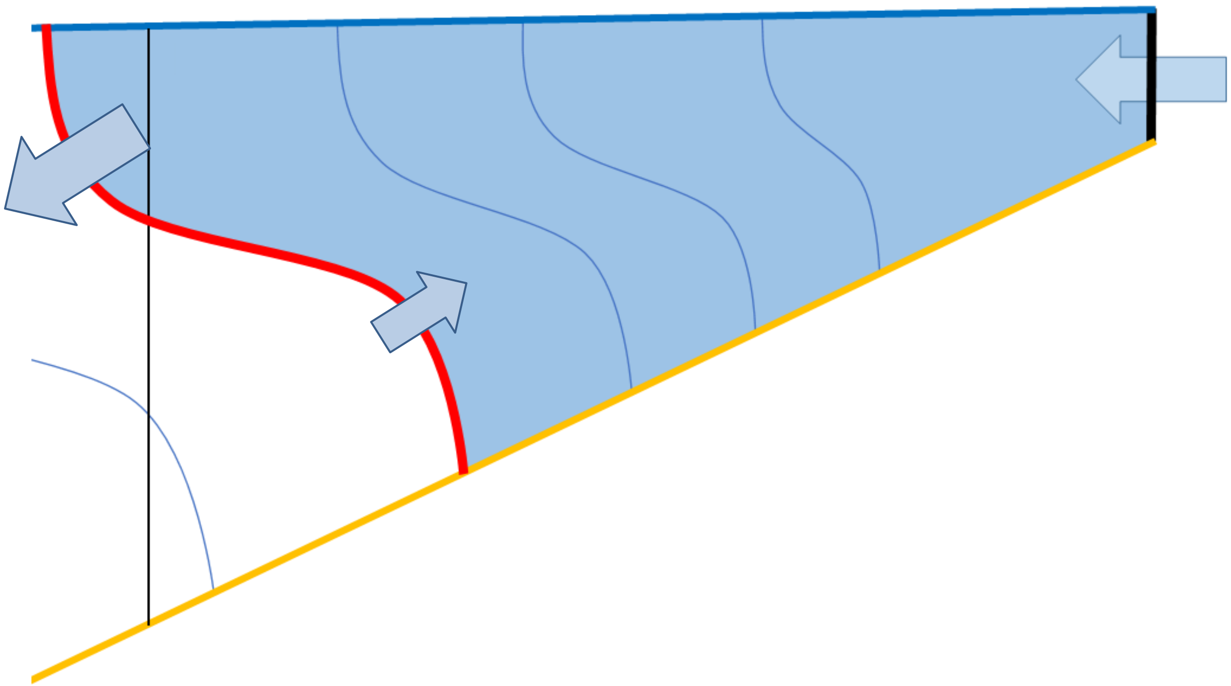
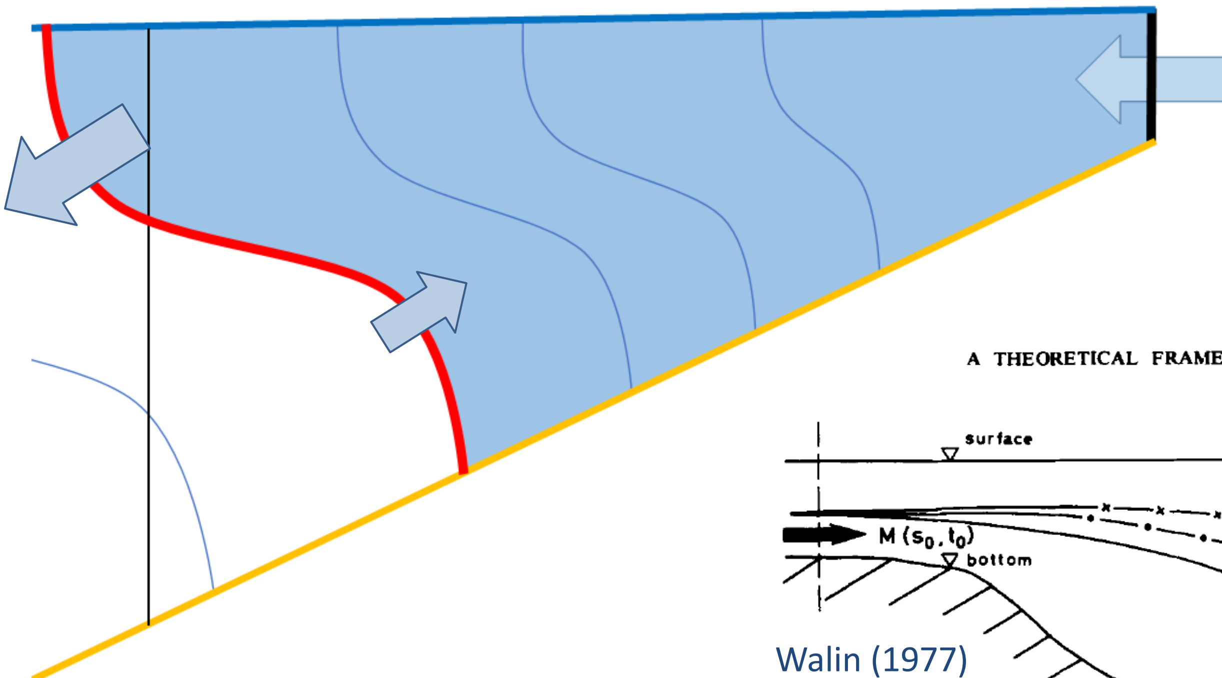
A universal law of estuarine mixing

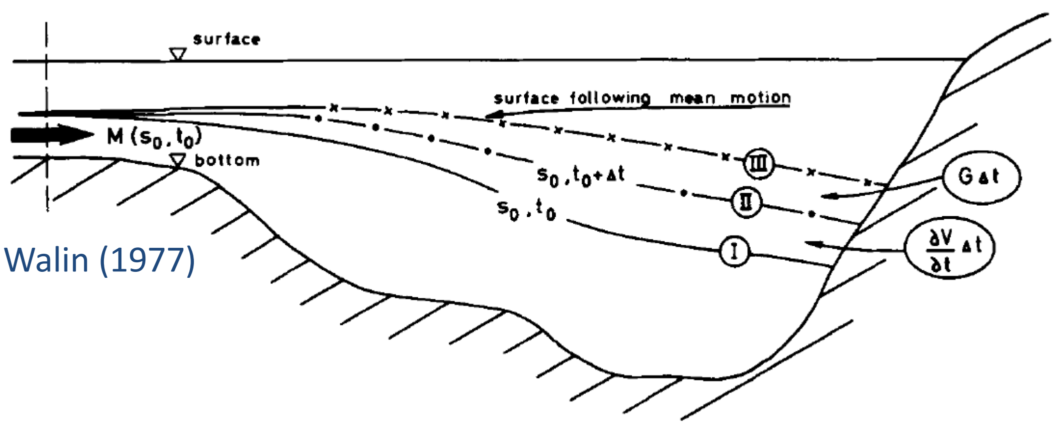


Hans Burchard[1], Ulf Gräwe[1], Knut Klingbeil[1], Xaver Lange[1], Xiangyu Li[1,2], Marvin Lorenz[1]

1. Leibniz Institute for Baltic Sea Research Warnemünde, Germany
2. School of Marine Sciences, Sun Yat-sen University, Zhuhai, China

A universal law of estuarine mixing





Walin (1977)

Hans Burchard[1], Ulf Gräwe[1], Knut Klingbeil[1], Xaver Lange[1], Xiangyu Li[1,2], Marvin Lorenz[1]

1. Leibniz Institute for Baltic Sea Research Warnemünde, Germany
2. School of Marine Sciences, Sun Yat-sen University, Zhuhai, China

Mixing in volume bounded by fixed transect

Long-term averaged estuary

Mixing in volume bounded by fixed transect

Classical Knudsen volume and salt relations:

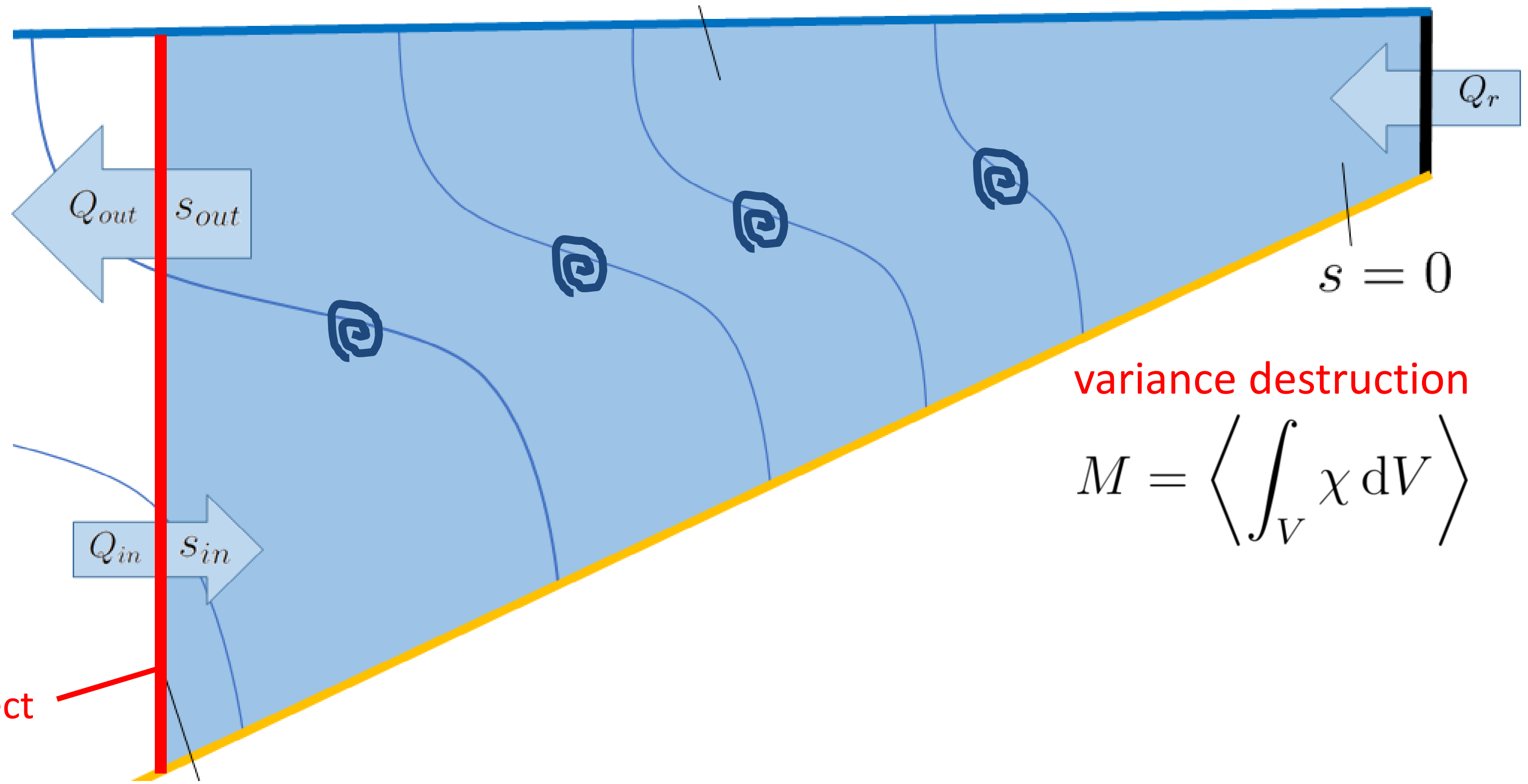
$$Q_{in} + Q_{out} + Q_r = 0 \qquad Q_{in} s_{in} + Q_{out} s_{out} = 0$$

