Bayesian Framework for Earthquake/Aftershock Forecasting and Testing

Robert Shcherbakov (rshcherb@uwo.ca) @roshcher

Department of Earth Sciences and Department of Physics and Astronomy, Western University, London, Ontario, Canada

vEGU21 – General Assembly – NH4.4 – 30 April 2021 (on-line)

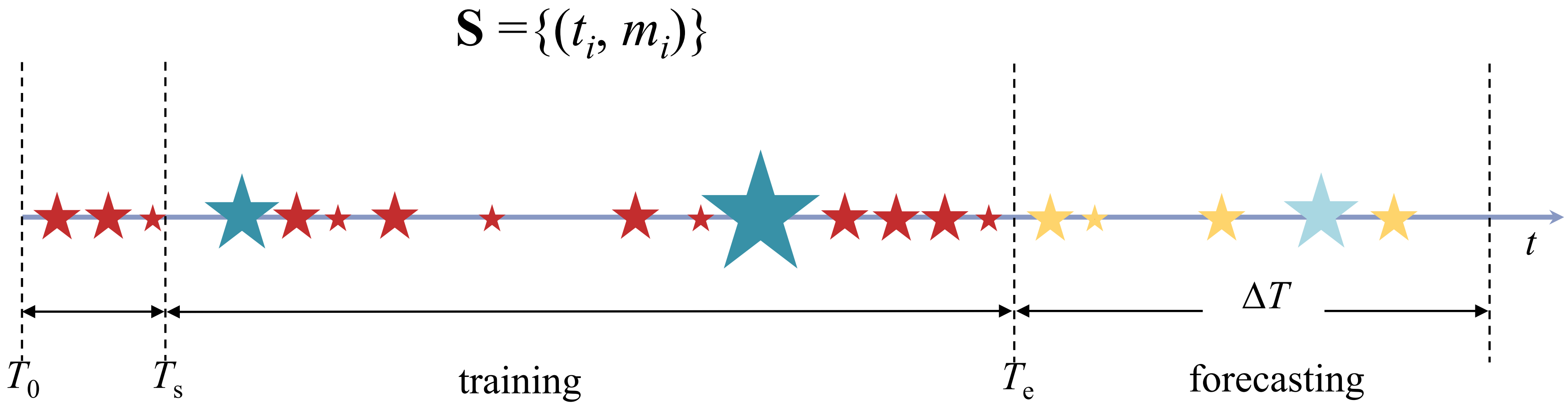
Western
UNIVERSITY · CANADA

Outline

- Research problem:
  - To compute the probabilities for the occurrence of the *largest expected events* (earthquakes) during a future time interval.
- Two approaches:
  - Extreme value distribution;
  - Bayesian predictive distribution.
- Application to the 2019 Ridgecrest, California, earthquake sequence.

Earthquake/Aftershock Forecasting

- Use the information from the early events in the interval $[T_0, T_e]$ to forecast the magnitudes of the *largest expected event* in $[T_e, T_e + \Delta T]$:

