MODELLING LARGE DEFORMATION SLOPE FAILURE USING THE SMOOTHED PARTICLE HYDRODYNAMICS (SPH) METHOD

Bunyamin Andreatama[1,*], Widjojo Adi Prakoso[1], Erly Bahsan[1], R.R. Dwinanti Rika Marthanty[1], Jessica Sjah[1]

[1]Department of Civil Engineering, University of Indonesia, Depok, Indonesia

Corresponding author: bunyamin.andreatama@ui.ac.id

Abstract

The slope stability analyses using limit equilibrium method (LEM) and finite element method (FEM) are mostly concerned about the factor of safety (FS) value of the slope. LEM cannot predict the soil behaviour after failure, while FEM can only be used to measure the material deformation before failure. Currently the Smoothed Particle Hydrodynamics (SPH) method has begun to be used as an alternative to overcome excess distortion of the mesh in FEM analysis due to post-failure large deformations in slope stability analysis. In this study, the behaviour of soil materials will be modelled as particles using the SPH method with reference to the previous research. The Bingham fluid model is used as a viscoplastic model of the soil material, and the Drucker-Prager soil constitutive model is used to describe the elastic-plastic behaviour of the soil. This modelling algorithm uses the equivalent viscosity of the Bingham fluid model as the initial stress between particles, and it uses the Drucker-Prager criterion with the associated flow rule to describe particle displacement due to slope failure. The soil particles are modelled as cohesive soil with a slope angle to the horizontal axis so that they can be compared with previous studies. The failure pattern is expected to be able to show areas of particles that are not deformed and particles that have collapsed. The FS value of the slope is obtained by the strength reduction method which seeks a non-convergent solution of each reduction in soil strength parameters.

Keywords: Smoothed Particle Hydrodynamics (SPH); Slope Stability; Bingham Fluid Model; Drucker-Prager Model; Strength Reduction Method

1. Introduction

The calculation approach method in analysing slope stability is generally carried out using the limit equilibrium method (LEM) and the finite element method (FEM), which both have their respective advantages and disadvantages. LEM can provide an estimation of the safety factor without requiring a stress-strain relationship from the soil, but requires assumptions to determine the critical slip surface on the slope. On the other hand, FEM does not require initial assumptions for the shape or location of the slip surface of a slope because the failure mechanism "looks for" the weakest path in the ground. However, this method also has a weakness in large deformation analysis, where excessive distortion of the FEM mesh can lead to unstable calculations.

The smoothed particle hydrodynamics (SPH) method as a mesh-free Lagrangian scheme-based particle method is an alternative that has begun to be used in analysing slope stability. The soil movement behaviour which is modelled as particles can overcome the excess distortion of the FEM mesh due to large deformation on slope failure. The computational procedure of the SPH method for geo-disaster application and modelling in the book of Huang et al. (2014) is divided into two main functions, namely Hydrodynamics SPH Procedure and Elasto-plastic SPH Procedure. Bingham fluid constitutive model is commonly used to analyse boil flow using the Hydrodynamics SPH Procedure. Meanwhile, the Drucker-Prager model is used for soil deformation problems by using the Elasto-plastic SPH Procedure.

Uzuoka et al. (1998) in their paper modelled liquefied soil as Bingham fluid, which is one of the viscoplastic models by considering the minimum undrained strength. In this study, to describe the behaviour of soil particles when a landslide occurs on a slope, the Elasto-plastic SPH Procedure is used in its programming algorithm with the Drucker-Prager model which refers to research that has been developed by Ha H Bui et al. (2008), as well as analysing the safety factor using the Strength Reduction Method from the paper Ha Hong Bui et al. (2011).

