

Higher-Order Ionospheric Corrections (HOIC) derived from realistic electron density fields for precise GNSS applications

F. Zus (1) and J. Wickert (1,2)

(1) GFZ Potsdam, Germany

(2) Technische Universität Berlin, Germany

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Motivation:

Provide **accurate** and **easy-to-use** HOICs for precise GNSS applications.

Content:

- The model for the HOICs
- The parameterized version for the HOICs
- The impact of the (parameterized) HOICs in Precise Point Positioning
- Summary

Model for the HOICs

We ignore the troposphere in this derivation. The observation equation reads as:

$$L_i = r - I_i + c t + \dots$$

L_i ... observed phase

r ... geometric distance

I_i ... ionospheric delay

t ... station clock error

c ... vacuum speed of light

i ... frequency index

$$O_i = r - I_i$$

O_i ... optical path length

The observation equation can be written as:

$$L_i = O_i + c t + \dots$$

The optical path length is given by ('quasi isotropic approximation'):

$$O_i = \int n_i ds_i$$

n_i ... refractive index

s_i ... arc-length of the ray-path

$$n_i = 1 - \frac{w}{f_i^2} N - \frac{q}{f_i^3} B \cos(\varphi) N$$

w & q ... constants

f_i ... frequency

N ... Electron density field

B ... Earth's magnetic field

φ ... angle between Earth's magnetic field vector and ray-tangent vector

Therefore

$$O_i = \int \left(1 - \frac{w}{f_i^2} N - \frac{q}{f_i^3} B \cos(\varphi) N \right) ds_i$$

and

$$I_i = r - \int \left(1 - \frac{w}{f_i^2} N - \frac{q}{f_i^3} B \cos(\varphi) N \right) ds_i$$

The standard linear combination reads as:

$$L = \frac{f_1^2}{f_1^2 - f_2^2} L_1 - \frac{f_2^2}{f_1^2 - f_2^2} L_2 = r - \delta I + c t + \dots$$

The higher order ionospheric residual can be written as

$$\delta I = \frac{f_1^2}{f_1^2 - f_2^2} I_1 - \frac{f_2^2}{f_1^2 - f_2^2} I_2 = \delta I_1 + \delta I_2 + \delta I_3$$

where

$$\delta I_1 = \frac{q}{f_1^2 - f_2^2} \left(\frac{1}{f_1} \int B \cos(\varphi) N ds_1 - \frac{1}{f_2} \int B \cos(\varphi) N ds_2 \right)$$

$$\delta I_2 = \frac{w}{f_1^2 - f_2^2} \left(\int N ds_1 - \int N ds_2 \right)$$

$$\delta I_3 = r - \left(\frac{f_1^2}{f_1^2 - f_2^2} \int ds_1 - \frac{f_2^2}{f_1^2 - f_2^2} \int ds_2 \right)$$

Ray-paths follow from Fermat's principle:

$$\delta O_i = 0$$

The equation for the ray-path $[x, y(x)]$ follows from calculus of variation ('no out of plane bending'):

$$\frac{d^2 y}{dx^2} = \frac{1}{\xi_i} \left[\frac{\partial \xi_i}{\partial y} - \frac{\partial \xi_i}{\partial x} \frac{dy}{dx} \right] \cdot \left[1 + \left(\frac{dy}{dx} \right)^2 \right] + \frac{q}{f_i^3 \xi_i} \left[\frac{\partial (N \cdot B_x)}{\partial y} - \frac{\partial (N \cdot B_y)}{\partial x} \right] \left[1 + \left(\frac{dy}{dx} \right)^2 \right]^{3/2}$$

$$\xi_i = 1 - \frac{w}{f_i^2} N$$

B_x & B_y ... x & y coordinate of the projection of Earth's magnetic field vector into the plane of constant azimuth.

The curvature term due to Earth's magnetic field is negligible small and can be discarded:

$$\frac{d^2 y}{dx^2} = \frac{1}{\xi_i} \left[\frac{\partial \xi_i}{\partial y} - \frac{\partial \xi_i}{\partial x} \frac{dy}{dx} \right] \cdot \left[1 + \left(\frac{dy}{dx} \right)^2 \right]$$

We distinguish two cases:

HOIC: Ray-bending is taken into account and the electron density field is at it is.
In this case the ionospheric residual reads as

$$\delta I = \delta I_1 + \delta I_2 + \delta I_3$$

$$\delta I_1 = \frac{q}{f_1^2 - f_2^2} \left(\frac{1}{f_1} \int B \cos(\varphi) N ds_1 - \frac{1}{f_2} \int B \cos(\varphi) N ds_2 \right)$$

$$\delta I_2 = \frac{w}{f_1^2 - f_2^2} \left(\int N ds_1 - \int N ds_2 \right)$$

$$\delta I_3 = r - \left(\frac{f_1^2}{f_1^2 - f_2^2} \int ds_1 - \frac{f_2^2}{f_1^2 - f_2^2} \int ds_2 \right)$$

SOIC: Ray-bending is not taken into account and all electrons are concentrated in a single thin shell at an altitude of $A = 450$ km. In this case, for some elevation angle e and azimuth angle a , the ionospheric residual can be written as

$$\delta I(e, a) = \frac{-q}{(f_1 f_2)(f_1 + f_2)} B(e, a) \cos(\varphi(e, a)) M(e) V(e, a)$$

V ... Vertical Total Electron Content

M ... Single layer model mapping function

$$M(e) = \frac{1}{\sqrt{1 - \left(\frac{R+H}{R+A} \right)^2 \cos^2(e)}}$$

R ... radius of the Earth

H ... altitude of the station

All quantities in the expression above are evaluated at the point where the straight line between the station and the satellite hits the altitude 450 km.

Parameterized version for HOICs

Direct application of ray-traced HOICs is impractical.

Parameterized version: For any location near Earth's surface (grid) a set of HOICs is computed (various elevation and azimuth angles) and the coefficients of a polynomial expansion are stored:

$$\delta l(e,a) = \sum A_m Z_m(e,a)$$

e ... elevation angle

a ... azimuth angle

Z ... Zernike polynomials

A ... coefficients of the polynomial expansion

δl is computed for the GPS frequencies f_1 and f_2 .

The conversion to other frequencies, say f_3 and f_4 , is given by:

$$\delta l_{34} \sim (f_1 f_2)^2 / (f_3 f_4)^2 \delta l$$

Zernike polynomials:

$z_0 = 1;$	Piston or Bias
$z_1 = \rho \cos[\theta];$	Tilt x
$z_2 = \rho \sin[\theta];$	Tilt y
$z_3 = -1 + 2 \rho^2;$	Power
$z_4 = \rho^2 \cos[2 \theta];$	Astig x
$z_5 = \rho^2 \sin[2 \theta];$	Astig y
$z_6 = \rho (-2 + 3 \rho^2) \cos[\theta];$	Coma x
$z_7 = \rho (-2 + 3 \rho^2) \sin[\theta];$	Coma y
$z_8 = 1 - 6 \rho^2 + 6 \rho^4;$	Primary Spherical
$z_9 = \rho^3 \cos[3 \theta];$	Trefoil x
$z_{10} = \rho^3 \sin[3 \theta];$	Trefoil y
$z_{11} = \rho^2 (-3 + 4 \rho^2) \cos[2 \theta];$	Secondary Astigmatism x
$z_{12} = \rho^2 (-3 + 4 \rho^2) \sin[2 \theta];$	Secondary Astigmatism y
$z_{13} = \rho (3 - 12 \rho^2 + 10 \rho^4) \cos[\theta];$	Secondary Coma x
$z_{14} = \rho (3 - 12 \rho^2 + 10 \rho^4) \sin[\theta];$	Secondary Coma y
$z_{15} = -1 + 12 \rho^2 - 30 \rho^4 + 20 \rho^6;$	Secondary Spherical
$z_{16} = \rho^4 \cos[4 \theta];$	Tetrafoil x
$z_{17} = \rho^4 \sin[4 \theta];$	Tetrafoil y
$z_{18} = \rho^3 (-4 + 5 \rho^2) \cos[3 \theta];$	Secondary Trefoil x
$z_{19} = \rho^3 (-4 + 5 \rho^2) \sin[3 \theta];$	Secondary Trefoil y
$z_{20} = \rho^2 (6 - 20 \rho^2 + 15 \rho^4) \cos[2 \theta];$	Tertiary Astigmatism x
$z_{21} = \rho^2 (6 - 20 \rho^2 + 15 \rho^4) \sin[2 \theta];$	Tertiary Astigmatism y
$z_{22} = \rho (-4 + 30 \rho^2 - 60 \rho^4 + 35 \rho^6) \cos[\theta];$	Tertiary Coma x
$z_{23} = \rho (-4 + 30 \rho^2 - 60 \rho^4 + 35 \rho^6) \sin[\theta];$	Tertiary Coma y
$z_{24} = 1 - 20 \rho^2 + 90 \rho^4 - 140 \rho^6 + 70 \rho^8;$	Tertiary Spherical
$z_{25} = \rho^5 \cos[5 \theta];$	Pentafoil x
$z_{26} = \rho^5 \sin[5 \theta];$	Pentafoil y
$z_{27} = \rho^4 (-5 + 6 \rho^2) \cos[4 \theta];$	Secondary Tetrafoil x
$z_{28} = \rho^4 (-5 + 6 \rho^2) \sin[4 \theta];$	Secondary Tetrafoil y
$z_{29} = \rho^3 (10 - 30 \rho^2 + 21 \rho^4) \cos[3 \theta];$	Tertiary Trefoil x
$z_{30} = \rho^3 (10 - 30 \rho^2 + 21 \rho^4) \sin[3 \theta];$	Tertiary Trefoil y
$z_{31} = \rho^2 (-10 + 60 \rho^2 - 105 \rho^4 + 56 \rho^6) \cos[2 \theta];$	Quaternary Astigmatism x
$z_{32} = \rho^2 (-10 + 60 \rho^2 - 105 \rho^4 + 56 \rho^6) \sin[2 \theta];$	Quaternary Astigmatism y
$z_{33} = \rho (5 - 60 \rho^2 + 210 \rho^4 - 280 \rho^6 + 126 \rho^8) \cos[\theta];$	Quaternary Coma x
$z_{34} = \rho (5 - 60 \rho^2 + 210 \rho^4 - 280 \rho^6 + 126 \rho^8) \sin[\theta];$	Quaternary Coma y
$z_{35} = -1 + 30 \rho^2 - 210 \rho^4 + 560 \rho^6 - 630 \rho^8 + 252 \rho^{10};$	Quaternary Spherical