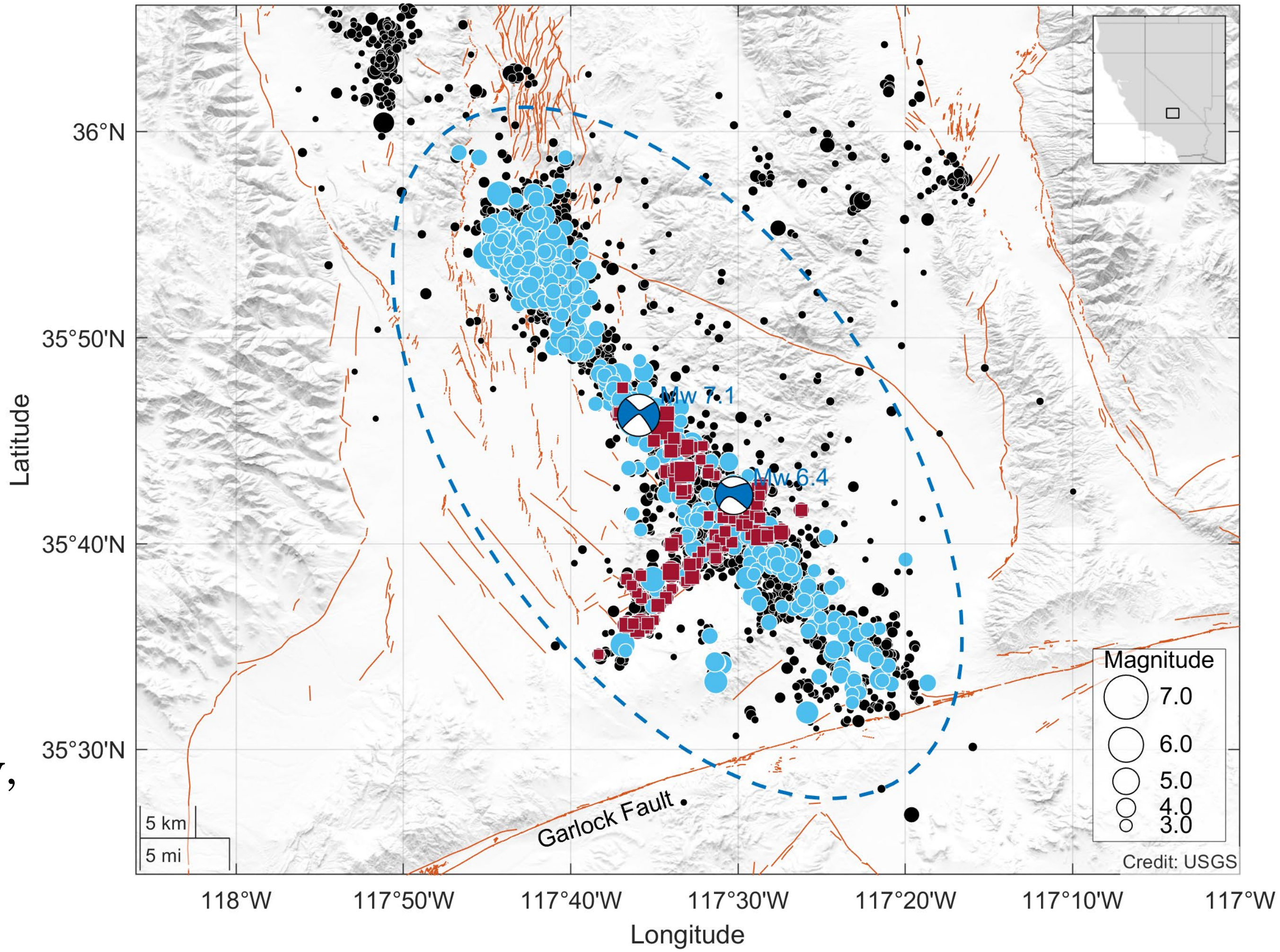


Two Approaches for Forecasting

- Using the extreme value theory and point estimates for the earthquake rate and the frequency-magnitude statistics (Reasenberg and Jones, 1989).
- Using the Bayesian approach combined with the Markov Chain Monte Carlo sampling of the posterior distribution to compute the Bayesian predictive distribution for the magnitude of the largest expected earthquake (Shcherbakov, et al., Nat. Comm., 2019; Shcherbakov, JGR, 2021).

The July 2019 Ridgecrest, California, Earthquake Sequence

- Earthquake epicentres.
- The focal mechanisms of the M7.1 (06/07/20) mainshock and M6.4 (04/07/20) foreshock (Shcherbakov, JGR, 2021).