2. Numerical Approaches

2.1. Basic Understanding of SPH

The SPH method was first discovered to solve astrophysical problems in three-dimensional open spaces, which was later developed for hydrodynamic problems in the form of partial differential equations (PDE) of field variables such as density, velocity, energy, and so on (Liu & Liu, 2003). Basically, the SPH method is an interpolation theory with two key stages, namely kernel approximation and particle approximation. Referring to Liu & Liu (2003), the concept of an integrated representation of a function f(x) used in the SPH method can be written with the following formula:

$$\langle f(x) \rangle = \int_{\Omega} f(x')W(x - x', h)dx' \quad (1)$$

Where Ω is the integral volume containing x, x is the position of the particle (or r), and W is the kernel function (smoothing kernel) with smoothing radius h. The function can be written numerically as:

$$f(x) = \sum_{j=1}^{n} \frac{m_j}{\rho_j} f(x_j) W(x - x_j, h) \tag{2}$$

The kernel smoothing approach greatly affects the stability, accuracy and speed of the SPH method. Ha H Bui et al. (2008) in their paper used the cubic spline function proposed by Monaghan and Lattanzio with the following formulations:

$$W_{ij} = \begin{cases} \frac{5}{14\pi h^5}(4h^3 - 6r^2h + 3r^3), & 0 \le r/h < 1 \\ \frac{5}{14\pi h^5}(2h - r)^3, & 1 \le r/h < 2 \\ 0, & r/h \ge 2 \end{cases} \tag{3}$$

2.2. Governing Equations

In the elasto-plastic SPH model, the soil governing equation consists of the equation for the conservation of mass and momentum. Conservation of mass is the basis of the following continuity equation:

$$\frac{\partial \rho}{\partial t} = -\rho \frac{\partial v^\alpha}{\partial x^\alpha} \tag{4}$$

and conservation of momentum is the basis of the equation for ground motion as follows:

$$\frac{\partial v^\alpha}{\partial t} = \frac{1}{\rho} \frac{\partial \sigma^{\alpha\beta}}{\partial x^\alpha} + F \tag{5}$$

where α and β are Cartesian components x, y and z; ρ is the density of the soil; v represents the velocity vector; σ is the total stress tensor and F external force vector which is the gravitational force in this study. The discretization of equations (4) and (5) in the formulation of SPH is shown in equations (6) and (7) respectively:

$$\frac{D\rho}{Dt} = -\sum_{\beta}^{N} m_j (v_i^\alpha - v_i^\alpha) \frac{\partial W_{ij}}{\partial x_j^\alpha} \tag{6}$$

$$\frac{Dv_i^\propto}{Dt} = \sum_{j=1}^{N} m_j \left(\frac{\sigma_i^{\alpha\beta}}{\rho_i^2} + \frac{\sigma_j^{\alpha\beta}}{\rho_j^2} - \delta^{\alpha\beta} \Pi_{ij} \right) \frac{\partial W_{ij}}{\partial x_j^\beta} + g^\alpha \tag{7}$$

$\delta^{\alpha\beta}$ is Kronecker's delta which has a value of $\delta^{\alpha\beta} = 1$ if $\alpha = \beta$, and $\delta^{\alpha\beta} = 0$ if $\alpha \neq \beta$. To prevent numerical oscillations and penetration between particles, Huang et al. (2014) added artificial viscosity Π_{ij} in terms of pressure to the equation of motion to convert kinetic energy into heat. This viscosity is defined as follows:

$$\Pi_{ij} = \frac{-a\bar{c}\mu_{ij} + b(\mu_{ij})^2}{\overline{\rho^{\alpha\beta}}} \tag{8}$$

$$\mu_{ij} = \frac{h(v_i - v_j)\sqrt{(x_i - x_j)^2 + (y_i - y_j)^2}}{(x_i - x_j)^2 + (y_i - y_j)^2 + kh^2} \tag{9}$$

$$\bar{c} = \frac{1}{2}\left(c_i - c_j\right) \quad \text{dan} \quad \bar{\rho} = \frac{1}{2}\left(\rho_i - \rho_j\right) \tag{10}$$

where a and b are constant parameters suggested by Monaghan with a value close to 1 for the most stable results. k is an independent parameter which avoids numerical drift when the particles approach each other with the value suggested by Liu & Liu (2003) of 0.01. c_i and c_j are the speed of sound at points i and j with a range of 450-600 m/s, which in the paper of Ha H Bui et al. (2008) take the maximum value of 600 m/s