with $\rho = 1 - \sin(e)$

Technical details:

Ray-trace software	DNS (Zus et al. 2012, Radio Sci.)
Ray-trace method	2D point-to-point
Electron density field	International Reference Ionosphere (IRI)
Earth's magnetic fields	International Geomagnetic Reference Field (IGRF)
Horizontal resolution	$2.5^{\circ} \times 5^{\circ}$
Temporal resolution	1h
Elevation angles per point	12 (3,5,7,10,15,20,25,30,40,50,70,90°)
Azimuth angles per point	24 ($\Delta=15^{\circ}$)

GFZ-HOIC

Higher-Order Ionospheric Corrections (HOIC) derived from realistic electron density fields

- (1) GFZ raytracing tool (Zus et. al 2017, Radio Sci.)
- (2) IRI-2016 provides the electron density field
- (3) IGRF-12 provides Earth's magnetic field

Parameterized version (polynomial expansion) freely available (1990-2019):

<ftp://ftp.gfz-potsdam.de/pub/home/GNSS/products/gfz-hoic/IFCz/>

Each month consists of one day only. Each day consists of 24 epochs (1h resolution).

GFZ-SOIC

Second-Order Ionospheric Correction (SOIC) is based on simplification:

- (1) Ray-bending is not taken into account
- (2) All electrons are concentrated in a single thin shell at an altitude of 450 km
- (3) IRI-2016 provides the VTEC
- (4) IGRF-12 provides Earth's magnetic field

Parameterized version (polynomial expansion) freely available (1990-2019):

<ftp://ftp.gfz-potsdam.de/pub/home/GNSS/products/gfz-hoic/SFCz/>

Each month consists of one day only. Each day consists of 24 epochs (1h resolution).

Impact of HOIC in Precise Point Positioning

PPP simulation

Least square solution of the linearized observation equation:

...

- 7° elevation cut-off angle
- elevation dependent weighting
- tropospheric parameters & clocks epochwise and station coordinates daily

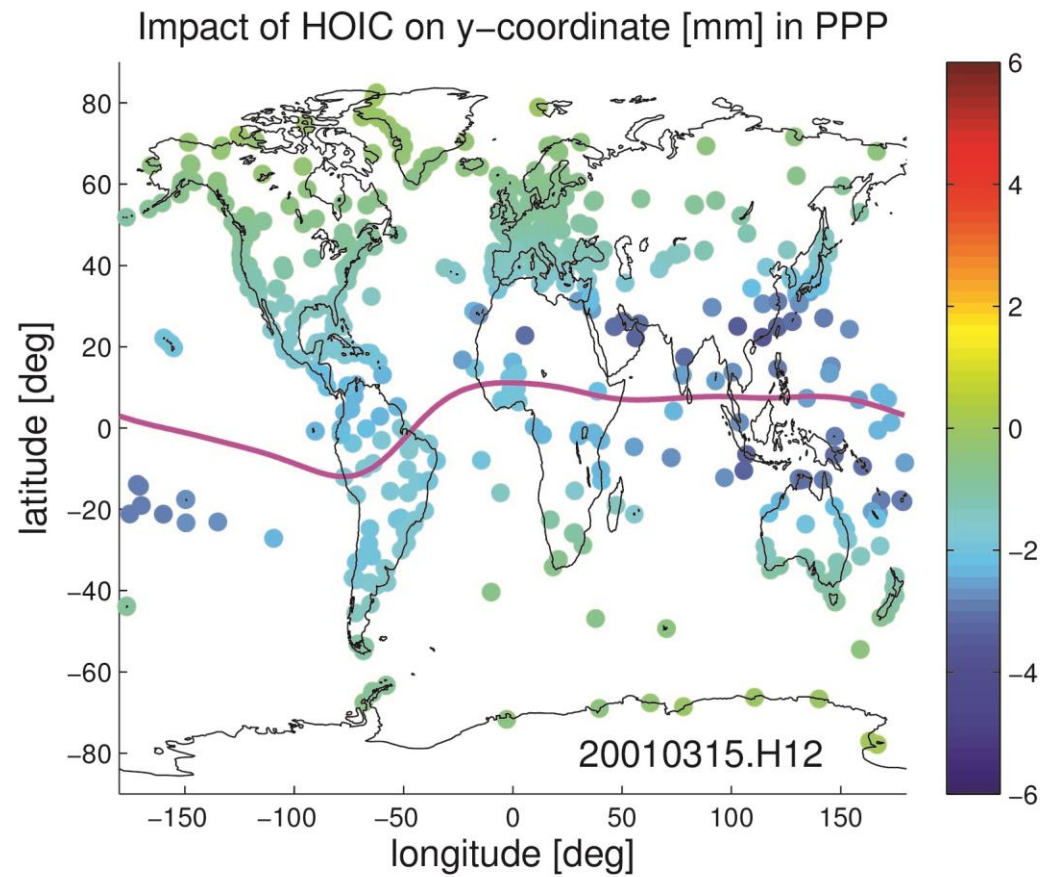
...

Stations: globally distributed TIGA (783) stations

Period: 1990-2019

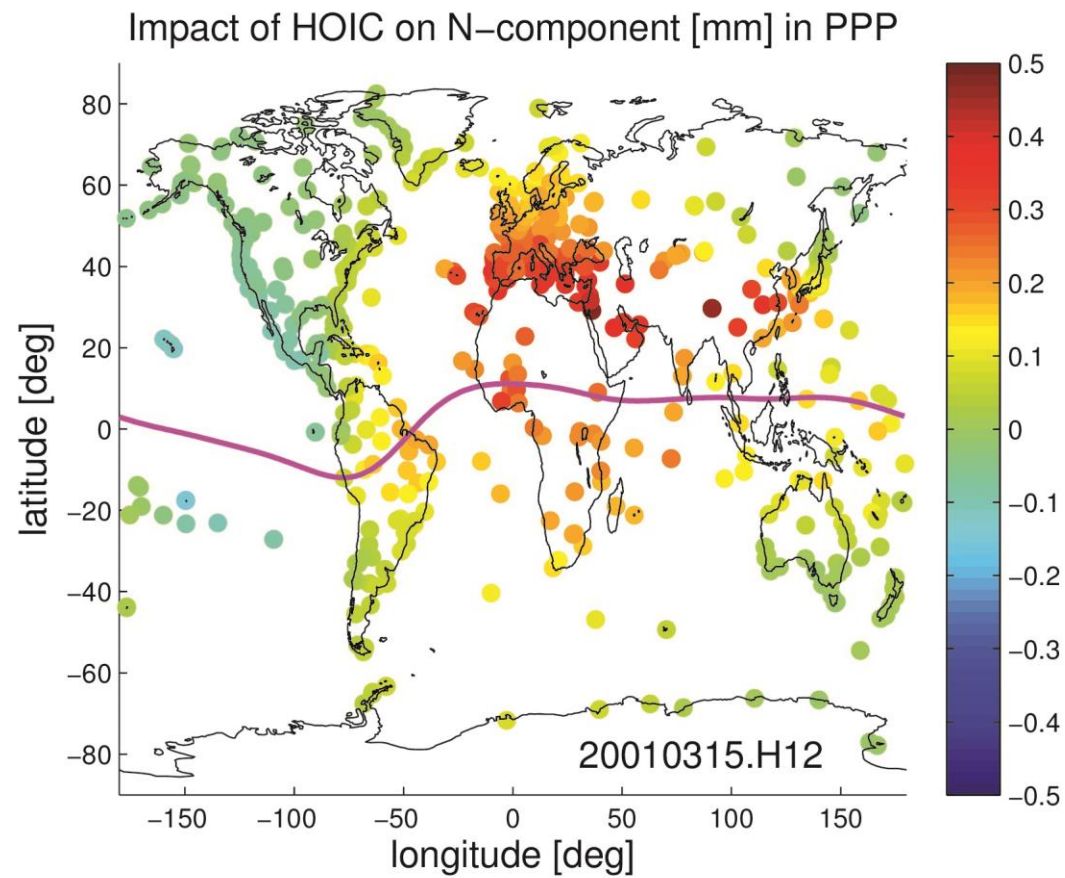
What happens if we apply HOIC in PPP?