Earthquake Rate Models

- The Omori-Utsu model (Utsu, 1961): $\lambda_\omega(t) = \dfrac{K_o}{\left(t + c_o\right)^{p_o}}$
- The compound Omori model (Ogata, 1984):

$$\lambda_\omega(t) = \frac{K_1}{\left(t + c_1\right)^{p_1}} + H(t - \tau_m)\frac{K_2}{\left(t - \tau_m + c_2\right)^{p_2}}$$

- The Epidemic Type Aftershock Sequence (ETAS) model (Ogata, 1988):

$$\lambda_\omega(t \mid H_t) = \mu + K \sum_{i=1:t_i<t}^{N} \frac{e^{\alpha(m_i - m_0)}}{\left[\left(t - t_i\right)/c + 1\right]^p}$$

Earthquake Magnitude Distribution

- A left-truncated exponential distribution for the earthquake magnitudes:

$$f_\theta(m) = \beta \exp\left[-\beta(m - m_c)\right]$$

$$F_\theta(m) = 1 - \exp\left[-\beta(m - m_c)\right]$$

- Gutenberg-Richter scaling and b-value:

$$\log_{10}\left(N_{\geq m}\right) = a - bm \qquad b = \frac{\beta}{\ln(10)}$$

Extreme value distribution

- Consider a non-homogeneous Poisson process.
- The extreme value distribution for the largest expected earthquake in the interval $[T_e, T_e + \Delta T]$ to be greater than m (Reasenberg and Jones, 1989; Shcherbakov et al., 2018, 2019):

$$P_{EV}(m_{ex} > m \mid \theta, \omega, \Delta T) = 1 - \exp\left\{-\Lambda_\omega(\Delta T)\exp\left[-\beta(m - m_0)\right]\right\}$$

where $\Lambda_\omega(\Delta T) = \int_{T_e}^{T_e+\Delta T} \lambda_\omega(t)\,dt$ is the productivity.

Bayesian Predictive Distribution

- The Bayesian predictive distribution for the largest expected earthquake in the interval $[T_e, T_e + \Delta T]$ to be greater than m (Shcherbakov, et al., Nature Comm., 2019):

$$P_B(m_{ex} > m \mid \mathbf{S}, \Delta T) = \int_{\Omega}\int_{\Theta} P_{EV}(m_{ex} > m \mid \theta, \omega, \Delta T)\, p(\theta, \omega \mid \mathbf{S})\, d\theta d\omega$$