2.3. Soil Constitutive Model

In the Elasto-plastic SPH Procedure, the Drucker-Prager constitutive model is chosen in this study as a reference to describe the soil flow behaviour in the soil yield criteria. This yield condition is stated as:

$$f(I_1, J_2) = \sqrt{J_2} + \alpha_\phi I_1 - k_c = 0 \tag{11}$$

$$I_1 = \sigma^{xx} + \sigma^{yy} + \sigma^{zz} \quad \text{and} \quad J_2 = \frac{1}{2} s^{\alpha\beta} s^{\alpha\beta} \tag{12}$$

$$\alpha_\phi = \frac{\tan\phi}{\sqrt{9 + 12\tan^2\phi}} \quad \text{and} \quad k_c = \frac{3c}{\sqrt{9 + 12\tan^2\phi}} \tag{13}$$

α_ϕ and k_c are Drucker-Prager constants, which correspond to the Coulomb material constants c (cohesion) and ϕ (angle of friction). $\sigma^{xx}, \sigma^{yy}, \sigma^{zz}$ are three normal stress components and $s^{\alpha\beta}$ are deviatoric shear stress rate tensor.

Deviatoric shear stress in the paper of Zubeldia et al. (2018) is referred to $\tau^{\alpha\beta}$ to find the second invariant J_2. The value of $\tau^{\alpha\beta}$ itself is obtained from the rheological constitutive model presented by Yang et al. (2020) with the following formulas:

$$\tau^{\alpha\beta} = 2\eta\dot{\varepsilon}^{\alpha\beta} \tag{14}$$

$$\eta = \frac{\mu P}{\sqrt{\dot{\varepsilon}^{\alpha\beta}\dot{\varepsilon}^{\alpha\beta}}} \tag{15}$$

$$P_i = c^2(\rho_i - \rho_0) \tag{16}$$

$$\mu = \mu_s + \frac{\mu_p - \mu_s}{\frac{I_0}{I_i} + 1} \tag{17}$$

$$I_i = \frac{d\sqrt{\dot{\varepsilon}^{\alpha\beta}\dot{\varepsilon}^{\alpha\beta}}}{\sqrt{P_i/\rho_0}} \tag{18}$$

where η is an apparent viscosity; P is the isotropic pressure; c is the kecepatan suara; ρ_i is the density of particle i; ρ_0 is the reference density of the material; μ is a frictional function that depends on the inertial number, I_i; d is the real grain diameter; dan μ_s, μ_p, dan I_0 are constants.

Soil behaviour in this constitutive model is then described by the relationship between stress and strain in the soil material. The full yield strain $\dot{\varepsilon}^{\alpha\beta}$ consists of 2 parts, namely the elastic strain $\dot{\varepsilon}_e^{\alpha\beta}$ and the plastic strain $\dot{\varepsilon}_p^{\alpha\beta}$ as in the following equation:

$$\dot{\varepsilon}^{\alpha\beta} = \dot{\varepsilon}_e^{\alpha\beta} + \dot{\varepsilon}_p^{\alpha\beta} \tag{19}$$

$$\dot{\varepsilon}^{\alpha\beta} = \frac{\dot{s}^{\alpha\beta}}{2G} + \frac{1-2v}{3E}\dot{\sigma}^{\gamma\gamma}\delta^{\alpha\beta} + \dot{\lambda}\frac{\partial g}{\partial \sigma^{\alpha\beta}} \tag{20}$$

The total stress tensor in equation (21) is then rearranged with equation (20) to produce a stress-strain relationship for a perfectly elastic-plastic material as in equation (22).