The stations appear to move southwards:



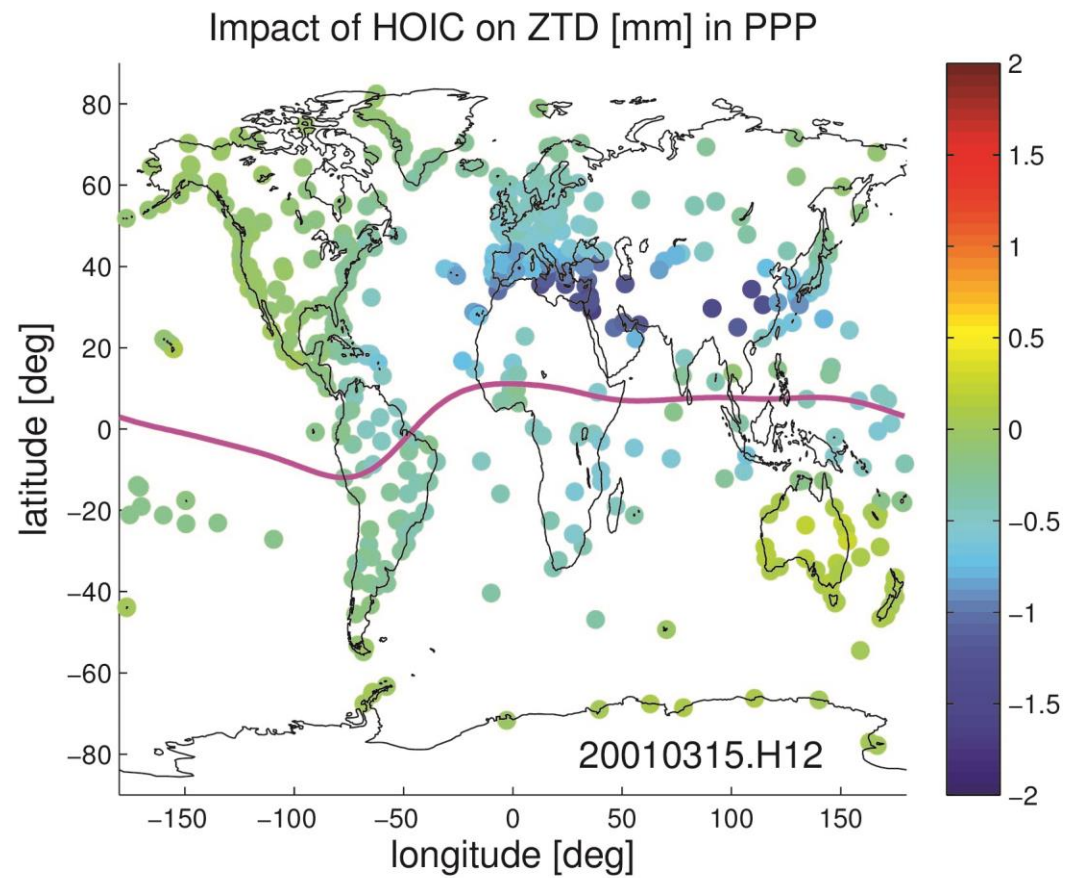
This is known for a very long time (Kedar et al. 2003, GRL).

The tilting of the troposphere is changing:



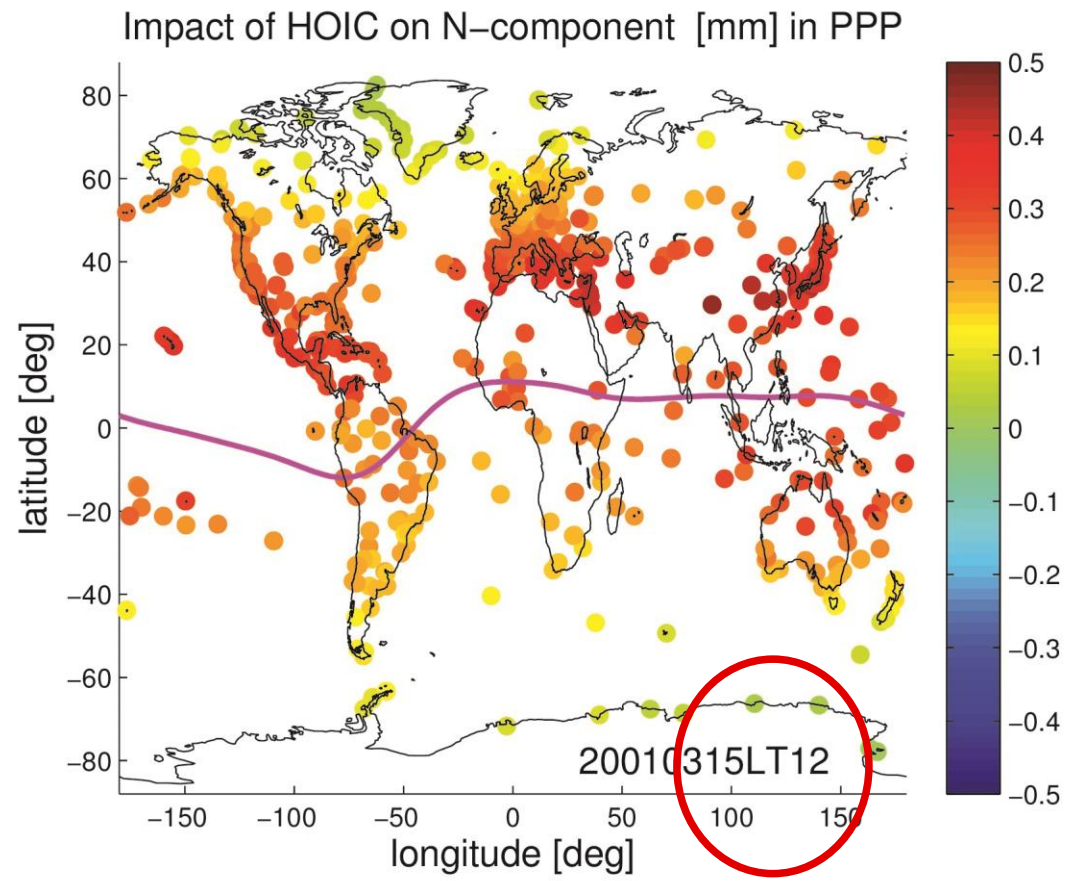
This was first shown by Zus et al. 2017, Radio Sci.

The ZTDs are affected as well:

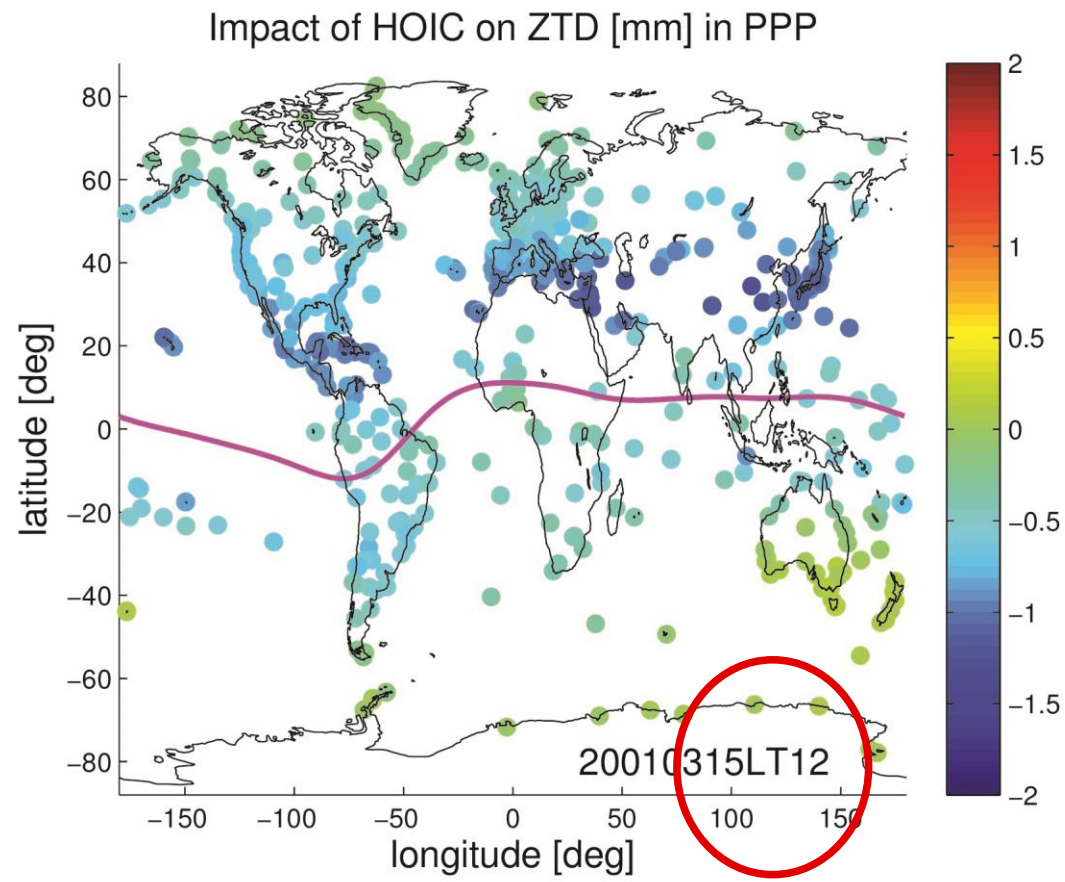


This is consistent with the results by Petrie et al., 2010, Journal of Geodesy

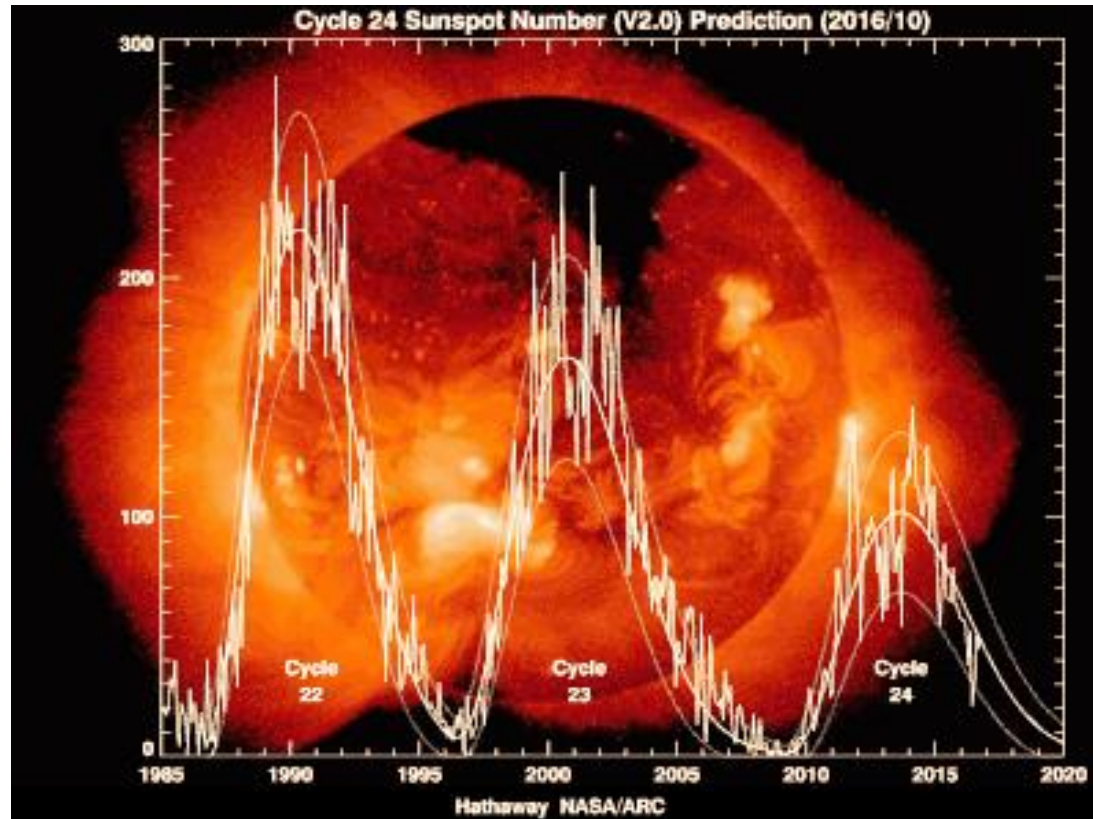
Same Local Time (LT):



Same Local Time (LT):

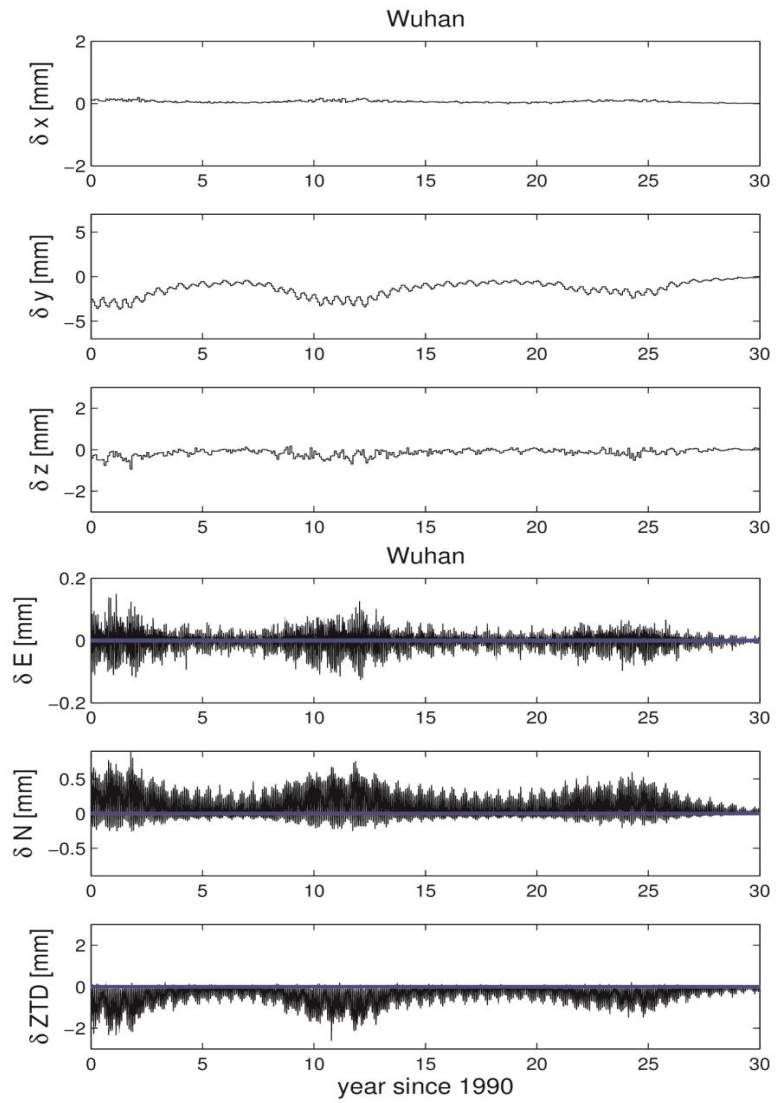
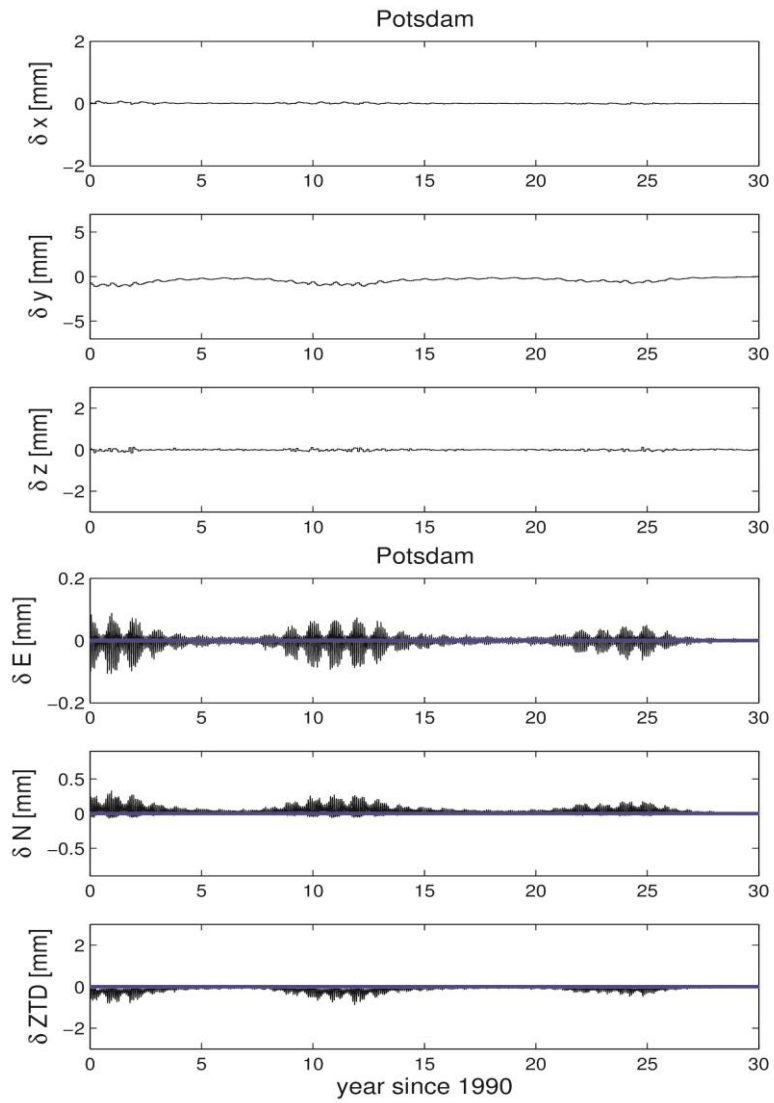