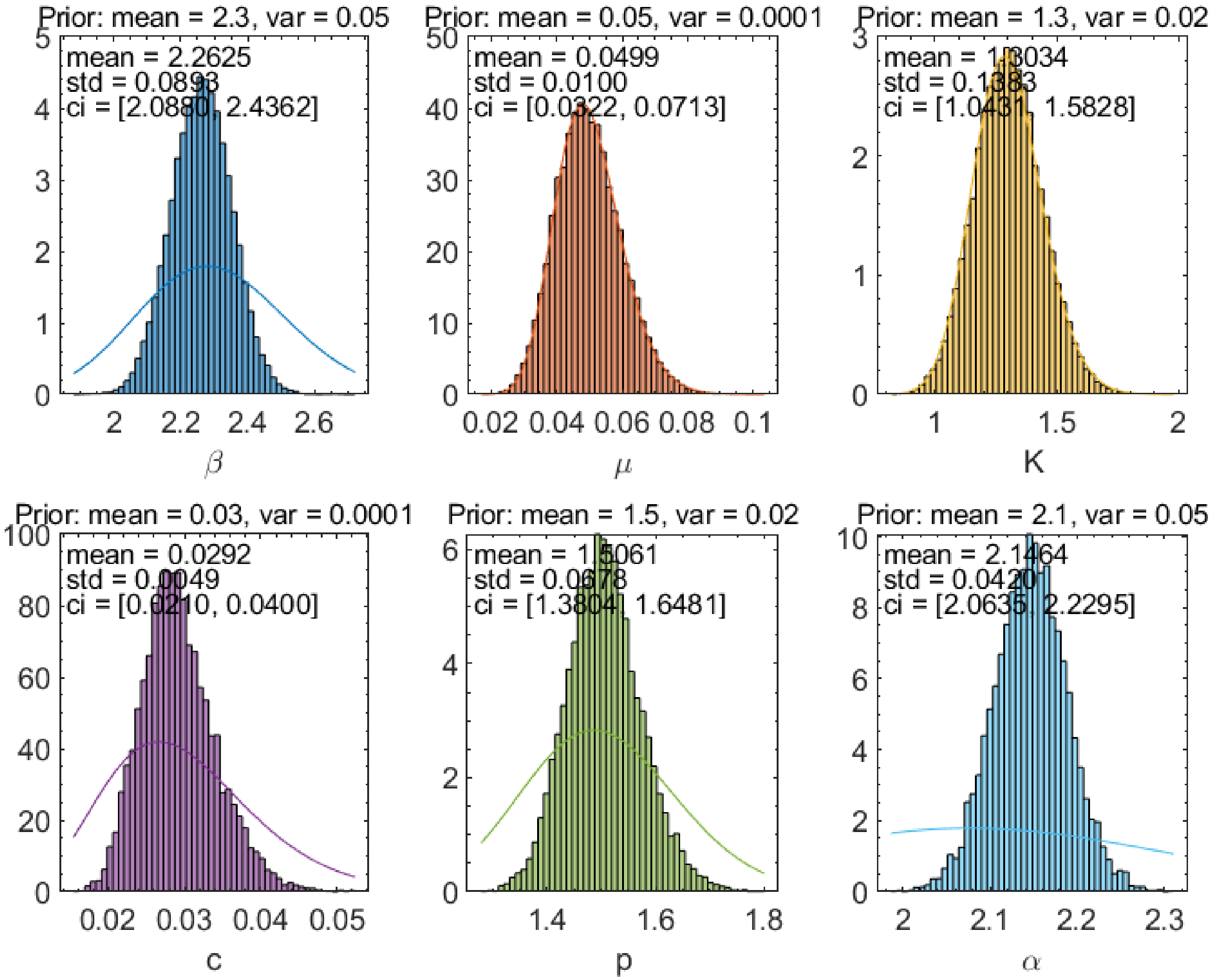
where $\mathbf{S} = \{m_i(t_i)\}$ is past seismicity during $[T_0, T_e]$.

$P_{EV}(m_{ex} > m)$ – is the extreme value distribution.

$p(\theta, \omega \mid \mathbf{S})$ – is the posterior distribution for the model parameters: $\{\theta, \omega\}$.

