



European Geosciences Union General Assembly 2021

HS8.1.1 Modern challenges and approaches to modeling subsurface flow and transport across multiple scales

vPICO presentation: Tue, 27 April 11:19-11:21 CEST

An efficient lubrication-based code for solving non-Newtonian flow in geological rough fractures

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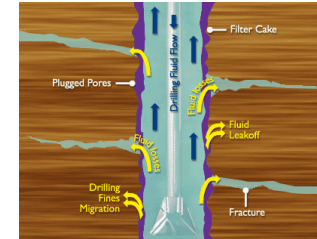
QR abstract
EGU21-7820



1. Flow of shear-thinning fluids in geological fractures

The **comprehension of the hydraulic behaviour** of fluids in geological formations is fundamental to the success of subsurface techniques aimed at:

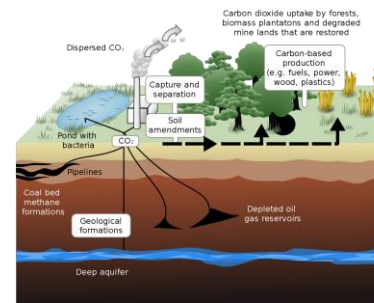
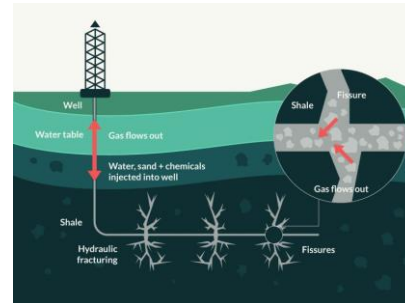
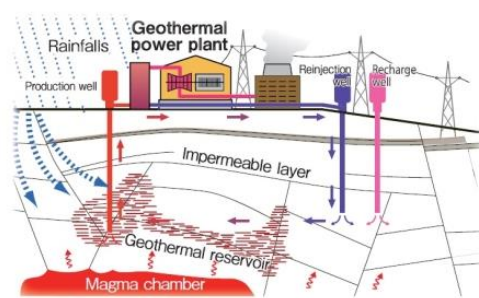
- resources recovery
- geothermal exploitation
- carbon sequestration
- soil reclamation



The **medium heterogeneity** and the **complex fluid rheology** are **source of non-linearity**, whose comprehension is still limited.

Several phenomena are linked with the **compound effects of heterogeneity and rheology**, such as:

- Flow localization (i.e. channeling)
- Anomalous transport



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2. Fracture geometry

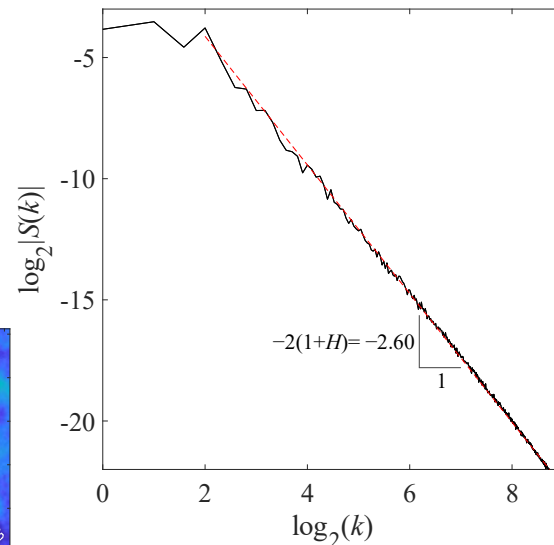
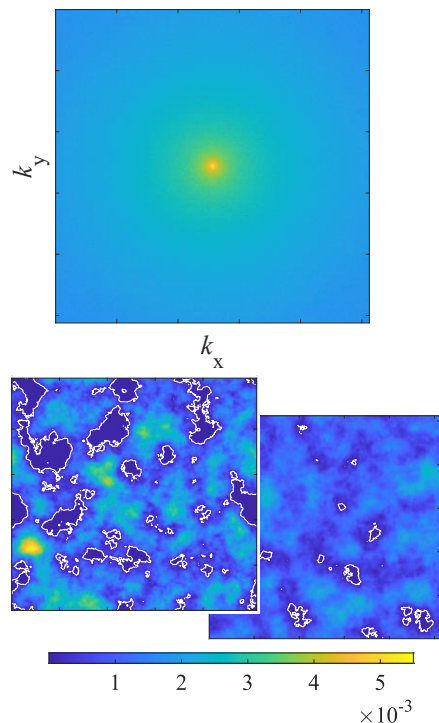
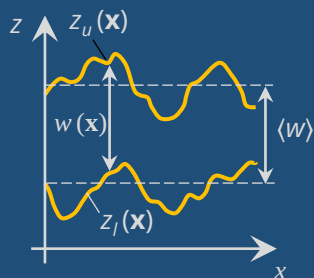
A synthetic aperture field can be obtained by mating two **isotropic self-affine surfaces** $h_i(\mathbf{x})$