The 11-year solar cycle:



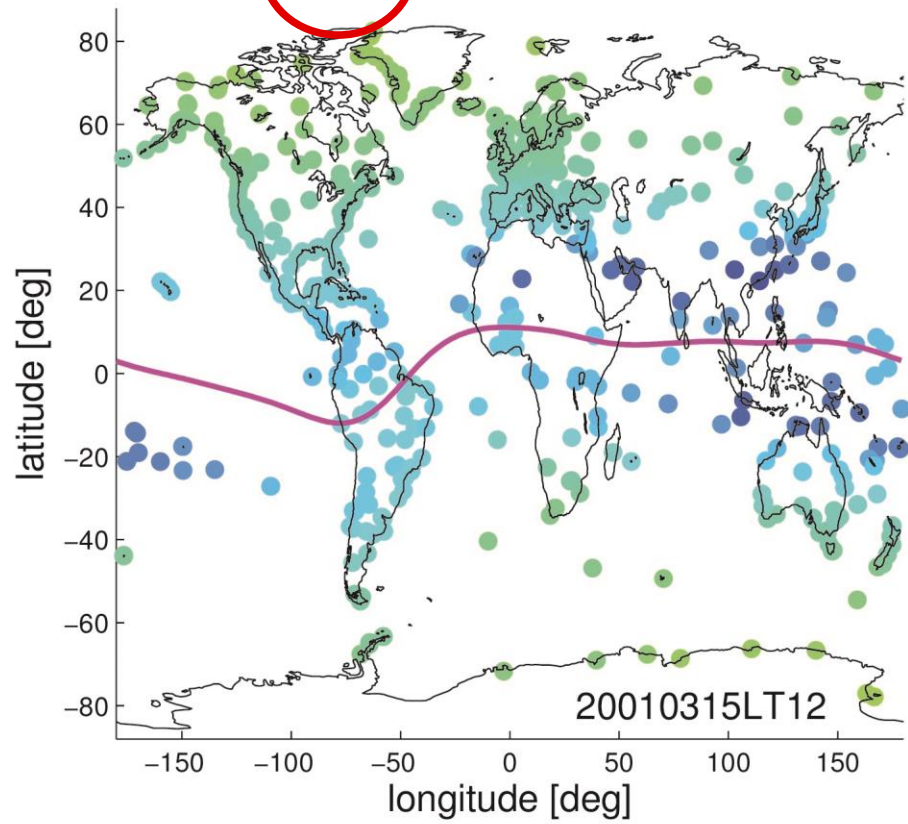
'...There are many, often mutually contradictory predictions, based on different methods, for the forthcoming solar cycle 25, ranging from very weak to moderate magnitude. At present, no definite prediction can be made...' (Wikipedia)

Impact of HOIC for two selected stations:

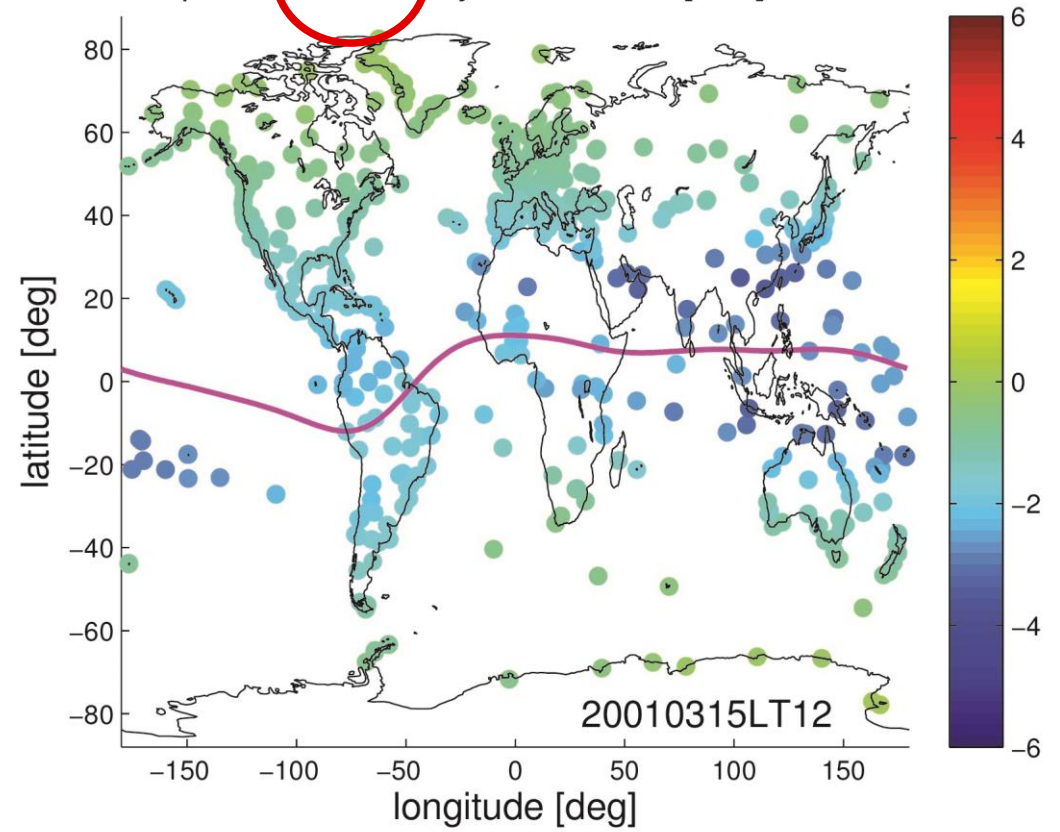


What is the difference between HOIC and SOIC in PPP?

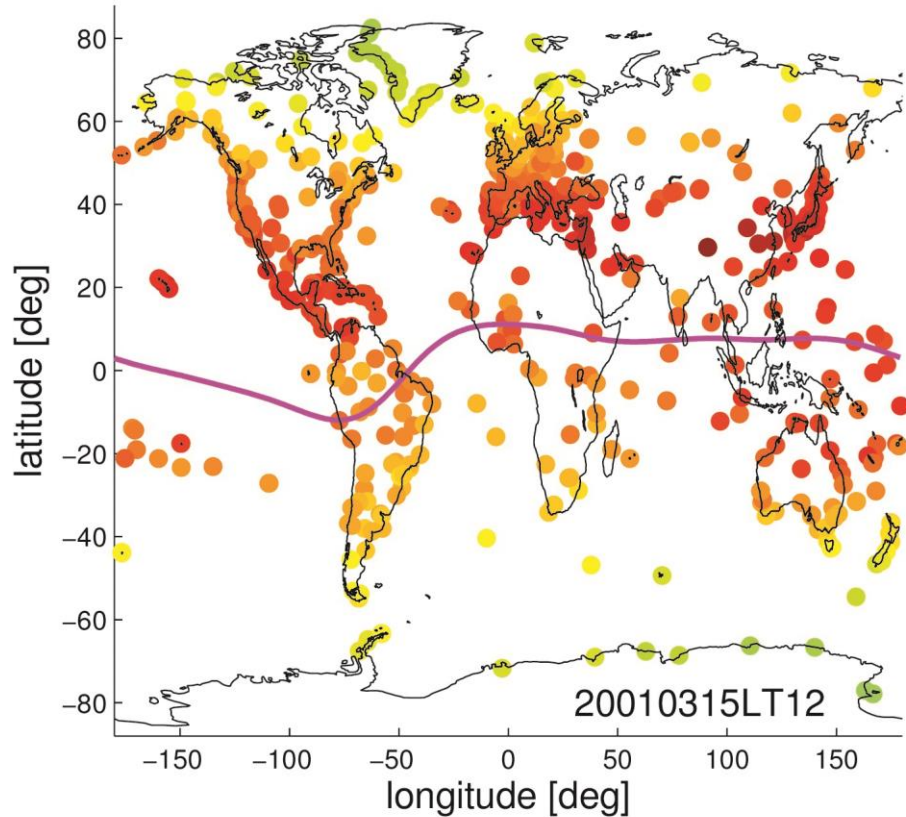
Impact of HOIC on y-coordinate [mm] in PPP