$$Q_{in} = \frac{s_{out}}{s_{in} - s_{out}} Q_r \qquad Q_{out} = -\frac{s_{in}}{s_{in} - s_{out}} Q_r$$

Knudsen (1900)

Long-term averaged estuary

Mixing in volume bounded by fixed transect



Classical Knudsen volume and salt relations:

$$Q_{in} + Q_{out} + Q_r = 0 \qquad Q_{in} s_{in} + Q_{out} s_{out} = 0$$

$$Q_{in} = \frac{s_{out}}{s_{in} - s_{out}} Q_r \qquad Q_{out} = -\frac{s_{in}}{s_{in} - s_{out}} Q_r$$

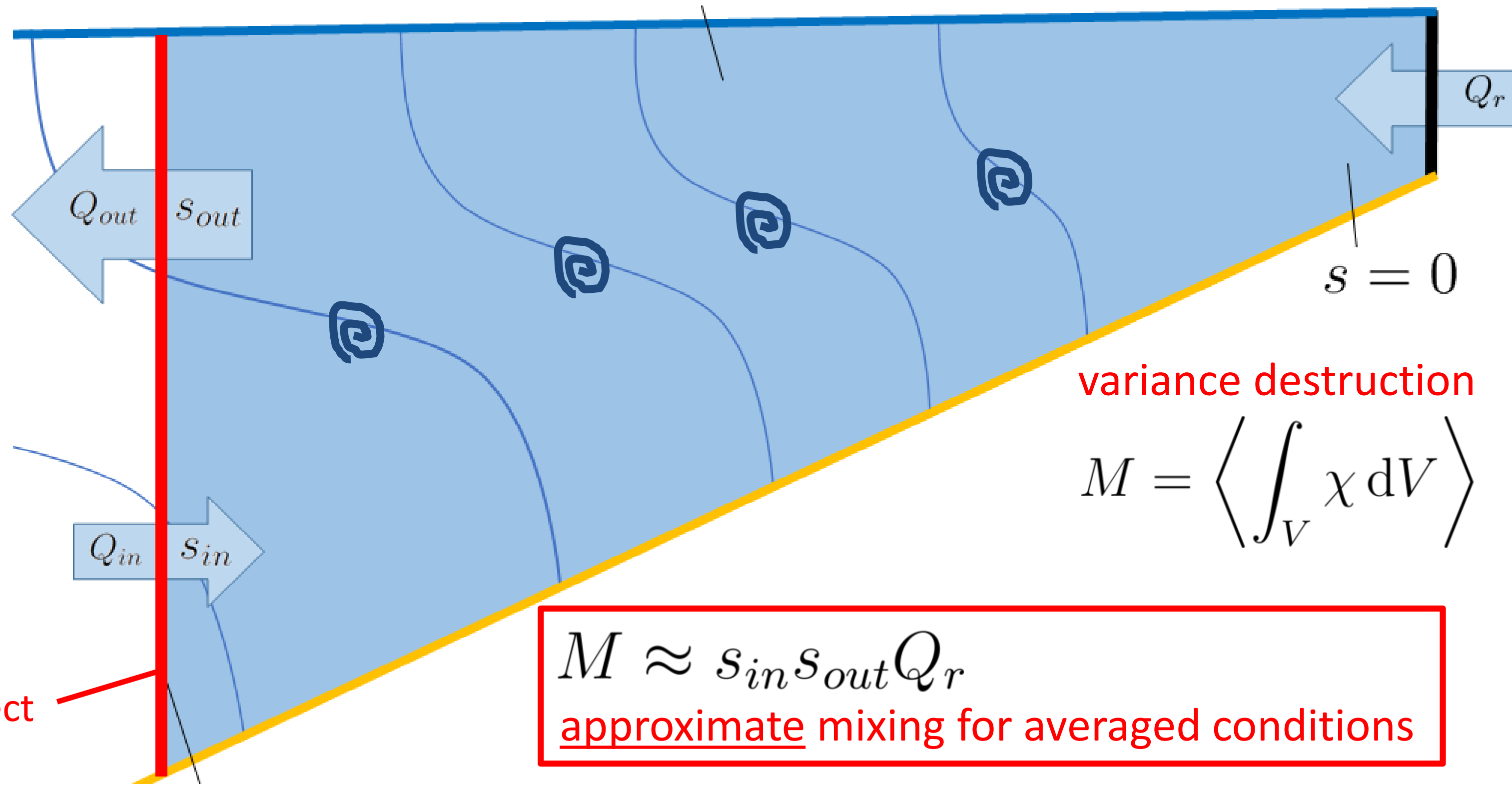
Knudsen (1900)

Knudsen mixing relation:

$$Q_{in} (s_{in})^2 + Q_{out} (s_{out})^2 = s_{in} s_{out} Q_r = M$$

MacCready et al. (2018)

Mixing in volume bounded by fixed transect



variance destruction

$$M = \left\langle \int_V \chi \, \mathrm{d}V \right\rangle$$

$$M \approx s_{in} s_{out} Q_r$$

approximate mixing for averaged conditions

Classical Knudsen volume and salt relations:

$$Q_{in} + Q_{out} + Q_r = 0 \qquad Q_{in} s_{in} + Q_{out} s_{out} = 0$$

$$Q_{in} = \frac{s_{out}}{s_{in} - s_{out}} Q_r \qquad Q_{out} = -\frac{s_{in}}{s_{in} - s_{out}} Q_r$$

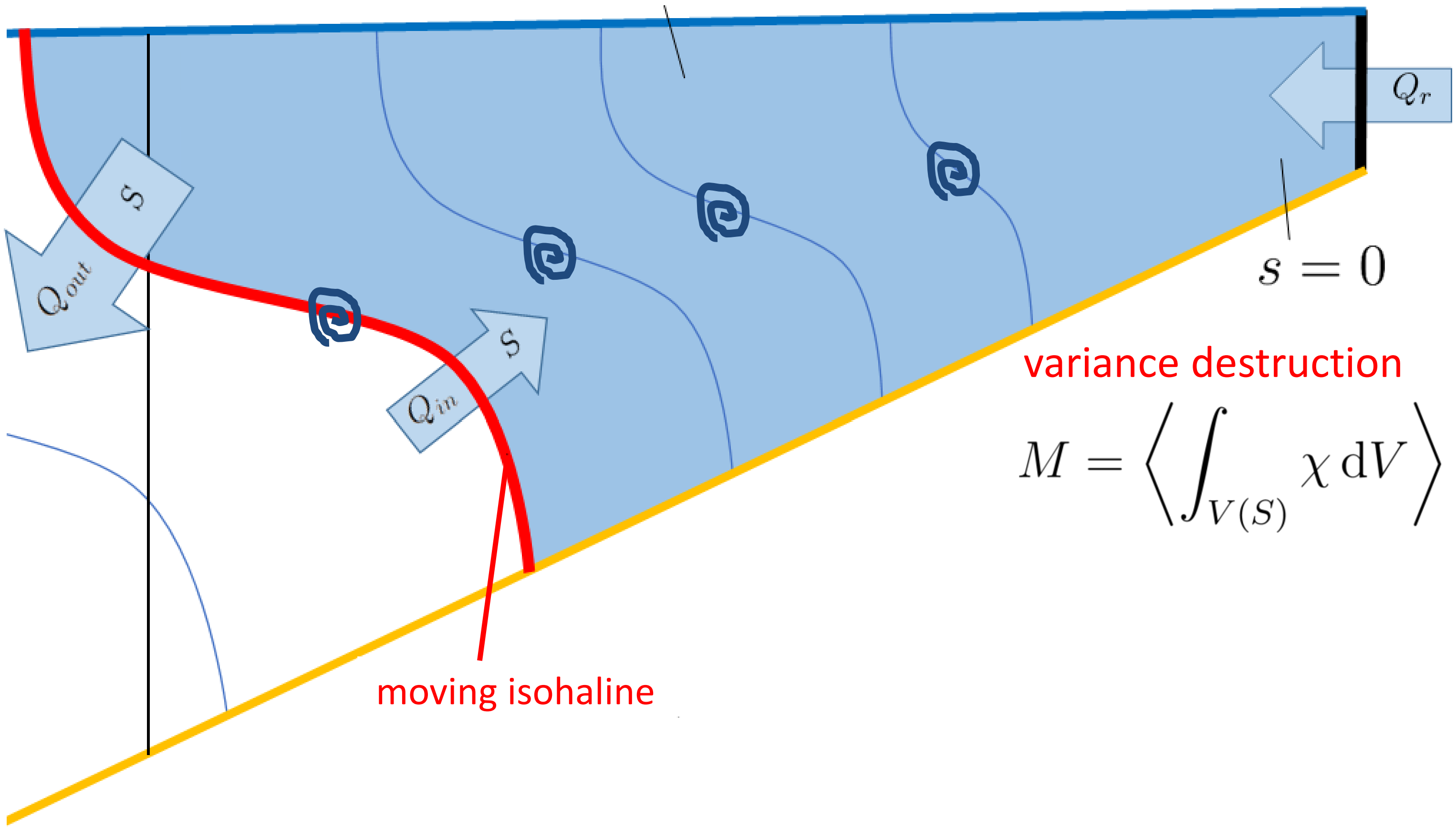
Knudsen (1900)