$$\sigma^{\alpha\beta} = s^{\alpha\beta} + \frac{1}{3}\sigma^{\gamma\gamma}\delta^{\alpha\beta} \tag{21}$$

$$\sigma^{\alpha\beta} = 2G\dot{e}^{\alpha\beta} + K\dot{\varepsilon}^{\gamma\gamma}\delta^{\alpha\beta} - \dot{\lambda}\left[\left(K - \frac{2G}{3}\right)\frac{\partial g}{\partial \sigma^{mn}}\delta^{mn}\delta^{\alpha\beta} + 2G\frac{\partial g}{\partial \sigma^{\alpha\beta}}\right] \tag{22}$$

where α, β are the free indices and m, n are the dummy indexes. $\dot{e}^{\alpha\beta}$ is a deviatoric shear strain rate tensor with $\dot{e}^{\alpha\beta} = \dot{\varepsilon}^{\alpha\beta} - \frac{1}{3}\dot{\varepsilon}^{\gamma\gamma}\delta^{\alpha\beta}$. K is the elastic bulk modulus, which corresponds to the shear modulus G and Poisson's ratio v in the following equation:

$$K = \frac{E}{3(1-2v)} \quad \text{and} \quad G = \frac{E}{2(1+v)} \tag{23}$$

The associated plastic flow rule which shows the plastic potential function of the Drucker-Prager material has the same shape as the yield criteria as follows:

$$g = \sqrt{J_2} + \alpha_\phi I_1 - k_c \tag{24}$$

Equation (24) is then substituted into equation (22) which produces the final stress-strain relationship of the Drucker-Prager perfect elastic-plastic soil model with the associated plastic flow rule (25).

$$\sigma^{\alpha\beta} = 2G\dot{e}^{\alpha\beta} + K\dot{\varepsilon}^{\gamma\gamma}\delta^{\alpha\beta} - \dot{\lambda}\left[3\alpha_\phi K\delta^{\alpha\beta} + \frac{G}{\sqrt{J_2}}s^{\alpha\beta}\right] \tag{25}$$

With the plastic multiplier change rate $\dot{\lambda}$ for the associated plastic flow rule which is shown in the following equation:

$$\dot{\lambda}_i = \frac{3\alpha_\phi K \dot{\varepsilon}_i^{\gamma\gamma} + \left(G/\sqrt{J_2}\right) s_i^{\alpha\beta} \dot{\varepsilon}_i^{\alpha\beta}}{9\alpha_\phi^2 K + G} \tag{26}$$

The strain and spin rate tensors of a particle are formulated in the SPH equation as follows

$$\dot{\varepsilon}^{\alpha\beta} = \frac{1}{2}\left[\sum_{j=1}^{N} \frac{m_j}{\rho_j}\left(v_j^\alpha - v_i^\alpha\right)\frac{\partial W_{ij}}{\partial x_i^\beta} + \sum_{j=1}^{N} \frac{m_j}{\rho_j}\left(v_j^\beta - v_i^\beta\right)\frac{\partial W_{ij}}{\partial x_i^\alpha}\right] \tag{27}$$

$$\dot{\omega}^{\alpha\beta} = \frac{1}{2}\left[\sum_{j=1}^{N} \frac{m_j}{\rho_j}\left(v_j^\alpha - v_i^\alpha\right)\frac{\partial W_{ij}}{\partial x_i^\beta} - \sum_{j=1}^{N} \frac{m_j}{\rho_j}\left(v_j^\beta - v_i^\beta\right)\frac{\partial W_{ij}}{\partial x_i^\alpha}\right] \tag{28}$$

So that the final form of the SPH formulation for a particular particle i in the stress-strain relationship for the associated flow rule soil model is then stated as follows:

$$\frac{D\sigma_i^{\alpha\beta}}{Dt} = \sigma_i^{\alpha\gamma}\dot{\omega}_i^{\beta\gamma} + \sigma_i^{\gamma\beta}\dot{\omega}_i^{\gamma\alpha} + 2G\dot{e}_i^{\alpha\beta} + K\dot{\varepsilon}_i^{\gamma\gamma}\delta_i^{\alpha\beta} - \dot{\lambda}_i\left[3\alpha_\phi K\delta^{\alpha\beta} + \frac{G}{\sqrt{J_2}} s_i^{\alpha\beta}\right] \tag{29}$$

The position of the particles in the SPH itself is moved according to the following equation:

$$\frac{dx_i^\alpha}{dt} = v_i^\alpha \tag{30}$$

2.4. Shear Strength Reduction Method

The reduced soil strength along with the movement of the soil on a slope in analysing the stability of the slope can be described by the shear strength reduction method. The use of this method in the SPH method is basically the same as FEM, where in the Griffiths & Lane (1999) paper the analysed value of the slope safety factor is determined when there is a drastic increase in dimensionless displacement so that the algorithm cannot converge within the iteration limits.