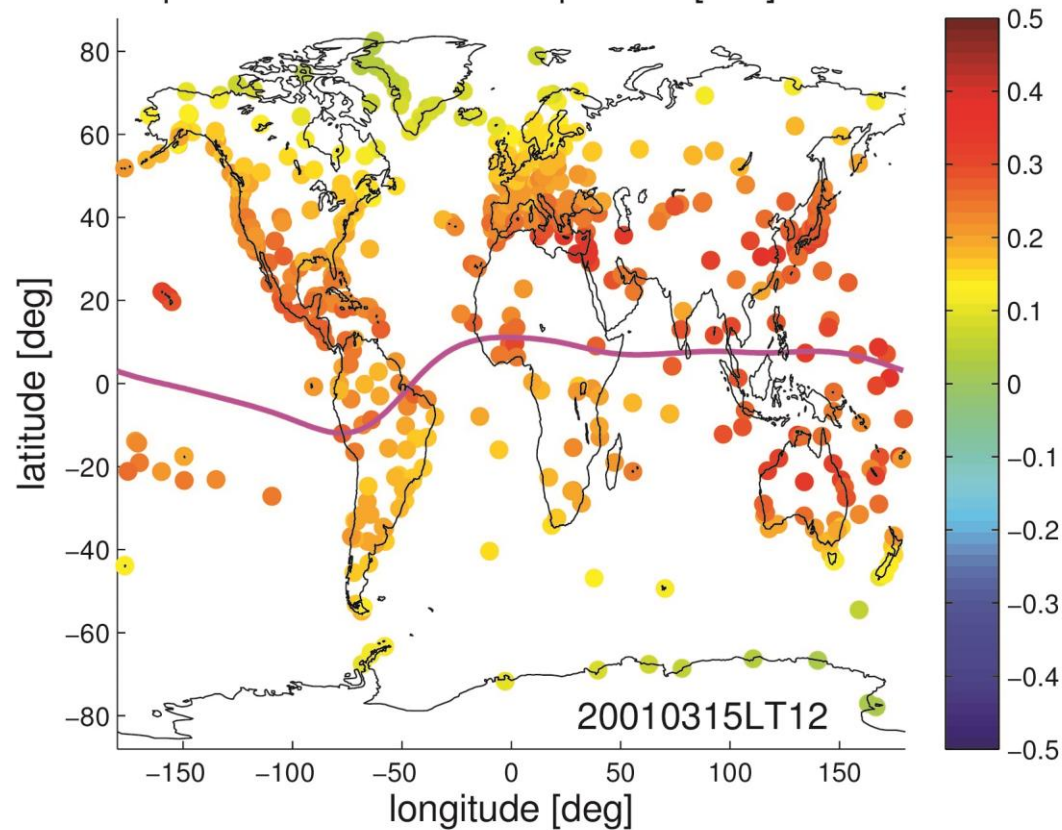
Impact of SOIC on y-coordinate [mm] in PPP



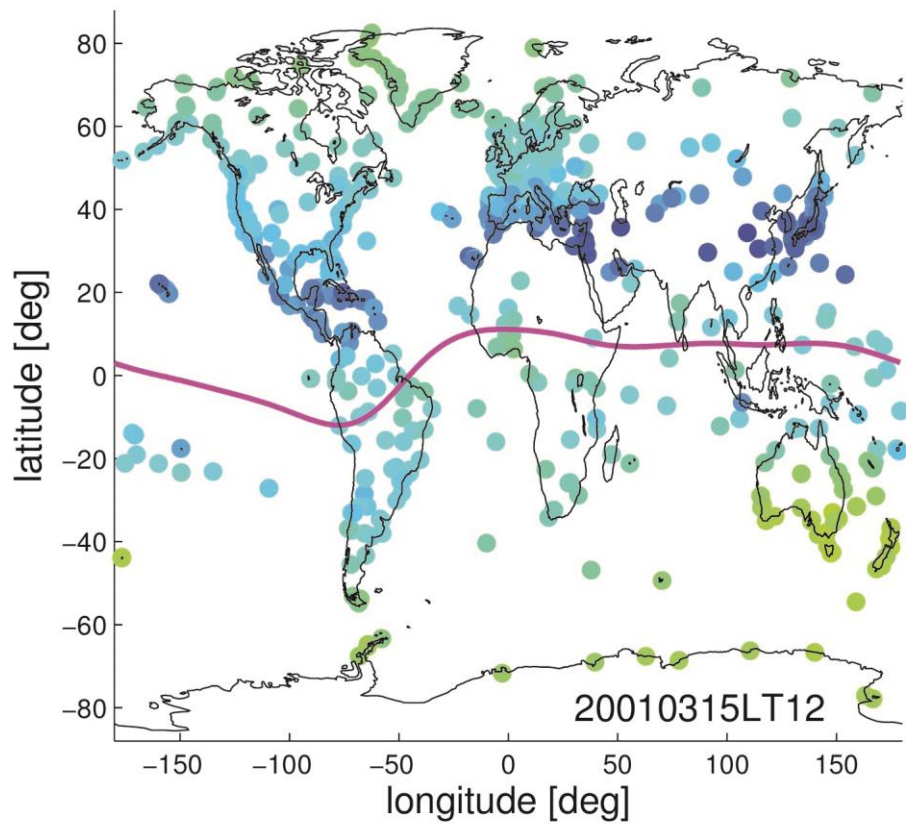
Impact of HOIC on N-component [mm] in PPP



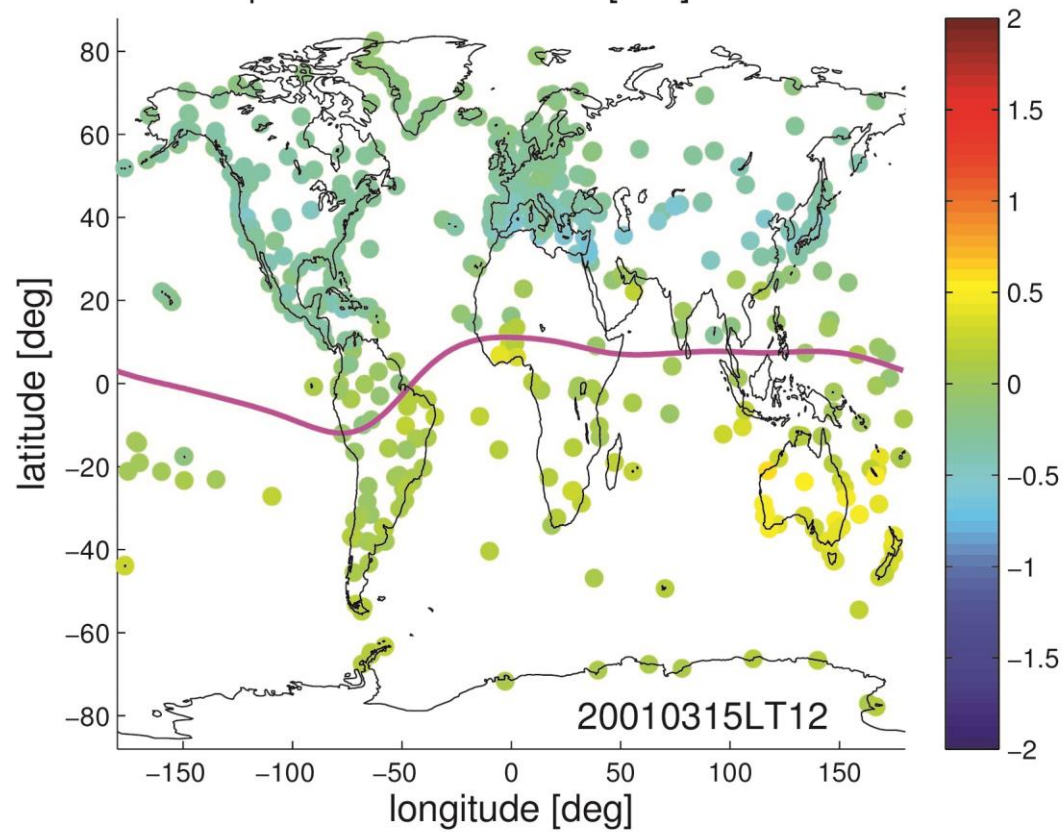
Impact of SOIC on N-component [mm] in PPP