Knudsen mixing relation:

$$Q_{in} \left(s_{in}\right)^2 + Q_{out} \left(s_{out}\right)^2 = s_{in} s_{out} Q_r = M$$

MacCready et al. (2018)

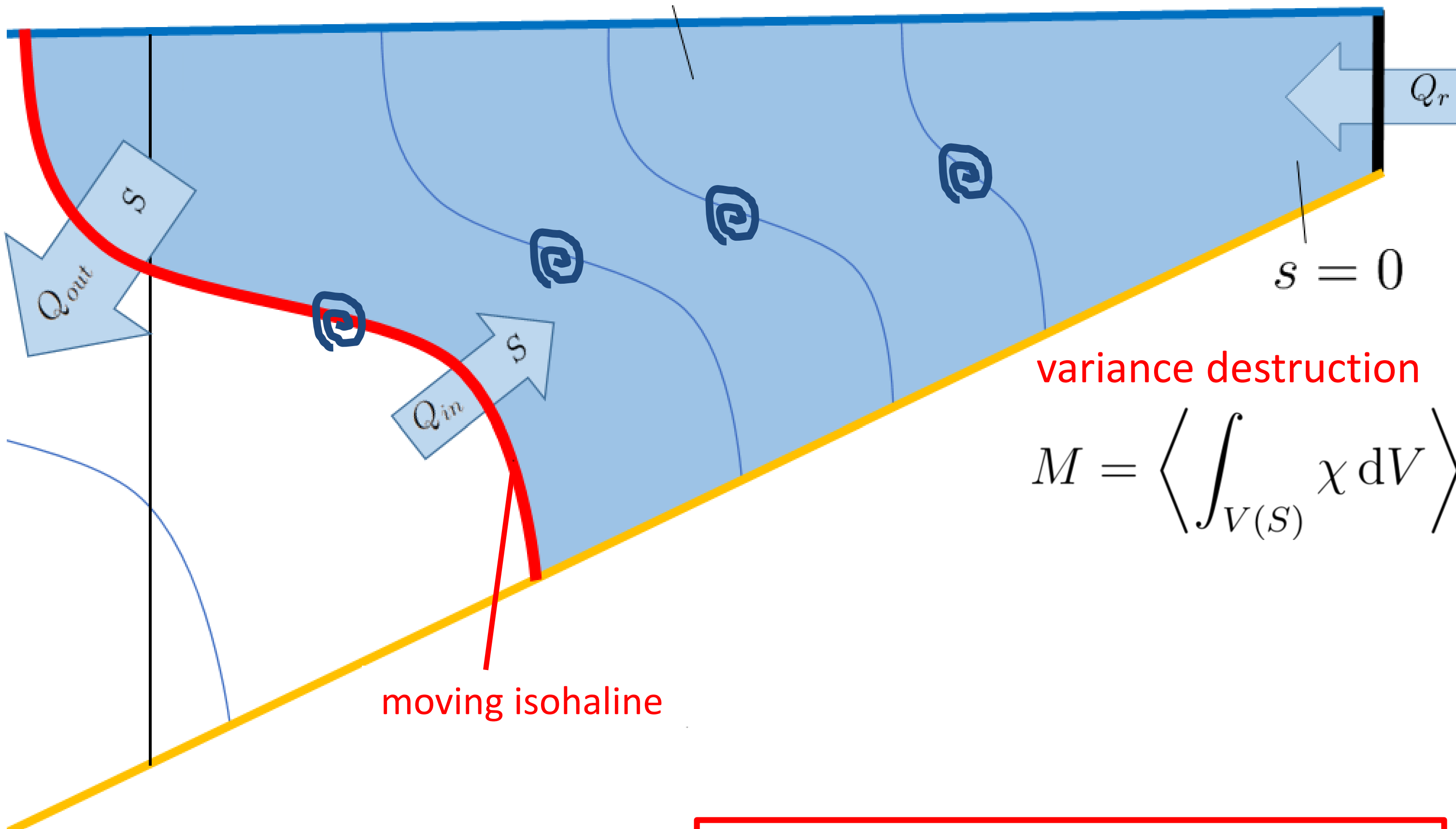
Mixing in volume bounded by moving isohaline



Burchard (2020)

Long-term averaged estuary

Mixing in volume bounded by moving isohaline



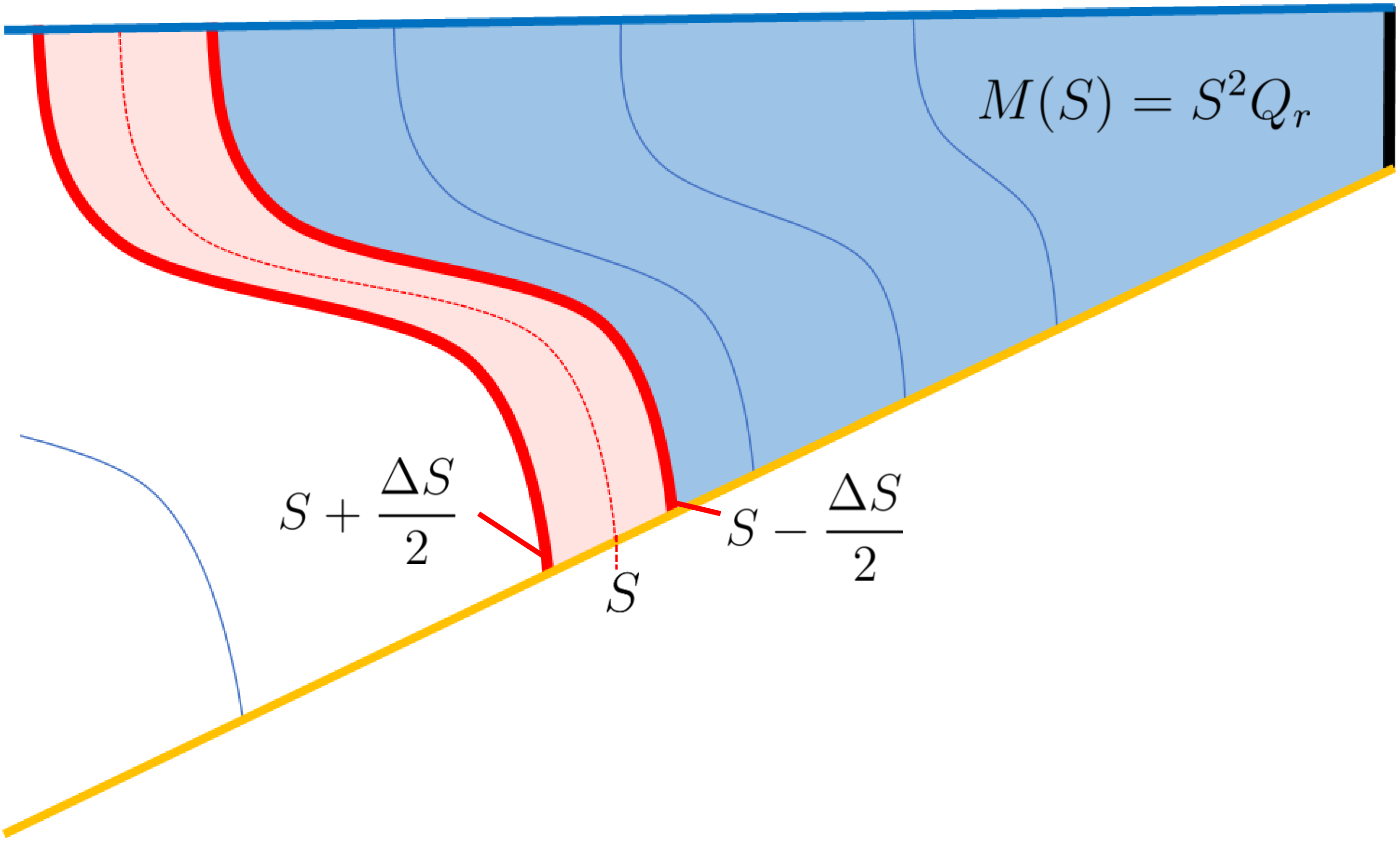
$$M = \left\langle \int_{V(S)} \chi \, \mathrm{d}V \right\rangle$$