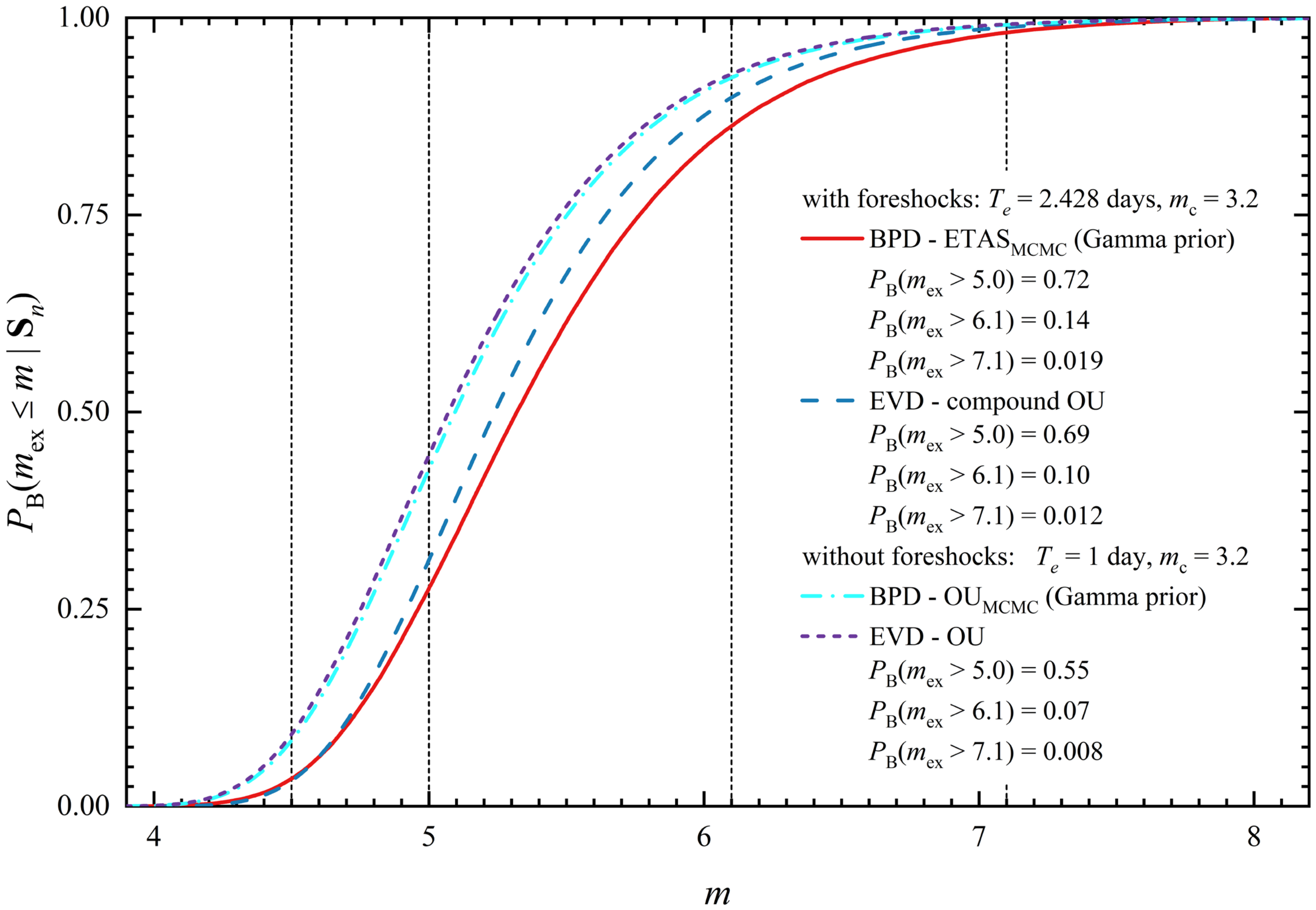
The 2019 Ridgecrest Sequence

- MCMC sampling of the posterior distribution (Shcherbakov, JGR, 2021).
- Posterior distribution of the ETAS parameters.