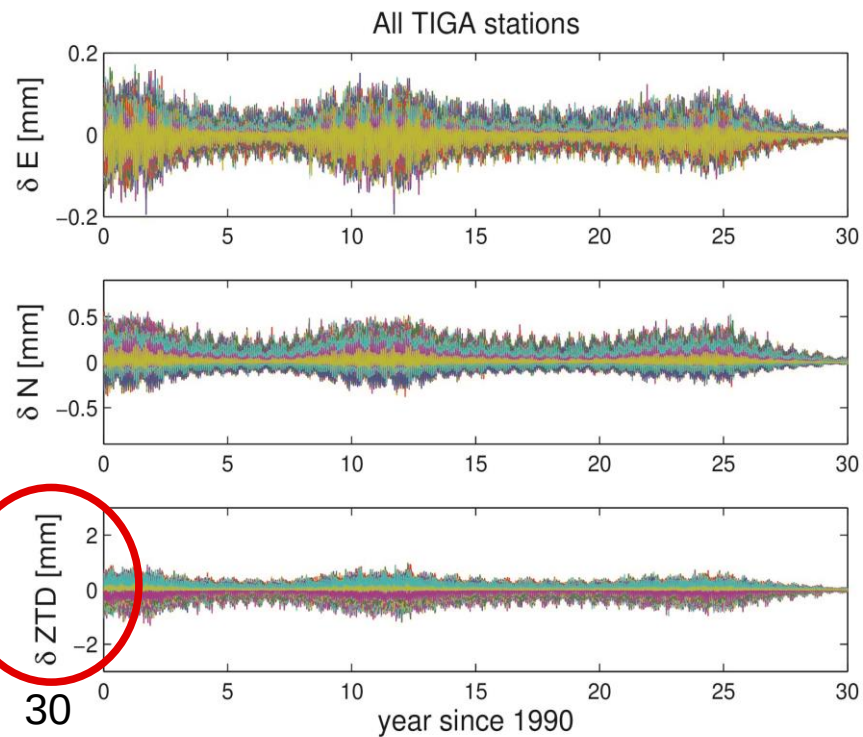
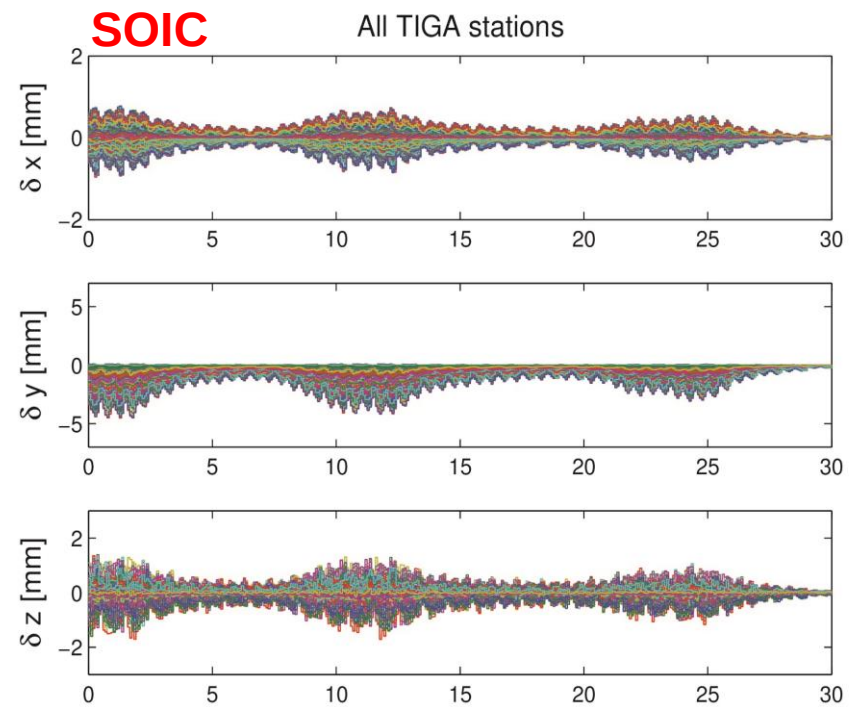
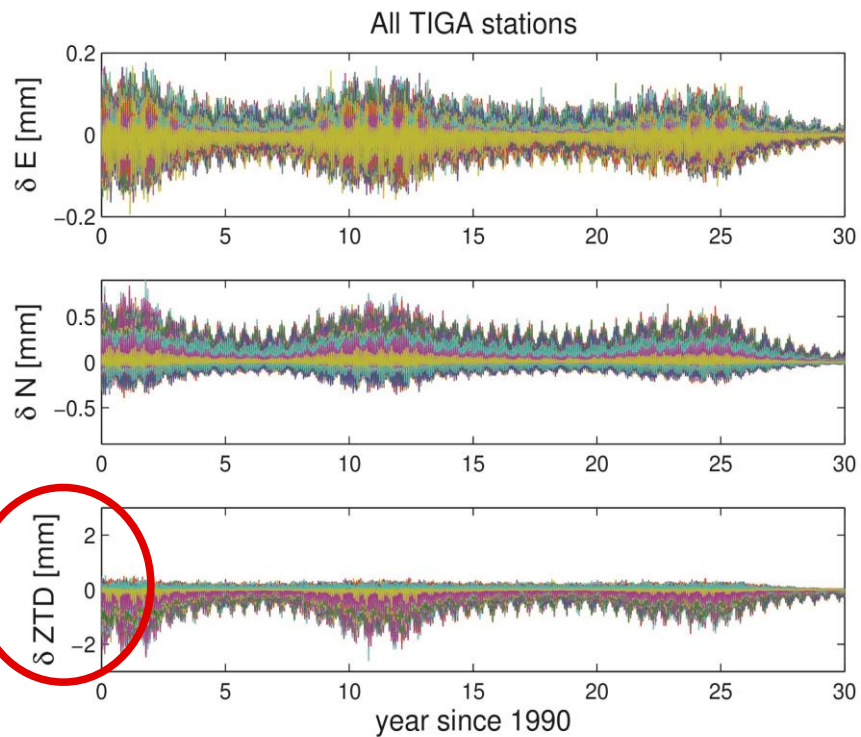
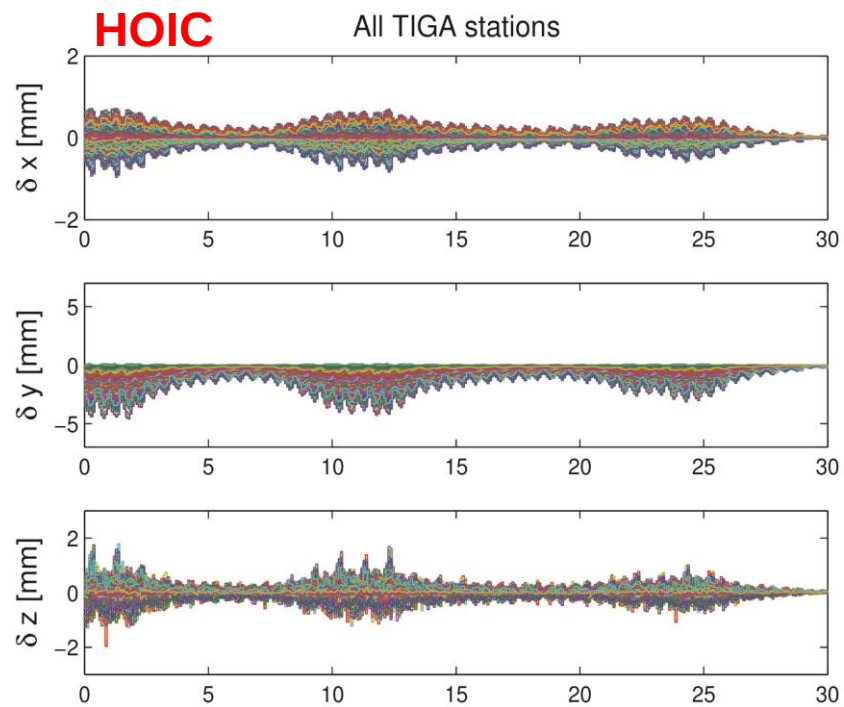


Impact of HOIC on ZTD [mm] in PPP



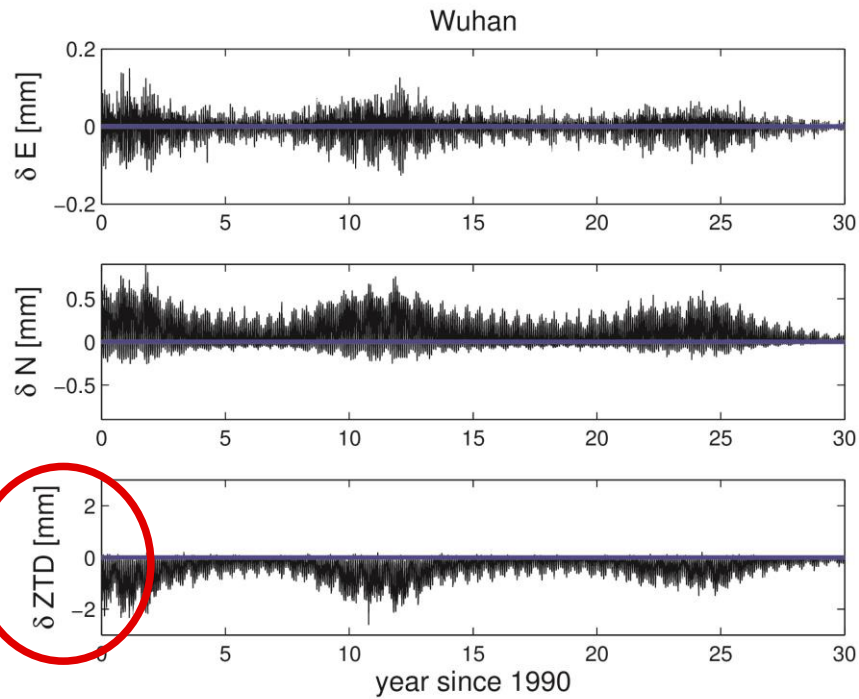
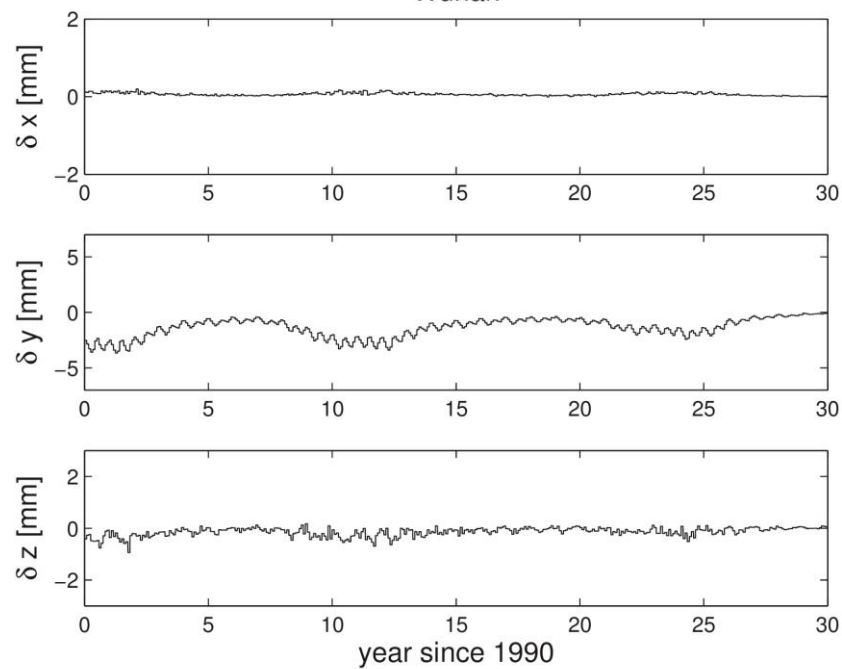
Impact of SOIC on ZTD [mm] in PPP