$$\boxed{M(S) = S^2 Q_r}$$

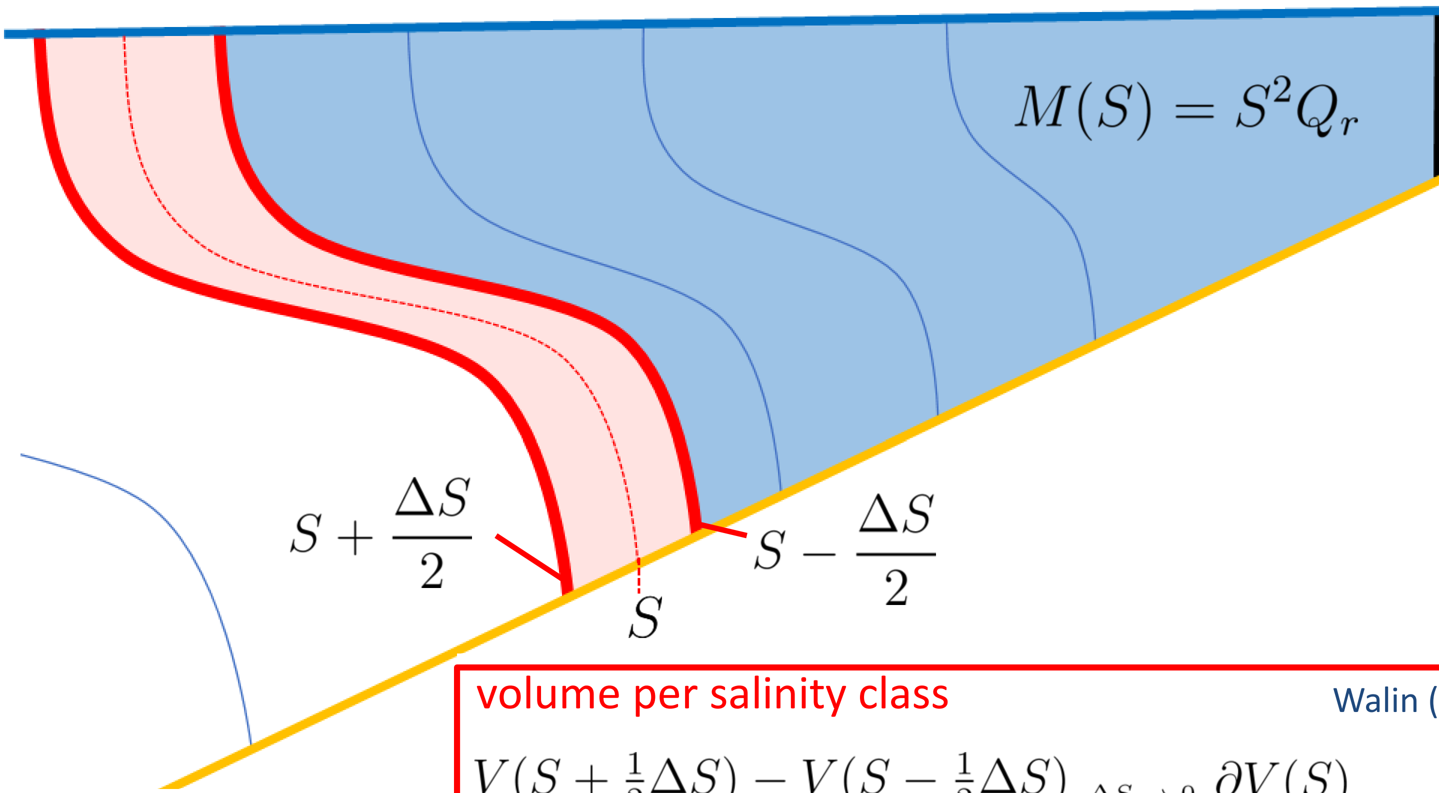
exact mixing for averaged conditions

Burchard (2020)

Defining properties per salinity class



Defining properties per salinity class

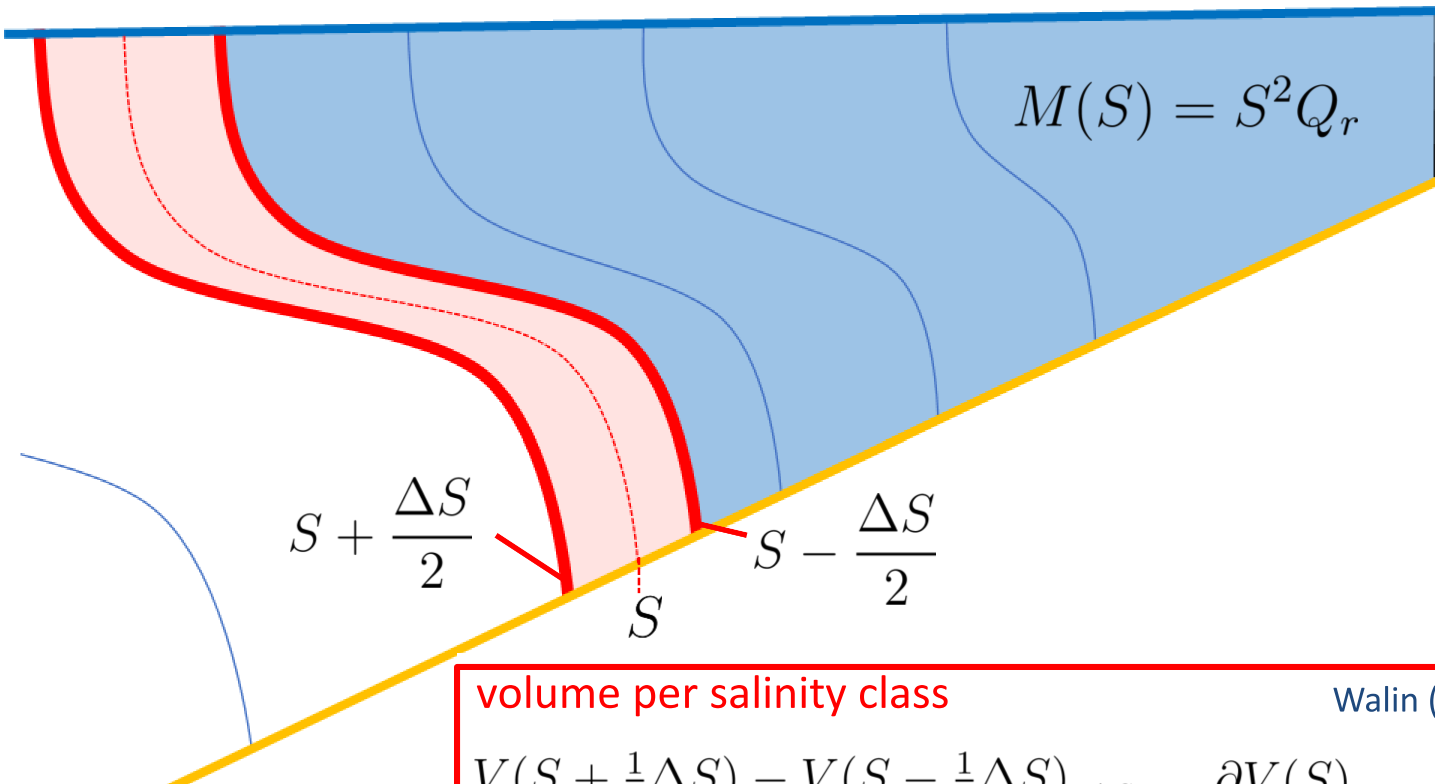


volume per salinity class

Walin (1977)

$$\frac{V(S+\frac{1}{2}\Delta S) - V(S-\frac{1}{2}\Delta S)}{\Delta S} \xrightarrow{\Delta S \to 0} \frac{\partial V(S)}{\partial S} = v(S)$$

Defining properties per salinity class



volume per salinity class Walin (1977)

$$\frac{V(S+\frac{1}{2}\Delta S) - V(S-\frac{1}{2}\Delta S)}{\Delta S} \xrightarrow{\Delta S \to 0} \frac{\partial V(S)}{\partial S} = v(S)$$

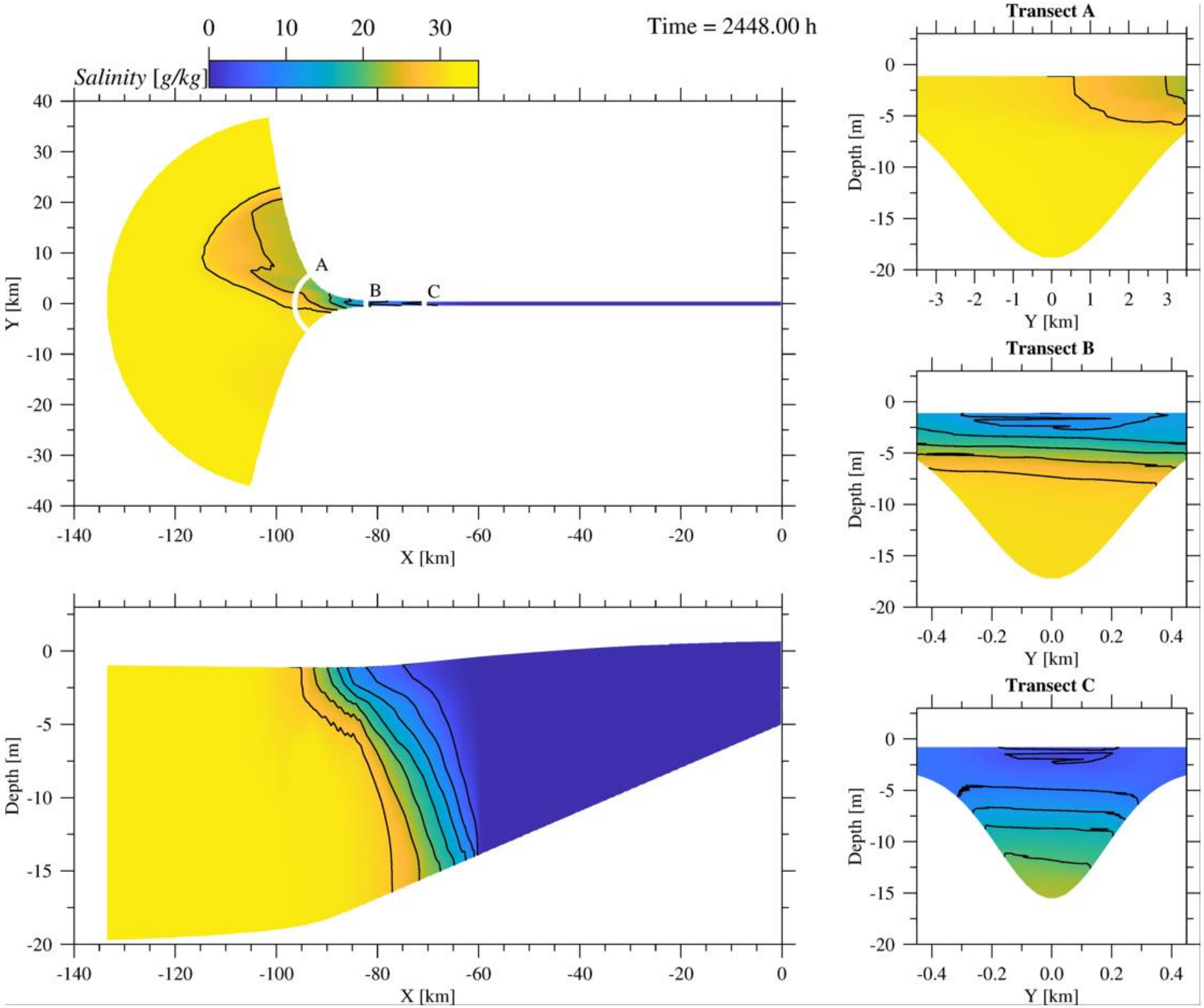
mixing per salinity class Wang et al. (2017), Burchard (2020)

