A model for multicomponent diffusive bubble growth in magmas
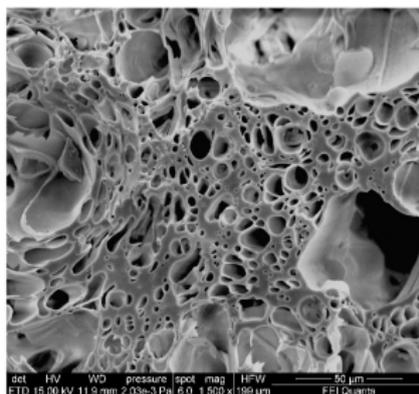
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SEM microphotographs of bubble-rich lapilli (Colucci et al. 2013)

► Volcanic eruptions are driven by the dynamics of the gas phase

► The rate of bubble growth determines the ascent rate of magma from the plumbing system to the atmosphere

► The rate of bubble growth controls the volcanic eruptive style (effusive vs. explosive)

▶ Most common volatiles in magmas: CO_2, H_2O, S

▶ Most previous models consider water only (Lyakhovsky, Hurwitz, and Navon 1996; Proussevitch and Sahagian 1998; Coumans et al. 2020; Hajimirza, Gonnermann, and Gardner 2021)

▶ CO_2 significantly affects H_2O saturation (non linear behaviour)

Does bubble growth dynamics change when multiple volatiles (e.g. CO_2 and H_2O) are included?

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Bubbles grow by mass diffusion, when the silicate melt is oversaturated in volatiles, **and by mechanical expansion** as a response to pressure decrease. The viscosity of the melt and the surface tension oppose a resistance to bubble growth (Proussevitch and Sahagian 1998).
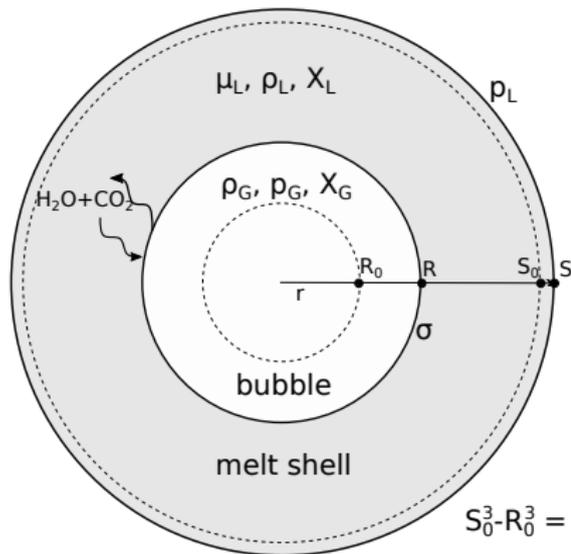
Model assumptions:
- ▶ spherical bubble
- ▶ incompressible melt
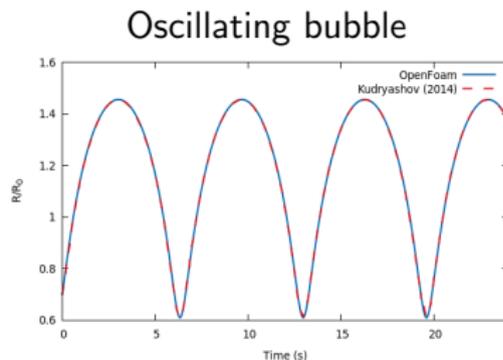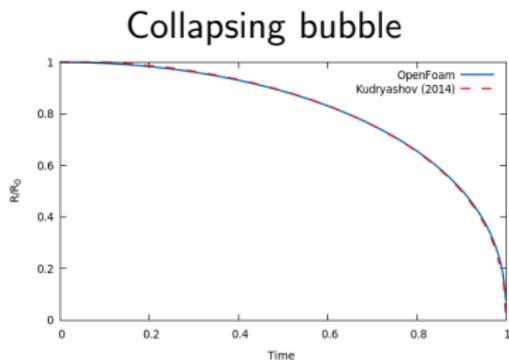- ▶ local thermodynamic equilibrium at bubble-melt interface