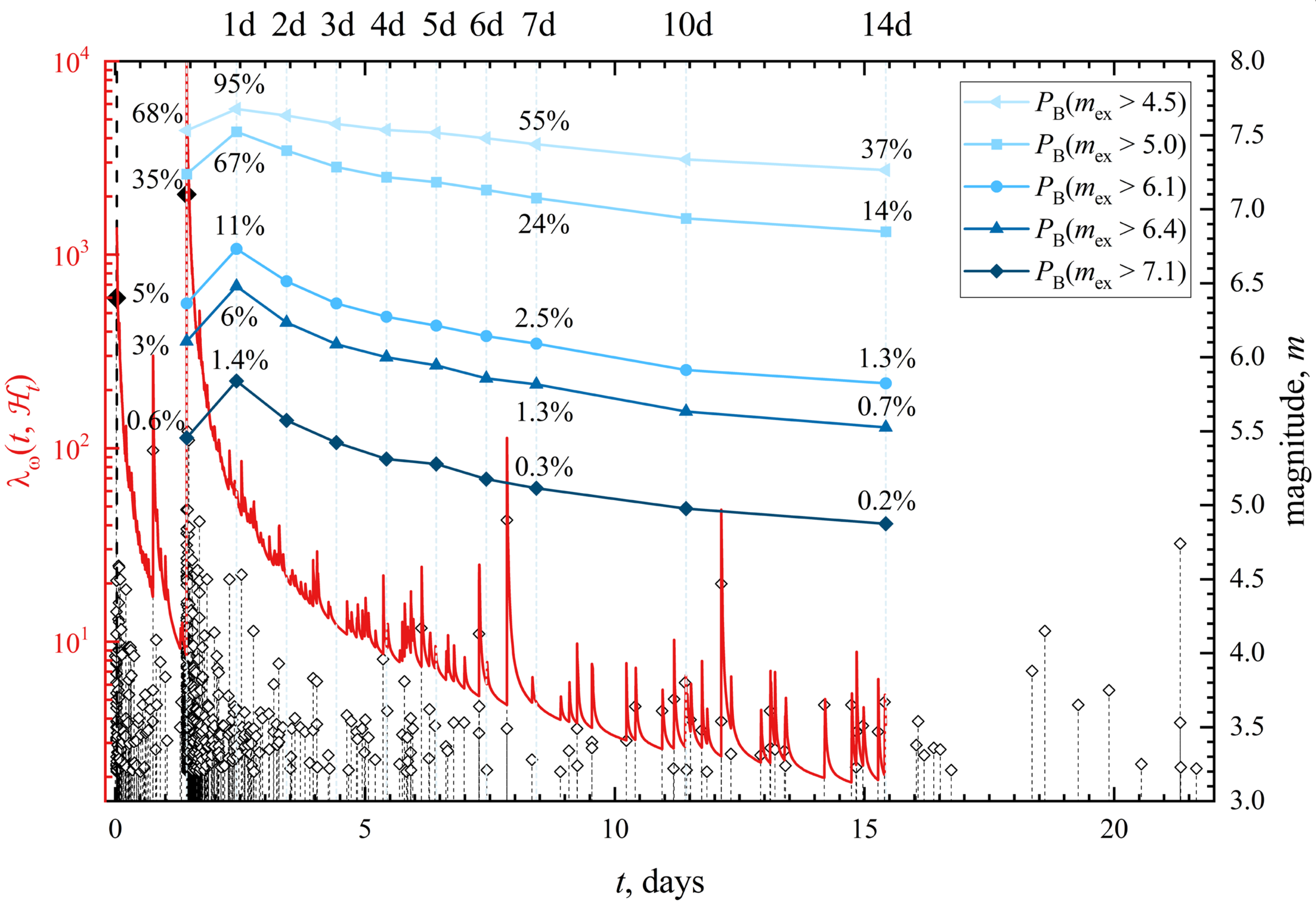
The 2019 Ridgecrest Sequence

- The extreme value and Bayesian predictive distributions (Shcherbakov, JGR, 2021):
- $\Delta T = 7$ days.