$$\frac{M(S+\frac{1}{2}\Delta S) - M(S-\frac{1}{2}\Delta S)}{\Delta S} \xrightarrow{\Delta S \to 0} \frac{\partial M(S)}{\partial S} = \boxed{m(S)}$$

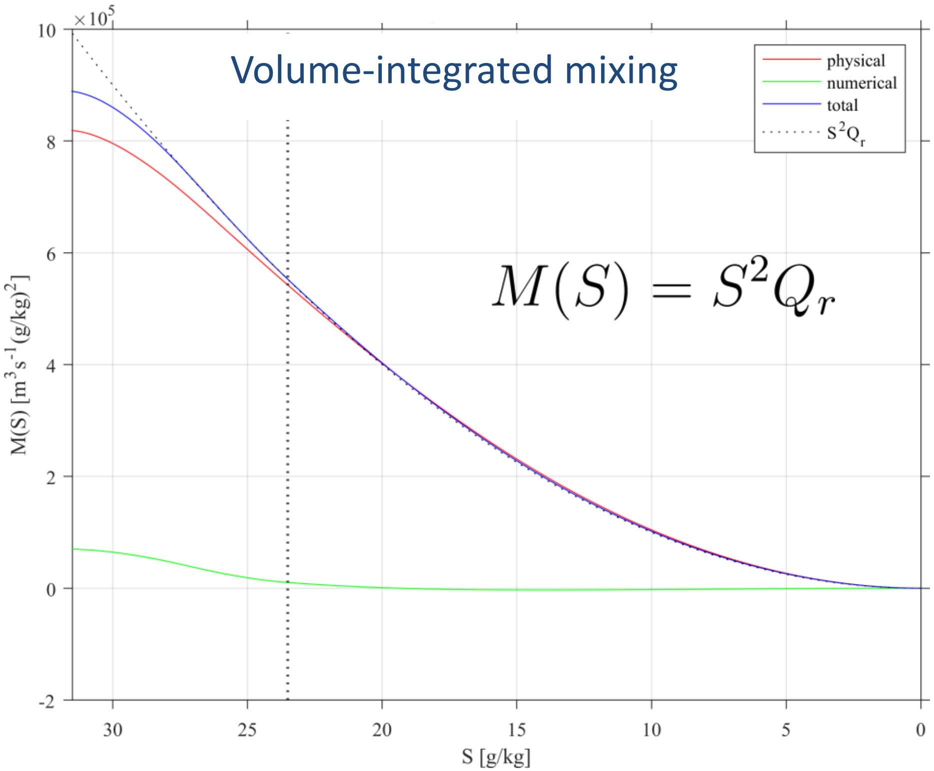
universal law

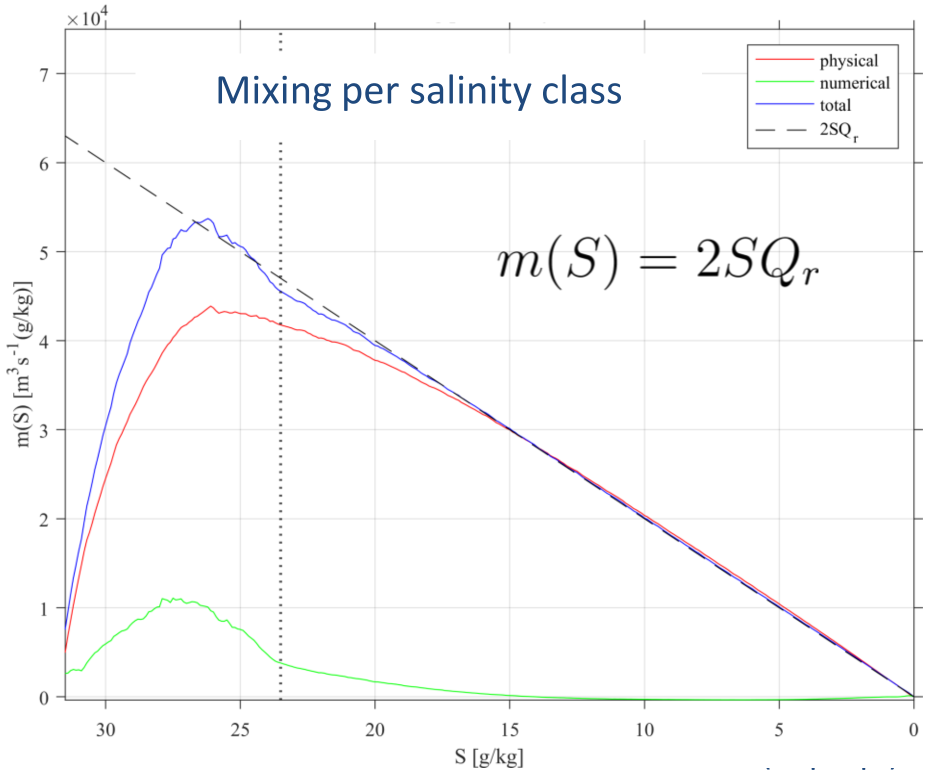
Exact for
- long averages
- P-E=0
- S fully inside domain