Rayleigh-Plesset equation

(Proussevitch and Sahagian 1998)

$$\underbrace{\rho R \frac{d^2 R}{dt^2} \left(1 - \frac{R}{S}\right) + \rho \left(\frac{dR}{dt}\right)^2 \left(\frac{3}{2} - \frac{2R}{S} + \frac{R^4}{2S^4}\right)}_{inertial\ terms}$$

$$= \underbrace{p_G - p_L}_{pressure\ term} + \underbrace{4 \frac{dR}{dt} R^2 \left(-3 \int_R^S \frac{\mu(r)}{r^4} dr\right)}_{viscous\ term} - \underbrace{\frac{2\sigma}{R}}_{surface\ tension\ term} \quad .$$

For a polytropic gas

$$p_G = p_{G,0} \left(\frac{R_0}{R} \right)^{3k}$$

Collapsing bubble

Oscillating bubble

Numerical model benchmark with analytical solutions from Kudryashov and Sinelshchicov 2014

For a perfect gas ($T = const.$)

$$p_G = \tilde{R} T / \rho_G$$

Gas mass conservation

$$\frac{d}{dt}\left(R^3 \rho_G\right) = \underbrace{3R^2 \rho_l \sum_{i=1}^{N} D_i \left[\frac{\partial C_i}{\partial r}\right]_{r=R}}_{\text{diffusive flux}}$$

Species mass conservation

$$\frac{d}{dt}\left(R^3 \rho_G C_i\right) = \underbrace{3R^2 \rho_l D_i \left[\frac{\partial C_i}{\partial r}\right]_{r=R}}_{\text{specie diffusive flux}}$$
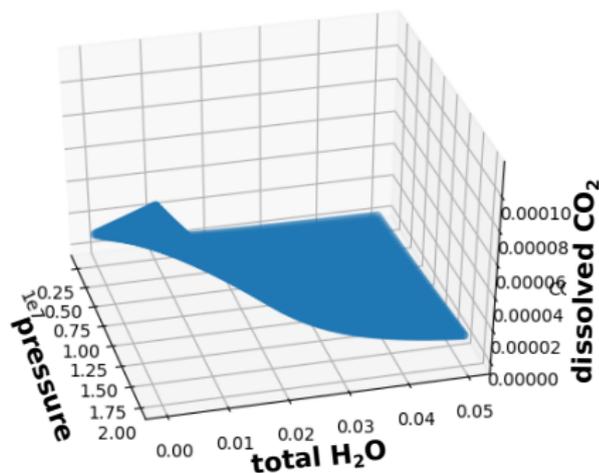
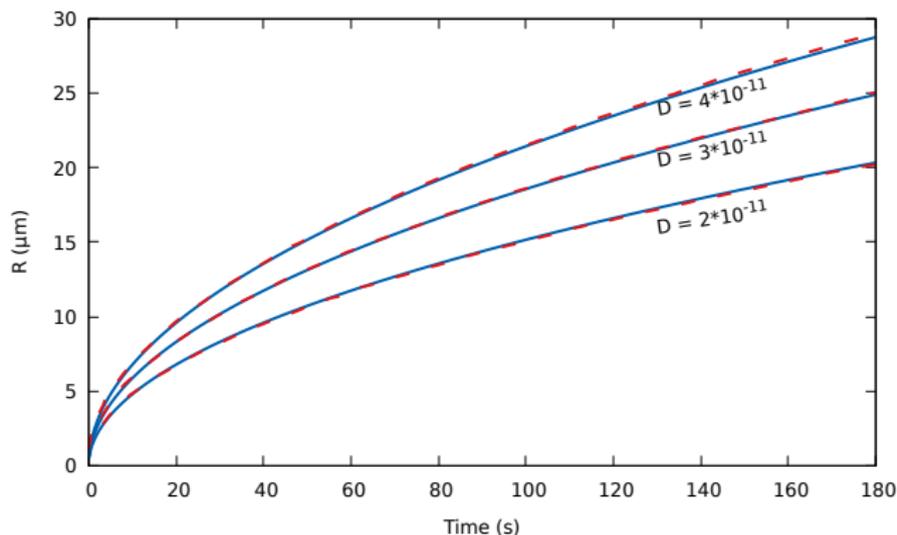**Advection-diffusion for volatile concentrations** C_i

$$\frac{\partial C_i}{\partial t} + u_r \frac{\partial C_i}{\partial r} = \frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 D_i \frac{\partial C_i}{\partial r} \right)$$