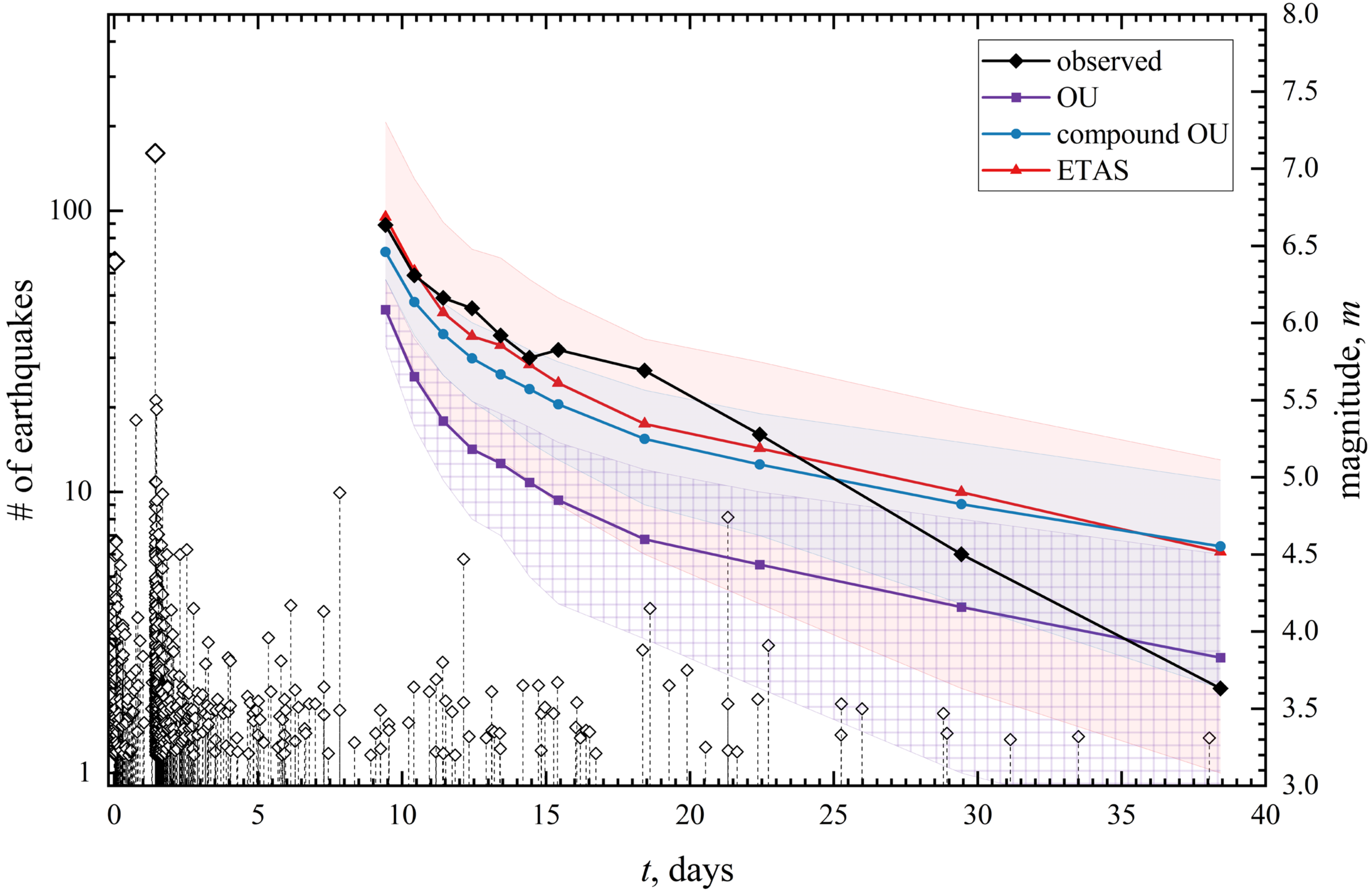
The 2019 Ridgecrest Sequence

- Probabilities for the largest expected earthquakes to be above $m \geq 4.5, 5.0, 6.1, 6.4, 7.1$ (Shcherbakov, JGR, 2021):



Forecast Verification: (N-, M-, R-, T-tests, Bayesian p-value)

- The observed and forecasted numbers of earthquakes (N-test).
- $\Delta T = 7$ days.



Conclusions

- An approach based on the Bayesian methods and extreme value theory is implemented to constrain the magnitudes of the largest expected earthquakes to occur during a specified future time interval.
- The Bayesian approach allows to incorporate the model uncertainties into the computation of probabilities.
- The 2019 Ridgecrest earthquake sequences was analyzed to illustrate the method (Shcherbakov, JGR, 2021).

References

Shcherbakov R. (2021). Statistics and forecasting of aftershocks during the 2019 Ridgecrest, California, earthquake sequence, *J. Geophys. Res.* 126, doi: 10.1029/2020JB020887.

Shcherbakov R., J. Zhuang, G. Zöller, Y. Ogata (2019). Forecasting the magnitude of the largest expected earthquake, *Nature Communications* 10, Art. No. 4051, doi: 10.1038/s41467-019-11958-4.

Shcherbakov R., J. Zhuang, Y. Ogata (2018). Constraining the magnitude of the largest event in a foreshock-mainshock-aftershock sequence, *Geophys. J. Int.* 212, 1-13, doi: 10.1093/gji/ggx407.