$$w(\mathbf{x}) = h_1(\mathbf{x}) - h_2(\mathbf{x}) + \langle w \rangle$$

generated as 2D white noises: **maintaining random phases** and **introducing spatial correlation** by multiplying the modulus of the Fourier transform by the modulus of the wave number $|k| = (k_x^2 + k_y^2)^{1/2}$ to the power $-(1+H)$

$$|h(\mathbf{k})| \rightarrow |k|^{-1-H} |h(\mathbf{k})|$$

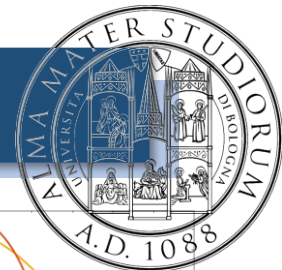
where H is the Hurst exponent



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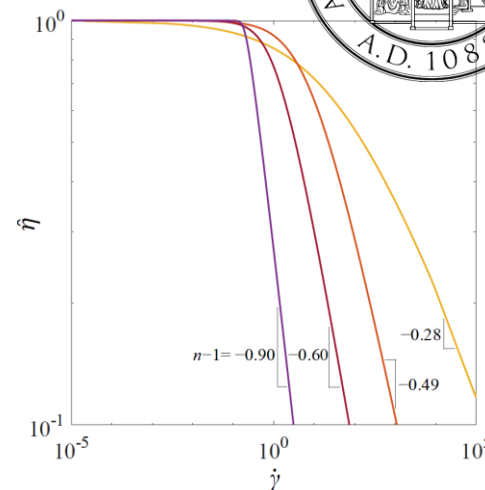


3. Fluid rheology: the Ellis Model

The Ellis rheologic model is a three-parameter shear-thinning model characterized by:

- Low-shear rates viscosity plateau ($\eta \rightarrow \mu_0$ for $\dot{\gamma} \rightarrow 0$)
- High-shear rates power-law behaviour with exponent (n)
- a continuous and smooth transition between the two trends regulated by a characteristic shear ($\tau_{1/2}$), defined as $\eta(\tau_{1/2}) = \mu_0/2$

$$\eta = \frac{\mu_0}{1 + \left(\frac{\tau_{xz}}{\tau_{1/2}} \right)^{\frac{1}{n} - 1}}$$

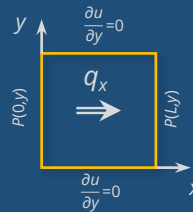


Fluid		μ_0	$\tau_{1/2}$	n
0.3 wt% CMC		0.05	4.07	0.72
0.5 wt% CMC		0.22	2.50	0.51
1.0 wt% CMC		2.99	5.14	0.40
VES		49.00	1.07	0.10

Ellis model admits a parallel-plate explicit expression of the flowrate

$$q_x = - \left[\frac{w^3}{12\mu_0} + \frac{n}{2n+1} \left(\frac{1}{2^{1+n}\mu_0^n\tau_{1/2}^{1-n}} \right)^{\frac{1}{n}} w^{\frac{2n+1}{n}} \left| \frac{\partial P}{\partial x} \right|^{\frac{1}{n}-1} \right] \frac{\partial P}{\partial x},$$

w is the constant aperture



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4. Lubrication theory: a generalized Reynolds equation

Considering isothermal-incompressible Stokes flow, under steady-state conditions

Depth-averaged mass balance

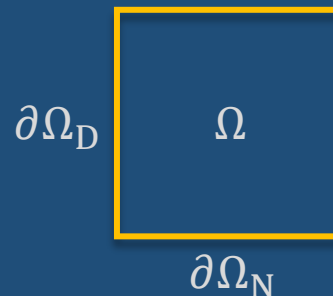
$$\nabla \cdot \mathbf{q} = 0$$

Lubrication approximation

- The aperture field does not change too abruptly $\nabla w \ll 1$
- Viscous forces dominate inertial forces $Re \ll 1$