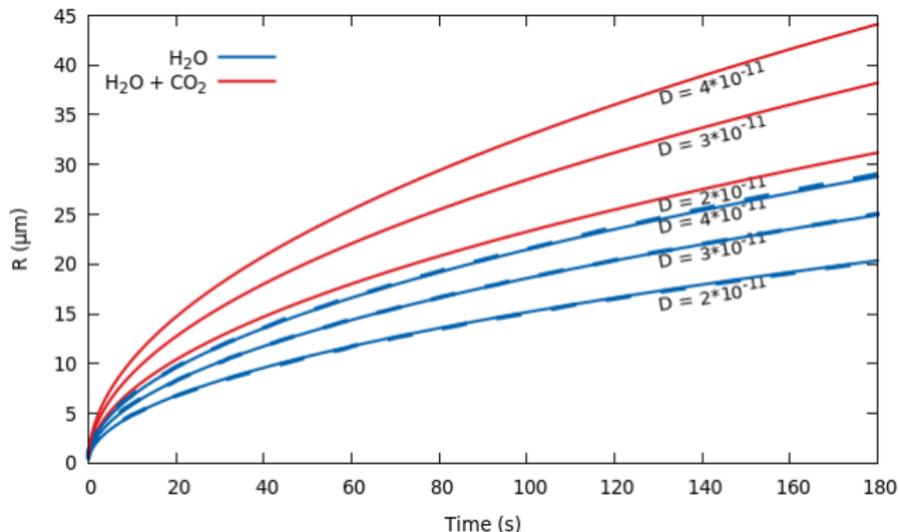**Condition at the gas-liquid interface** (thermodynamic equilibrium):

$$C_i(r = R, t > 0) = C_{i,sat}(p_G)$$

Equilibrium concentrations $C_{i,sat}(p_G)$ **at the interface** for H_2O and CO_2 are calculated with SOLWCAD (Papale, Moretti, and Barbato 2006)

Comparison with the numerical solutions of Lyakhovsky, Hurwitz, and Navon 1996 for different values of the diffusion coefficient D

By adding 1 wt% of CO_2 the bubble radius increases by $\approx 50\%$ and **the gas volume fraction triplicates**

Implementation in a modular framework

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The model can easily be combined with a variety of fluid dynamics solvers and several constitutive equations and sub-grid models within the MagmaFOAM environment (Brogi et al. 2018)

Eulerian-Lagrangian simulations

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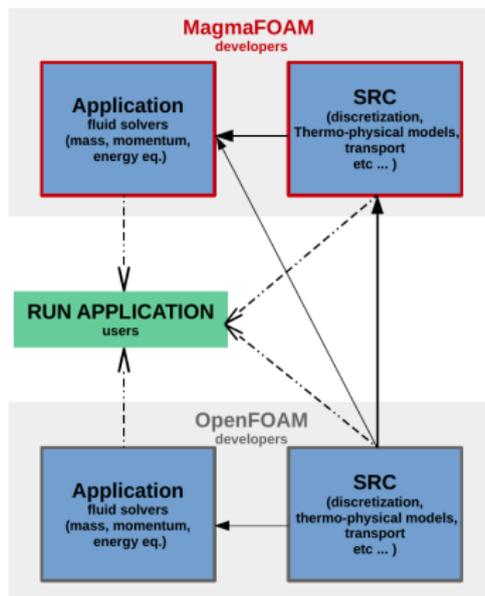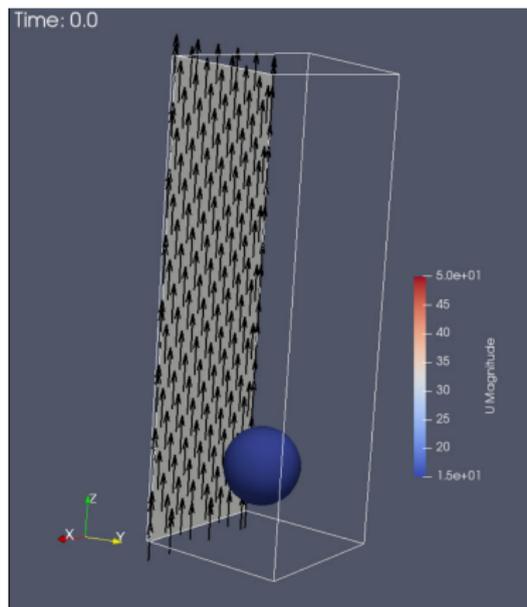
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