3D exponential estuary – from neap tide into spring tide



Analysing mixing

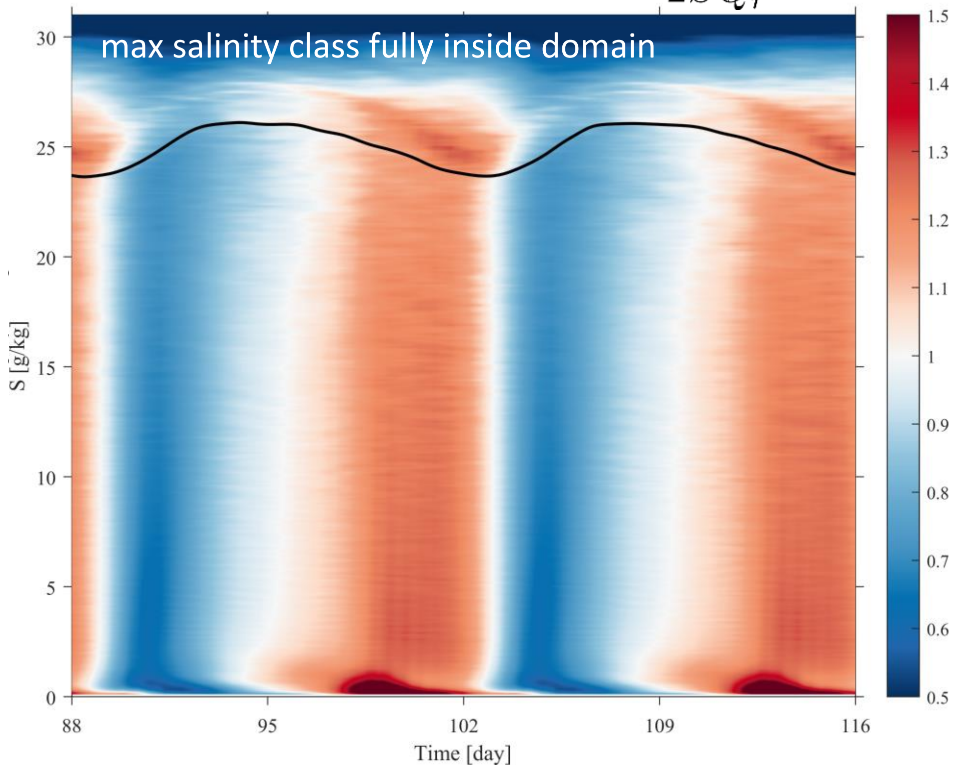




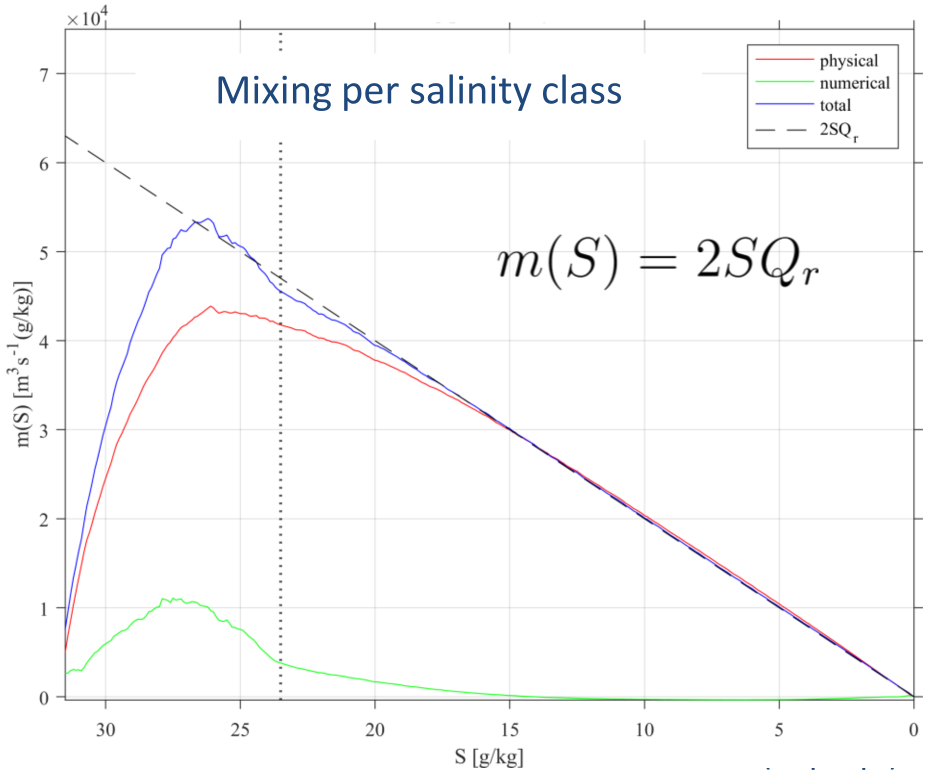
Mixing averaged over 3 spring-neap cycles

Analysing mixing

Tidally-averaged mixing $\frac{m(S,t)}{2SQ_r}$



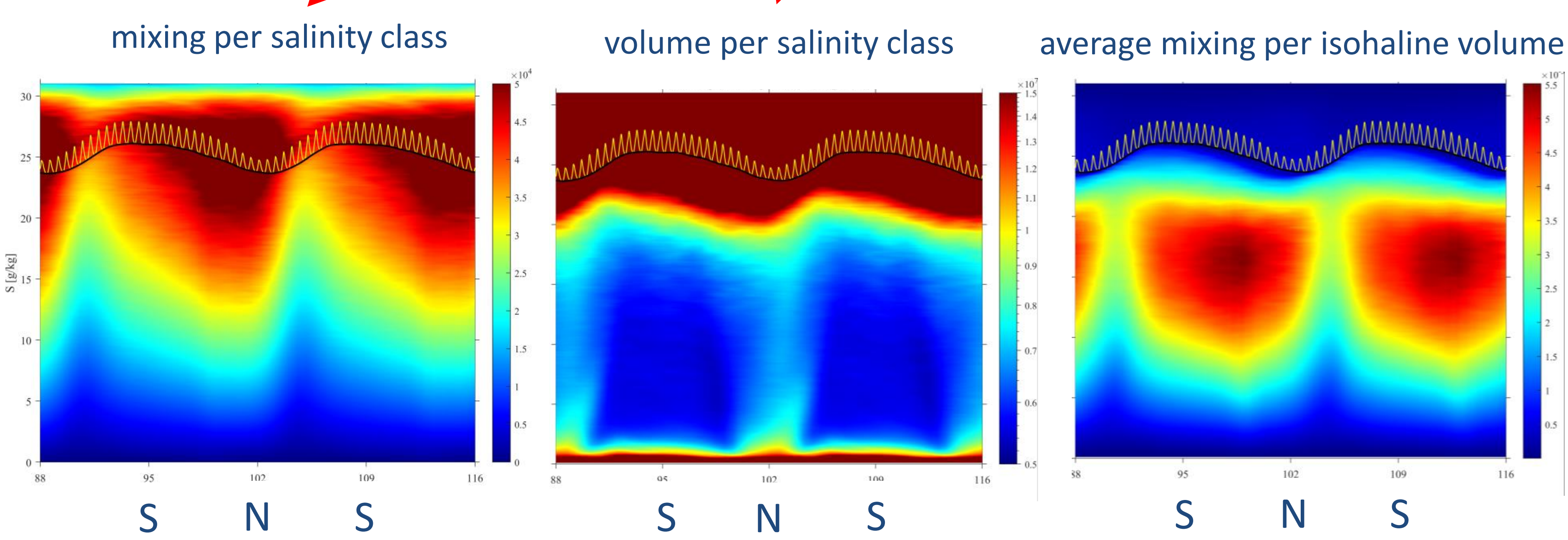
S N S



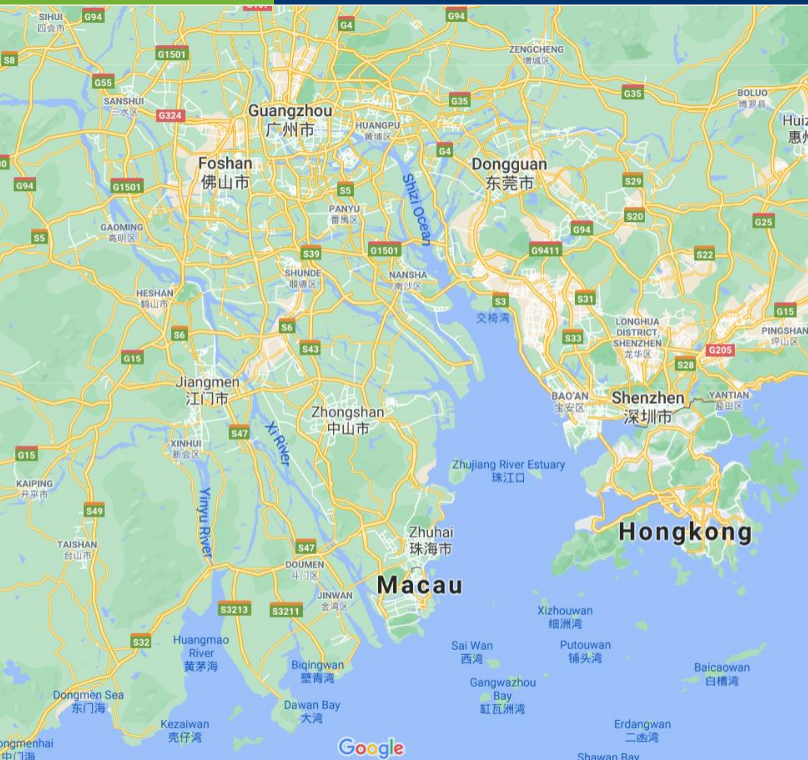
Mixing averaged over 3 spring-neap cycles