→ The parallel plate expression, which satisfy momentum conservation, is locally valid

$$-\nabla \cdot \left[\frac{w(\mathbf{x})^3}{12\mu_0} + \frac{n}{(2n+1)} \left(\frac{1}{2^{1+n} \mu_0^n \tau_{1/2}^{1-n}} \right)^{\frac{1}{n}} w(\mathbf{x})^{\frac{2n+1}{n}} |\nabla P|^{\frac{1}{n}-1} \right] \nabla P = 0$$



Problem formulation

$$\begin{cases} -\nabla \cdot \left(a(\mathbf{x}) + b(\mathbf{x}) |\nabla P|^{t-2} \right) \nabla P = 0 & x \in \Omega \\ P = P_0 & x \in \partial\Omega_D \\ \nabla P \cdot \nu = 0 & x \in \partial\Omega_N \end{cases}$$

→ The solution **exists** and is **unique** for $2 < t < +\infty$
(i.e. $0 < n < 1$)



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5. Numerical modeling: finite volume scheme

Scheme

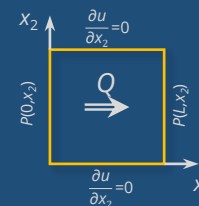
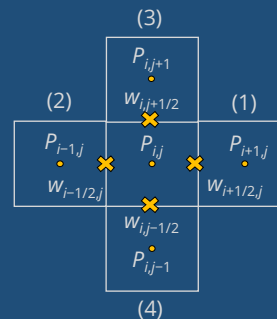
A square fracture $\Omega \in (0, L) \times (0, L)$ is discretised with a regular partition

$$\left[\frac{w_{i+\frac{1}{2},j}^3}{12} + \frac{n}{2n+1} \left(\frac{1}{2^{1+n}(\tau_{1/2}\Delta)^{1-n}} \right)^{\frac{1}{n}} w_{i+\frac{1}{2},j}^{\frac{2n+1}{n}} |P_{i+1,j} - P_{i,j}|^{\frac{1}{n}-1} \right] (P_{i+1,j} - P_{i,j}) + \quad (1)$$

$$+ \left[\frac{w_{i-\frac{1}{2},j}^3}{12} + \frac{n}{2n+1} \left(\frac{1}{2^{1+n}(\tau_{1/2}\Delta)^{1-n}} \right)^{\frac{1}{n}} w_{i-\frac{1}{2},j}^{\frac{2n+1}{n}} |P_{i-1,j} - P_{i,j}|^{\frac{1}{n}-1} \right] (P_{i-1,j} - P_{i,j}) + \quad (2)$$

$$+ \left[\frac{w_{i,j+\frac{1}{2}}^3}{12} + \frac{n}{2n+1} \left(\frac{1}{2^{1+n}(\tau_{1/2}\Delta)^{1-n}} \right)^{\frac{1}{n}} w_{i,j+\frac{1}{2}}^{\frac{2n+1}{n}} |P_{i,j+1} - P_{i,j}|^{\frac{1}{n}-1} \right] (P_{i,j+1} - P_{i,j}) + \quad (3)$$

$$+ \left[\frac{w_{i,j-\frac{1}{2}}^3}{12} + \frac{n}{2n+1} \left(\frac{1}{2^{1+n}(\tau_{1/2}\Delta)^{1-n}} \right)^{\frac{1}{n}} w_{i,j-\frac{1}{2}}^{\frac{2n+1}{n}} |P_{i,j-1} - P_{i,j}|^{\frac{1}{n}-1} \right] (P_{i,j-1} - P_{i,j}) = 0; \quad (4)$$



Algorithm

- The system of non-linear equation is solved via inexact Newton-Krylov method, providing the Jacobian Matrix
- The solution of the linearised system is obtained with variable-fill-in incomplete Cholesky preconditioned conjugate gradient algorithm, calibrated by trial and error.
- A continuation strategy is adopted for strongly non-linear cases.