The soil movement in the SPH method is described by the movement of soil particles that gets farther along with the addition of a certain time step in the calculation. Each of these calculations is carried out with a reduced shear strength parameter value, namely c_t and ϕ_t which is stated by the following formula:

$$c_t = \frac{c}{SRF} \quad \text{dan} \quad \phi_t = \tan^{-1}\left(\frac{\tan\phi}{SRF}\right) \tag{31}$$

Where c and ϕ are the parameters of the initial shear strength values, and SRF is the strength reduction factor. This SRF value will be inputted with a low value, then gradually increased as the slope conditions become increasingly unstable until slope failure occurs.

3. Methodology

SPH modelling in this study is carried out using the FORTRAN programming language using the Absoft Pro-FORTRAN program, where the source code used is a recycle from previous research at the University of Indonesia. The modelling algorithm is depicted in the following flow chart:

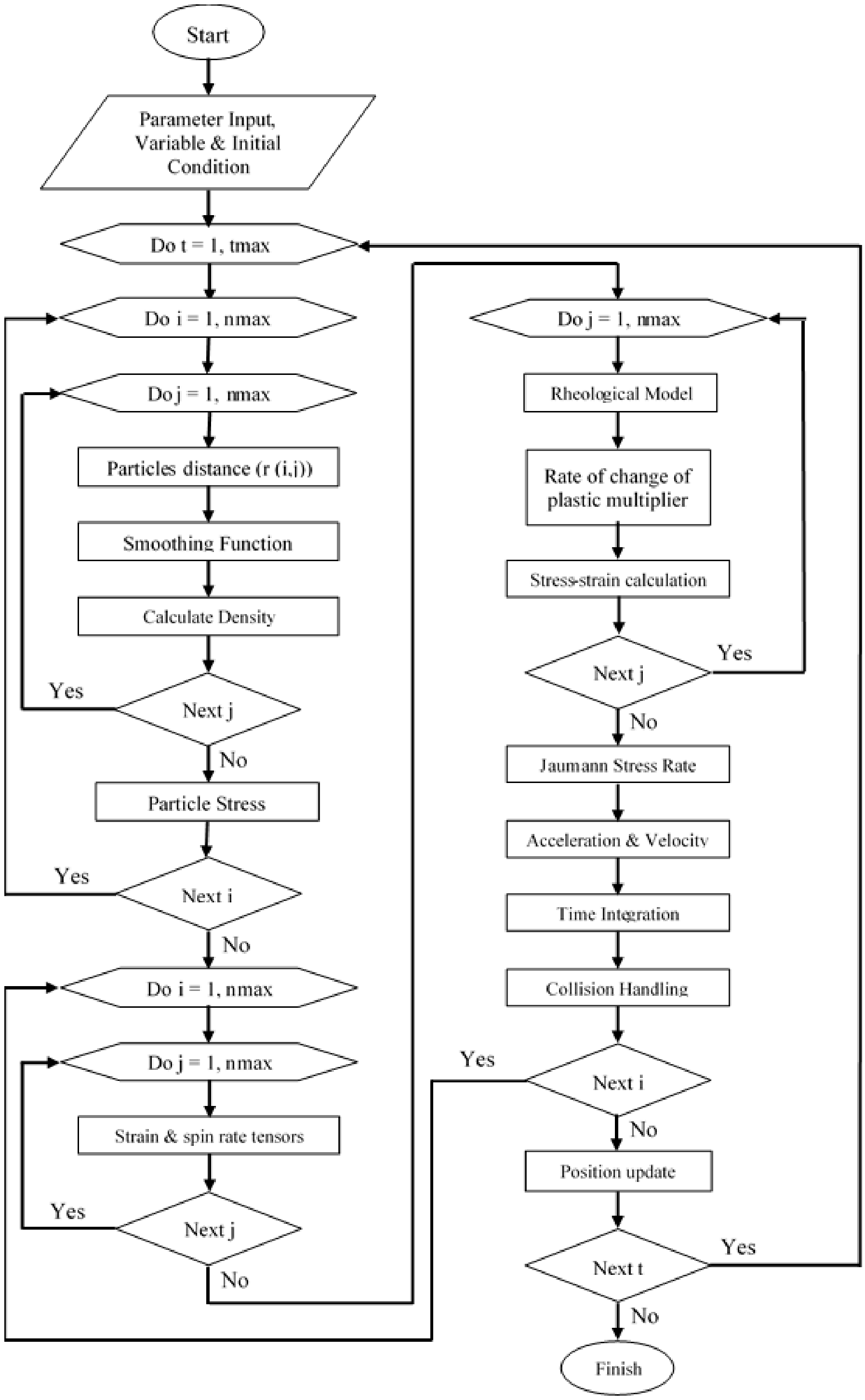

Figure 1. Program Algorithm

In the first stage, identification of computational problems is carried out by inputting parameters, determining variables, and the initial condition of the particles before the start of the simulation. The calculation of the Drucker-Prager constants, bulk modulus and shear modulus is carried out at this initial stage before the iteration begins. The t notation is the number of iterations performed in the simulation of this program. Particle i is the main particle under study, while particle j is the neighboring particle that is within the radius of the reviewed particle's kernel.

The iteration of the program starts at the next stage which begins with calculating the distance of the particles that will be used in the smoothing function to calculate density. Particle stress calculation is done after the density is obtained.

The next step is to calculate the strain rate and spin rate tensors of each particle. Rheological model calculations are performed to obtain deviatoric stress, and the rate of change of plastic multiplier is calculated for use in stress-strain relationships. The stress that occurs on the particles is obtained from the calculation of the stress-strain relationship.

The Jaumann stress rate is then used with respect to the rigid-body rotation that occurs in the particle when it is subjected to stress. Acceleration and velocity calculations are performed after the particle stress is obtained. Then performed time integration and collision handling for particle movement. The update of the particle position is then carried out for each iteration time to see the particle displacement.