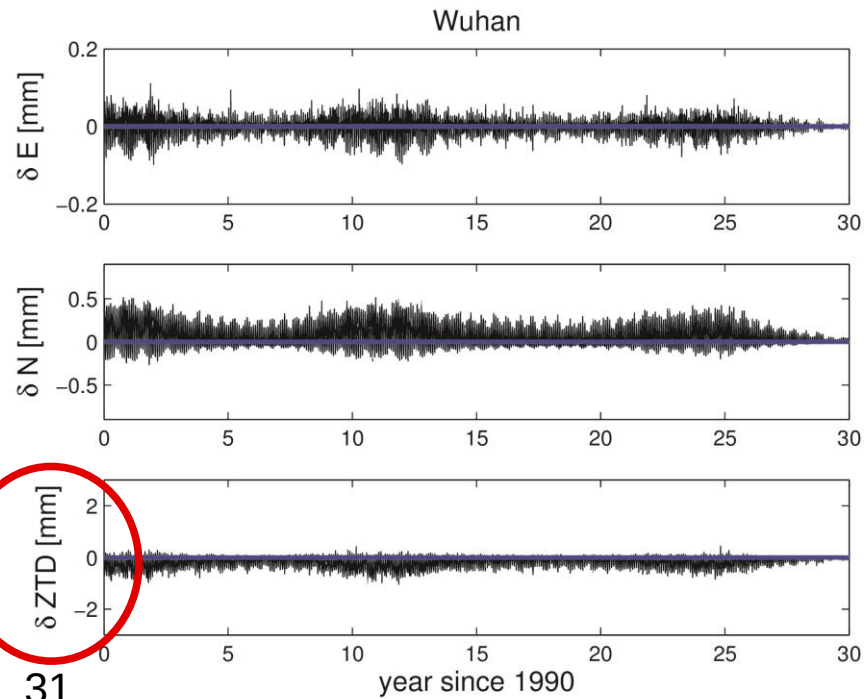
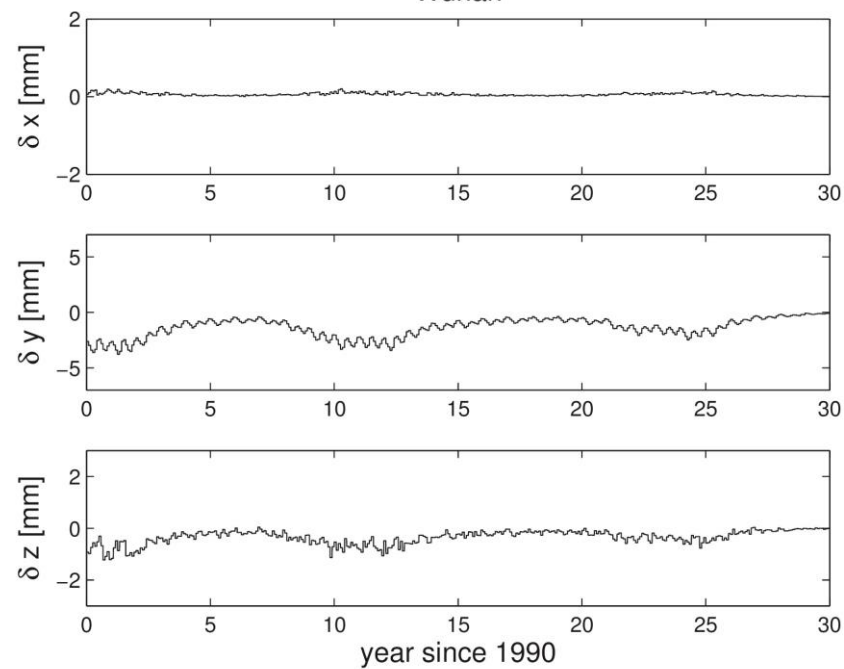


HOIC

Wuhan

**SOIC**

Wuhan



Summary

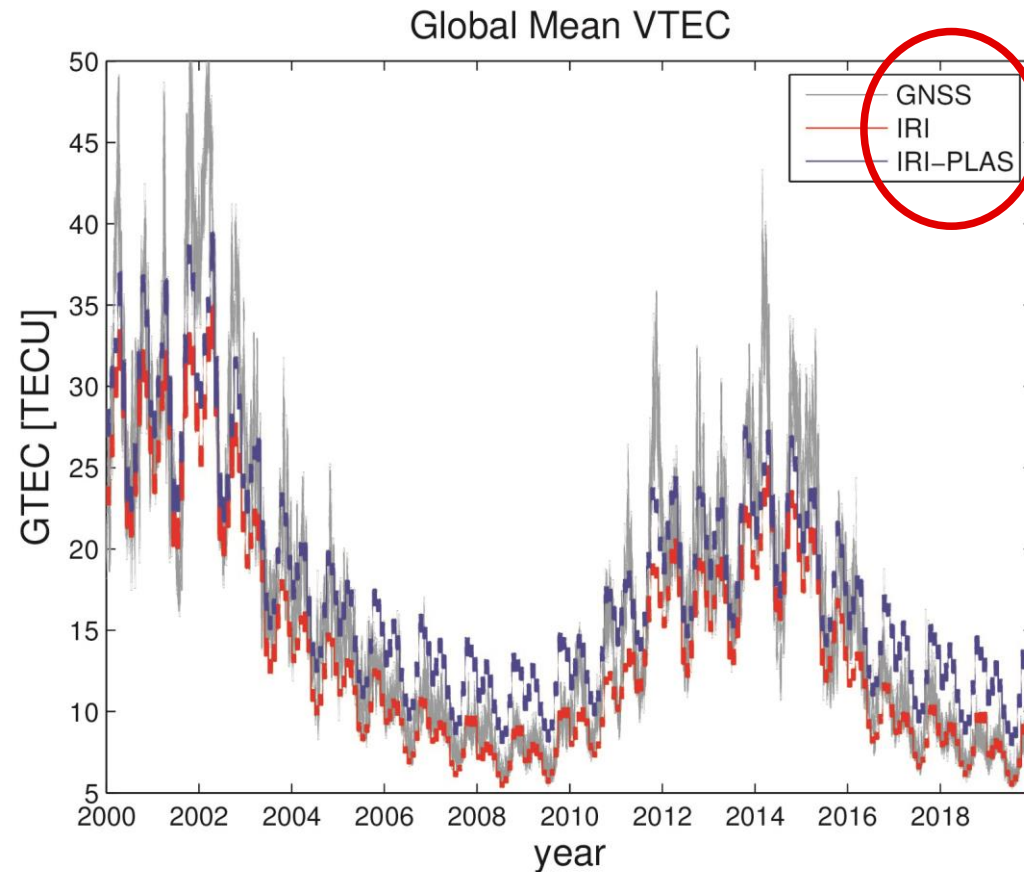
Result:

- Stations appear to move southwards by up to 5 mm
- North gradient components are systematically affected by up to 0.5 mm
- ZTDs are systematically affected by up to 2 mm; the ray-path bending effects are responsible for this and not the higher-order terms in the formula for the refractive index of the ionosphere

Recommendation:

Make use of HOICs (and not SOICs) in precise GNSS applications.

Note: Under high solar activity HOIC derived from IRI is probably too small



IRI-PLAS ... Plasmaspheric extension of IRI (Gulyaeva and Bilitza 2012)

Future of GFZ-HOIC:

- Optimize parameterization (data compression)
- Scalable version utilizing GNSS TEC data
- Version available for real time applications

Auxiliary material (animations etc):

<ftp://ftp.gfz-potsdam.de/pub/home/GNSS/products/gfz-hoic/>

References:

Gulyaeva, T.L., and Bilitza, D. (2012) Towards ISO Standard Earth Ionosphere and Plasmasphere Model. In: New Developments in the Standard Model, edited by R.J. Larsen, pp. 1-39, NOVA, Hauppauge, New York.

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