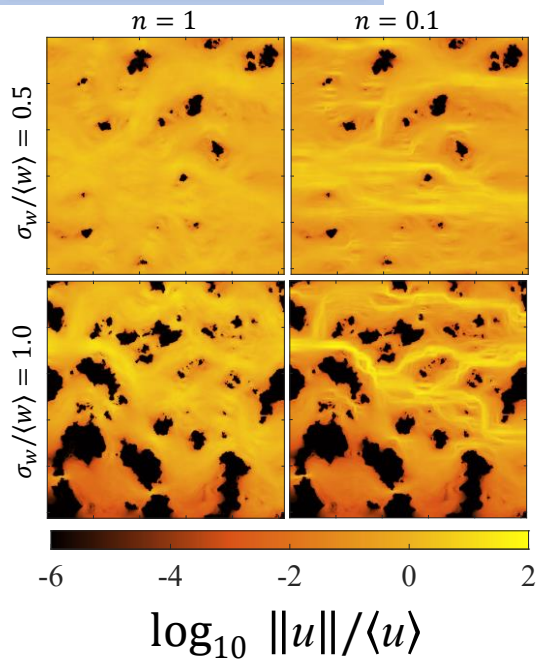
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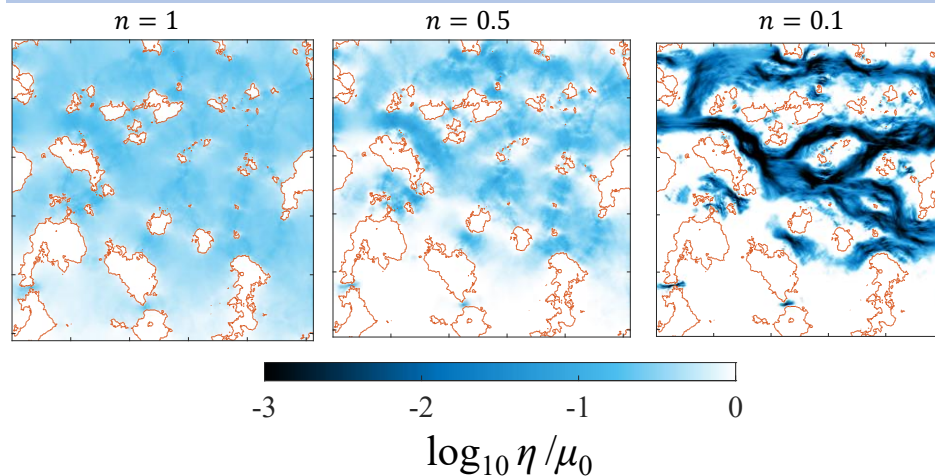
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6. Results

Velocity Magnitude



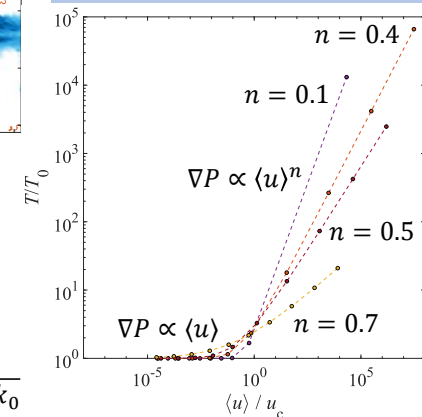
Apparent viscosity



$$T = \frac{Q\mu_0}{\nabla P}$$

$$u_c = \gamma_c \sqrt{k_0}$$

Transmissivity



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7. Conclusions

- The numerical code **generates self-affine fractures** and **solves the 2D flow of a shear-thinning fluid** described by the Ellis rheologic model;
- The **code is able to handle strongly non-linear cases** (small values of n) on large meshes (e.g. $2^{12} \times 2^{12}$) in fractures presenting a **high degree of heterogeneity** (e.g. $\frac{\sigma_w}{\langle w \rangle} = 1$);
- The **inexact Newton-Krylov method** guarantees **optimal convergence**, avoiding oversolving and leading to a good trade-off between **robustness** and **efficiency** is achieved;
- The algorithm requires **low computational time** to complete a simulation in respect to other CFD techniques and lubrication-based algorithms (e.g. Picard's method).



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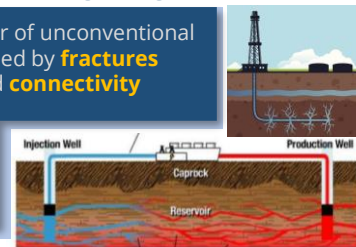
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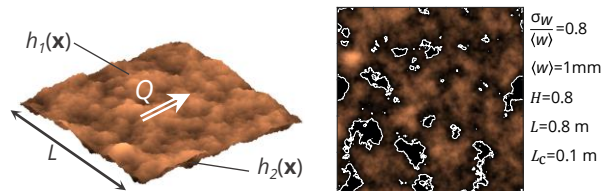
1. Shear-thinning flow in geological fractures

The hydraulic behaviour of unconventional reservoirs is governed by **fractures permeability** and **connectivity**