4. Discussion

The SPH modelling program carried out in this study is actually still in progress. If the program using the elasto-plastic SPH procedure has been completed, it is expected that the displacement of collapsed particles and particles that are still stable can be seen as a slip-surface of the slope stability being modelled.

The shear strength reduction method will then be performed by analysing the convergent / non-convergent properties of the change in particle displacement at a certain iteration stage. If the displacement change is smaller than the previous iteration stage, where the displacement-iteration relationship graph shows a convex shape which means providing a convergent SPH solution, then the SPH calculation process is carried out again with a larger SRF (Strength Reduction Factor) value. If the displacement change at a certain iteration stage reaches a higher value than the previous iteration stage, where the displacement-iteration relationship graph shows a concave shape which means providing a non-convergent SPH solution, then the SRF value at that stage is taken as a safety factor.

There are several obstacles faced in the development of this program, where the stress calculation on modelling that has been made at the University of Indonesia using the hydrodynamics SPH procedure, is only done in the x, y and z directions. Whereas in the elasto-plastic SPH procedure as has been done by Ha H Bui et al. (2008) used a rank-2 tensor in the calculation of stress-strain relationships, so it was quite confusing when calculating the acceleration and velocity of the particles.

5. Conclusion

The development of slope failure modelling using Smoothed Particle Hydrodynamics is still very open to improvement. The use of the elasto-plastic SPH procedure in this study is an attempt to simulate slope failure as closely as possible to the behaviour of soil particles when subjected to large deformations. The use of the Drucker-Prager constitutive model is an alternative to the use of the Bingham fluid model that has been developed at the University of Indonesia. The use of soil parameters such as cohesion, shear angle, modulus of elasticity, and Poisson's ratio which had not been used in previous source program were then used in this study. The search for the value of the safety factor using the strength reduction method will also be carried out in line with the current development of the program.

6. References

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