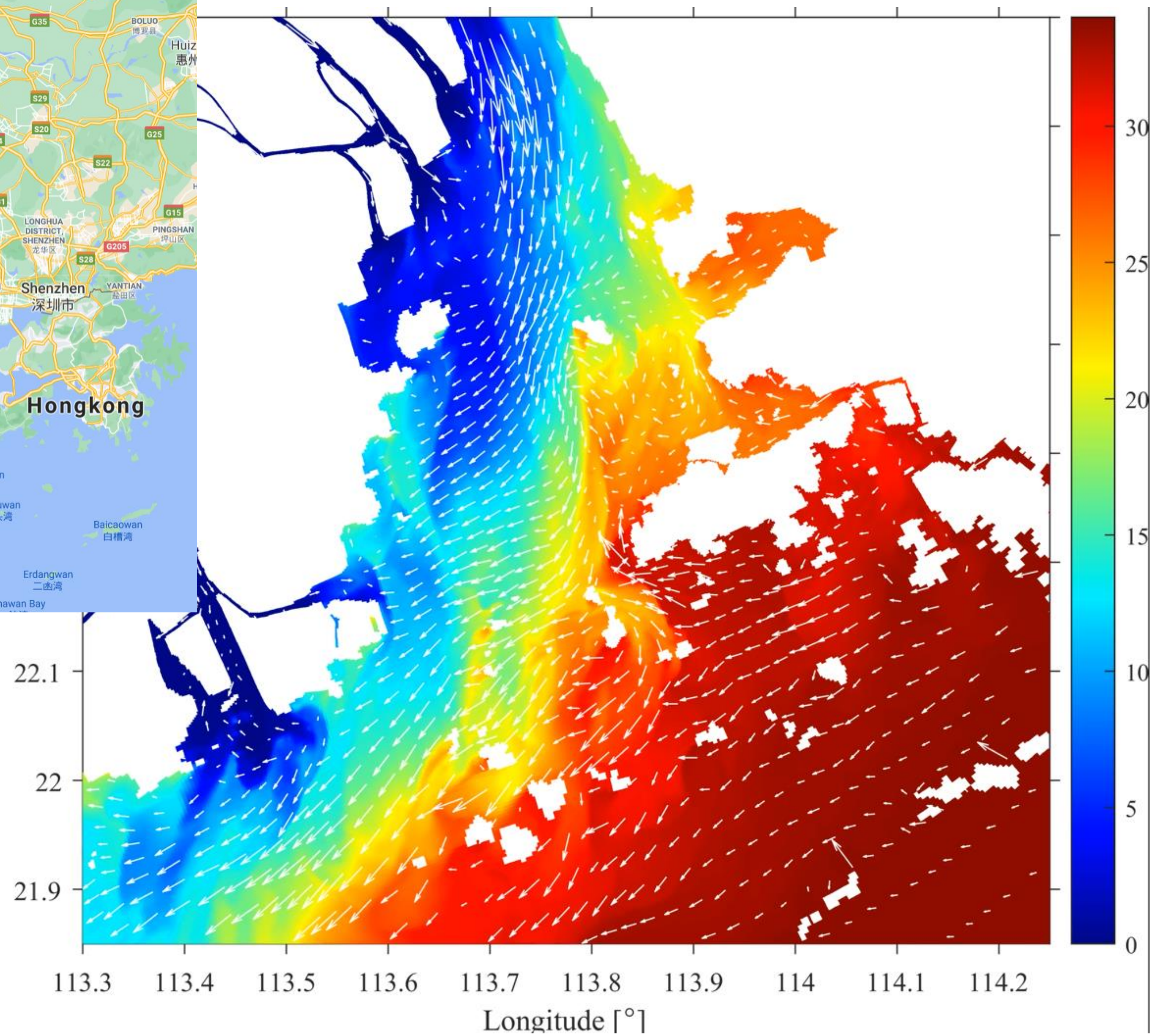
Analysing mixing

$$m(S,t) = v(S,t) \cdot \bar{\chi}(S,t)$$



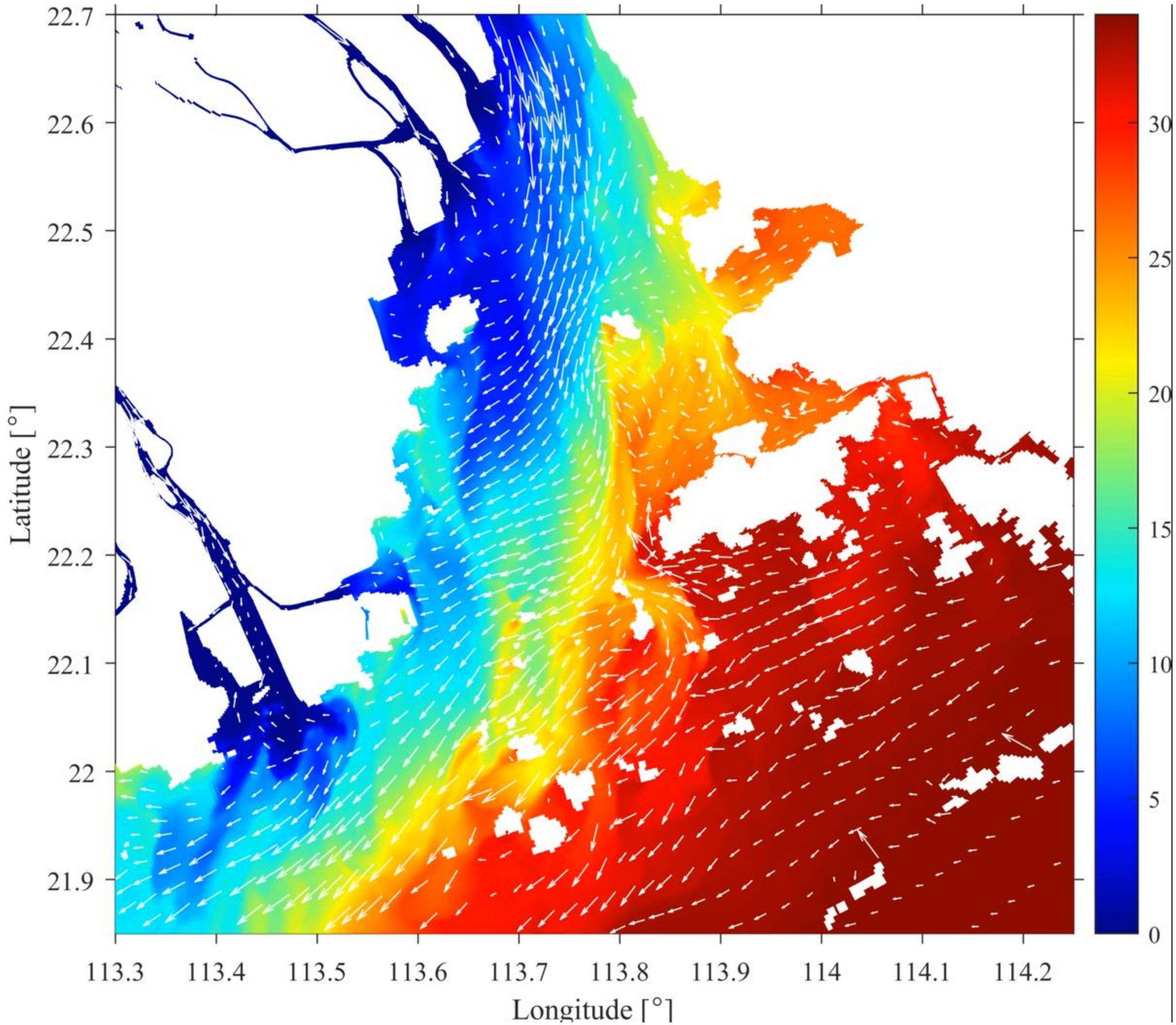
Salinity mixing in Pearl River Estuary





GETM setup, simulation, and analysis by Xiangyu Li

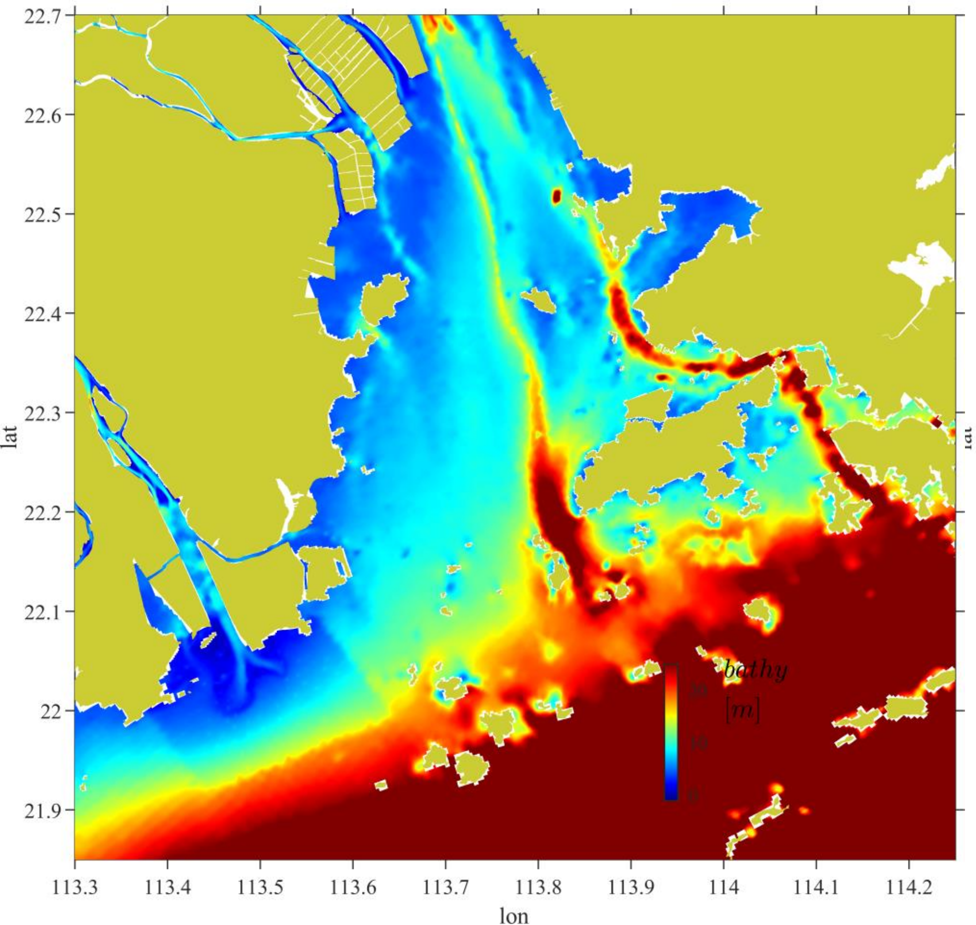
Salinity mixing in Pearl River Estuary



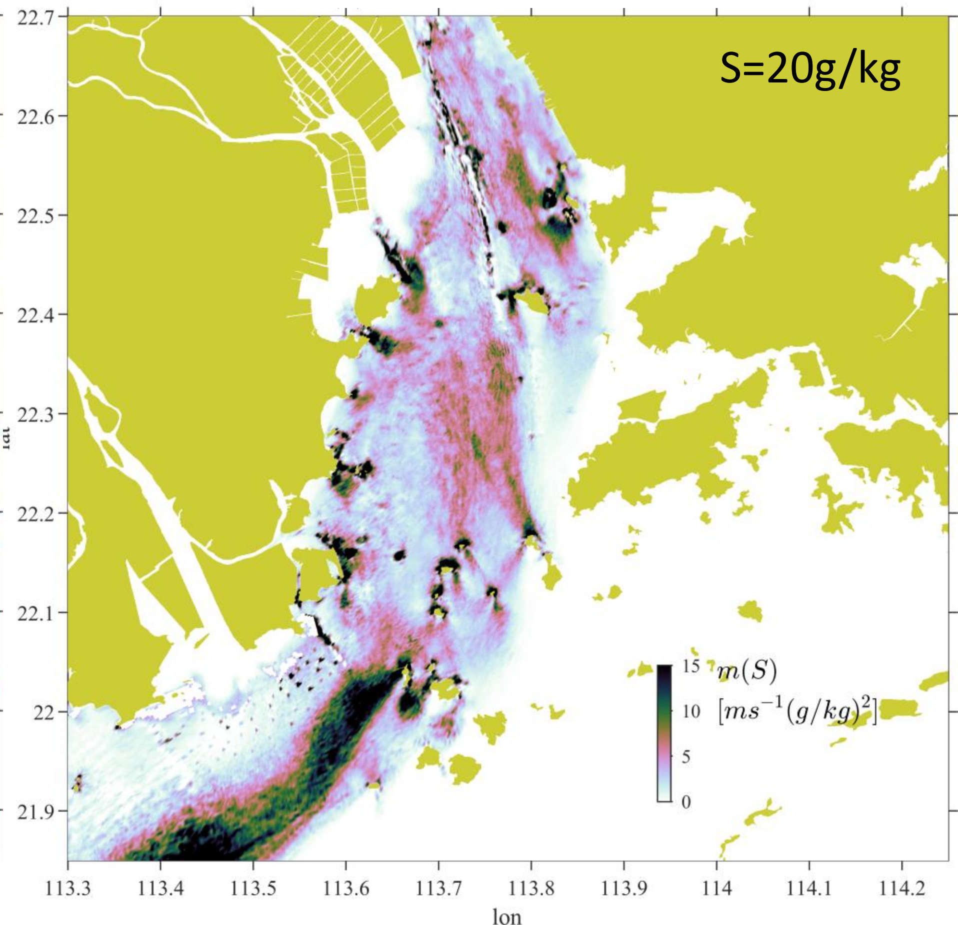
GETM setup, simulation, and analysis by Xiangyu Li

