Cross-diffusion Triggered Wave Instabilities

M.M. Hu\textsuperscript{1}, Q.P. Sun\textsuperscript{1}, C. Schrank\textsuperscript{2}, K. Regenauer-Lieb\textsuperscript{3}
\textsuperscript{1}School of Civil Engineering, The University of Hong Kong, Hong Kong
\textsuperscript{2}Science and Engineering Faculty, Queensland University of Technology, Australia
\textsuperscript{3}School of Minerals and Energy Resources Engineering, UNSW Sydney, Australia

“All is flux” -- Heraclitus

• Consider a concurrent scenario of primary and secondary consolidation processes in a porous viscoplastic medium\textsuperscript{1}:

<table>
<thead>
<tr>
<th>1-D Hydro-Poro-Mechanics based on thermodynamics</th>
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<tbody>
<tr>
<td>Therodynamic Force (1-D)</td>
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<td>--------------------------</td>
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<tr>
<td>$F_H = \frac{\partial p_f}{\partial x}$</td>
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<tr>
<td>$F_M = \frac{\partial p_s}{\partial x}$</td>
</tr>
</tbody>
</table>

• How to manifest the possible time-dependent interactions “around” the solid-fluid interface?

- Cross-diffusion in a complex system: the phenomenon when a generalized thermodynamic force induces a generalized thermodynamic flux of another kind.

\[
\frac{\partial p_s}{\partial t} = D_M \frac{\partial^2 p_s}{\partial x^2} + d_H \frac{\partial^2 p_f}{\partial x^2} + R_1,
\]

\[
\frac{\partial p_f}{\partial t} = d_M \frac{\partial^2 p_s}{\partial x^2} + D_H \frac{\partial^2 p_f}{\partial x^2} + R_2,
\]

Mixture theory + Inter-constituent mass transfer + Relaxing the adiabatic constraints on the reaction part of the system