- Enhanced oil recovery (EOR)
- Enhanced geothermal systems (EGS)
- Carbon sequestration (CO₂-EOR and CO₂-EGS)



2. Synthetic fracture generator



A synthetic aperture field can be obtained by mating two **isotropic self-affine surfaces** $h_i(\mathbf{x})$

$$w(\mathbf{x}) = h_1(\mathbf{x}) - h_2(\mathbf{x}) + \langle w \rangle$$

generated as 2D white noises: **maintaining random phases** and **introducing spatial correlation** by multiplying the modulus of the Fourier transform by the modulus of the wave number $|k| = (k_x^2 + k_y^2)^{1/2}$ to the power $-(1+H)$

$$|h(\mathbf{k})| \rightarrow |k|^{-1-H} |h(\mathbf{k})|$$

where H is the Hurst exponent

3. Flow solver

ELLIS RHEOLOGY

$$\eta = \frac{\mu_0}{1 + \left(\frac{\tau_{xz}}{\tau_{1/2}} \right)^{\frac{1}{n-1}}} \begin{cases} \mu_0 & \text{low-shear rate viscosity } (\eta \rightarrow \mu_0 \text{ for } \dot{\gamma} \rightarrow 0) \\ n & \text{shear-thinning index} \\ \tau_{1/2} & \text{characteristic shear stress} \end{cases}$$

$$\eta(\tau_{1/2}) = \mu_0/2$$

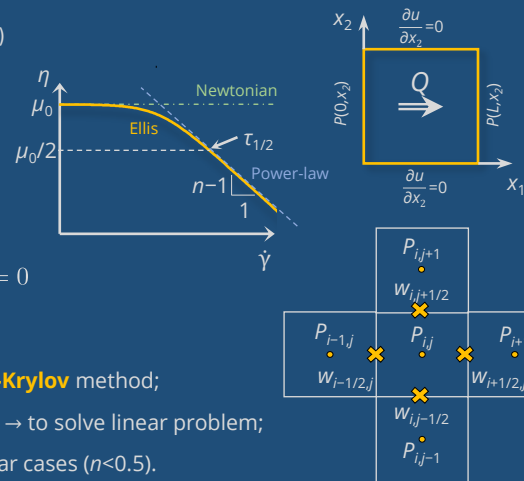
GENERALIZED REYNOLDS EQUATION

- **lubrication theory** holds ($\nabla w \ll 1$ and $Re \ll 1$)

$$-\nabla \cdot \left[\frac{w(\mathbf{x})^3}{12\mu_0} + \frac{n}{(2n+1)} \left(\frac{1}{2^{1+n}\mu_0^n\tau_{1/2}^{1-n}} \right)^{\frac{1}{n}} w(\mathbf{x})^{\frac{2n+1}{n}} |\nabla P|^{\frac{1}{n-1}} \right] \nabla P = 0$$

Numerical modeling via finite volume method:

- The non-linear system of equation is solved via inexact **Newton-Krylov** method;
- **Variable-fill-in Cholesky preconditioned conjugate gradient** → to solve linear problem;
- A **continuation strategy** is adopted to handle strongly non-linear cases ($n < 0.5$).



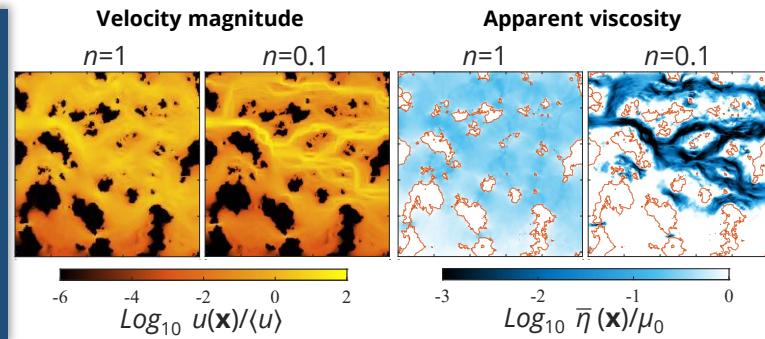
4. Results

A **fast** and **robust** code has been implemented to simulate the flow of shear-thinning fluids in geological fractures.

Optimal convergence is achieved for realistic sets of parameters that contribute the most to problem non-linearity

$$0.1 \leq n \leq 1 \quad 0 \leq \frac{\sigma_w}{\langle w \rangle} \leq 1$$

σ_w is the aperture standard deviation



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