Spatial distribution

bathymetry

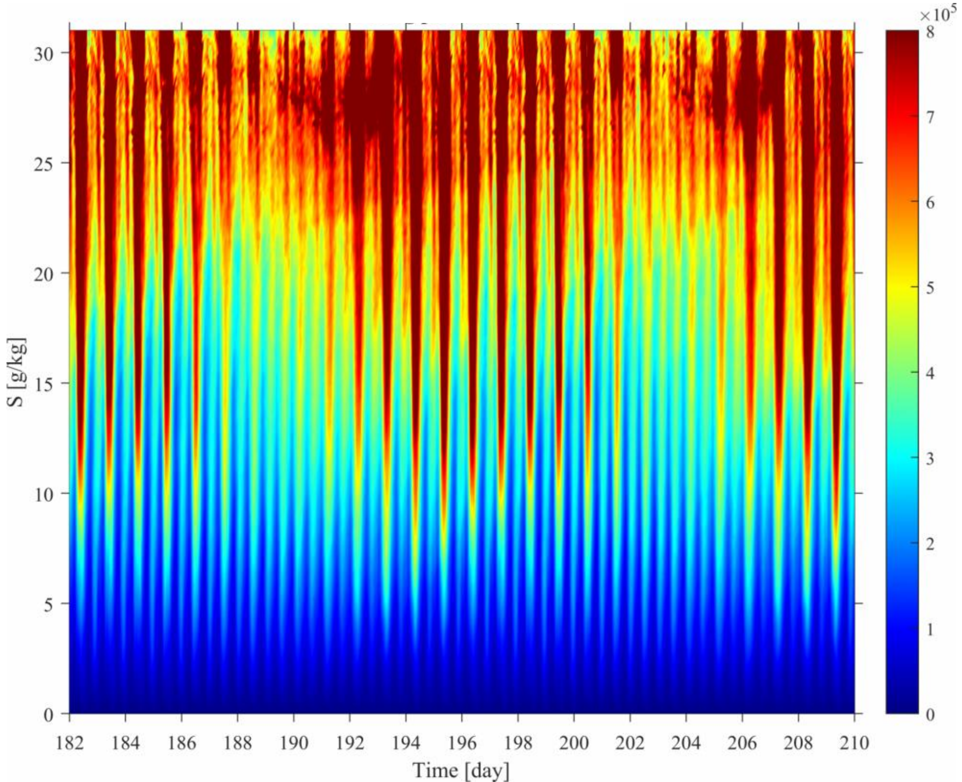


mixing per salinity class

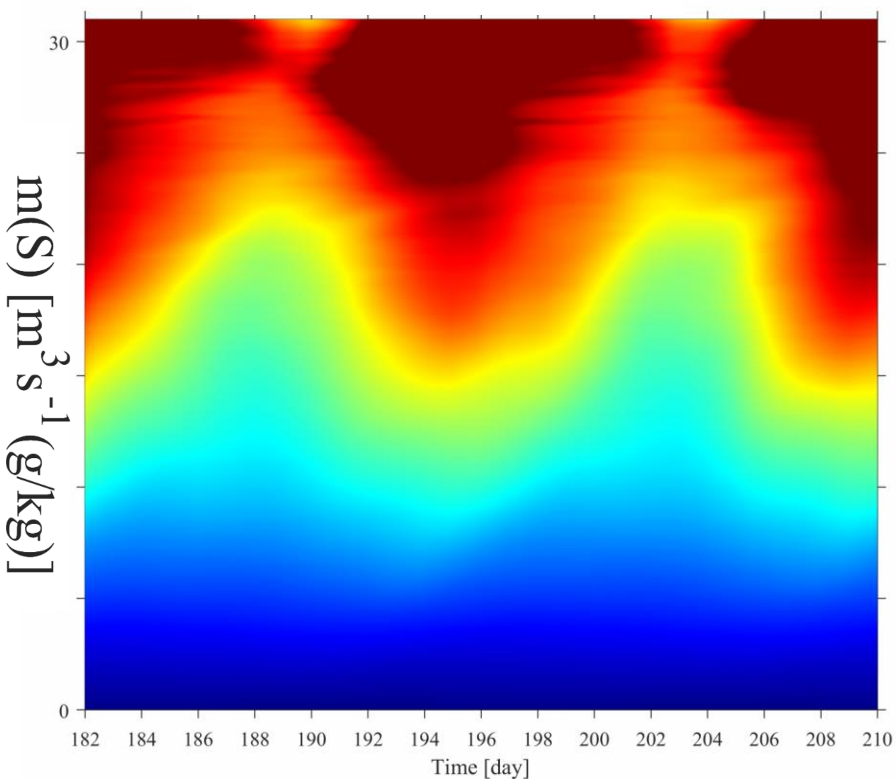


Mixing per salinity class

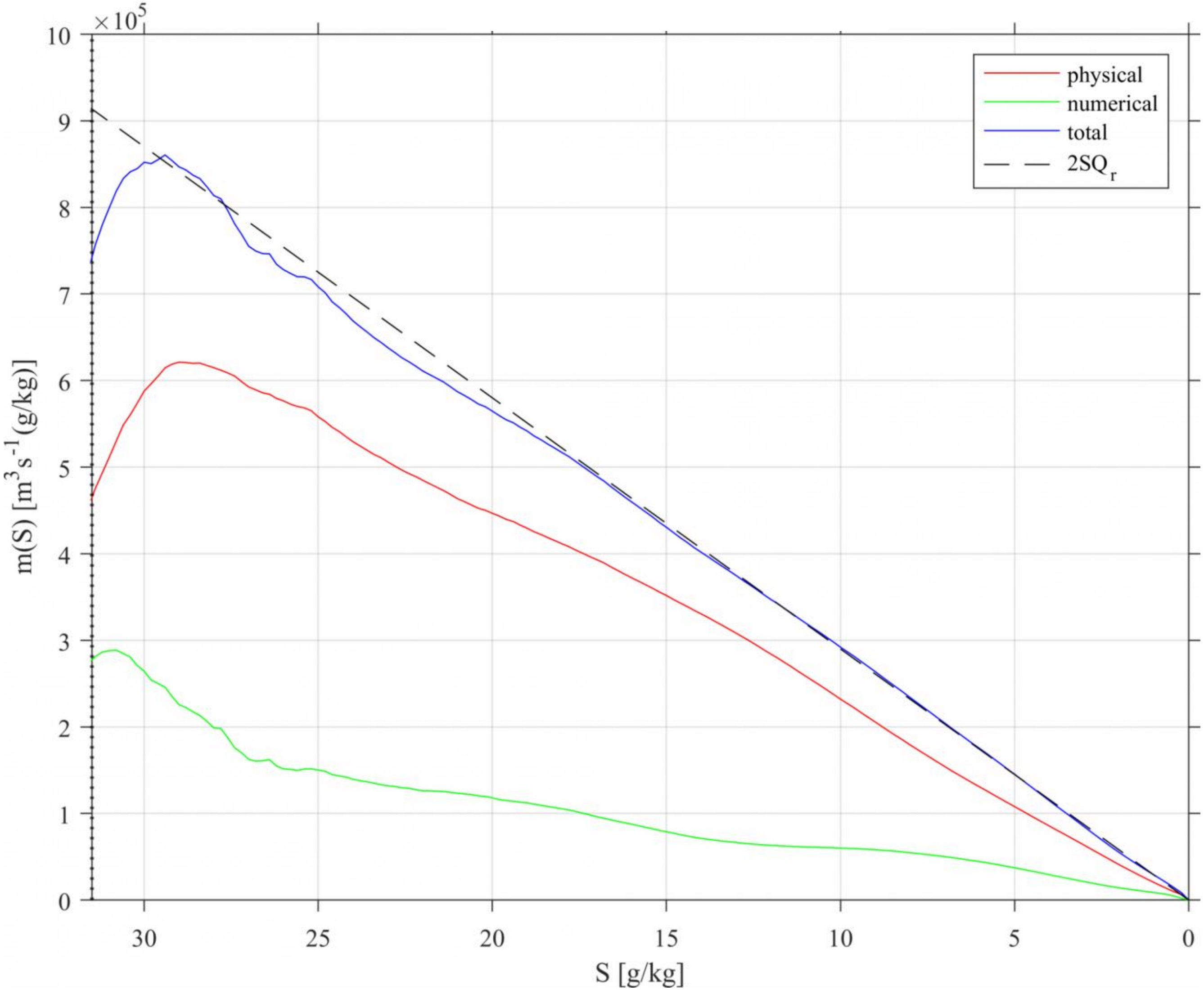
tidally resolved



tidally averaged



Mixing per salinity class (time averaged)



Conclusions

- Calculating the mixing per salinity class, *m(S),* provides a useful parameter to analyse estuarine mixing.
- Long term averages of the effective mixing per salinity class should result as the **universal law of estuarine mixing,** $m(S)=2SQ_r$.
- The law is valid even for complex multi-channel estuaries with islands such as the Pearl River Estuary.
- The effective mixing per salinity class is exactly composed of physical plus numerical mixing.
- What is of interest are the short-term deviations from this estuarine mixing law.

Burchard, H., A universal law of estuarine mixing. *J. Phys. Oceanogr*., 50, 81-93, **2020**.