

Time dependent sliding laws for soft-bedded glaciers

¹Institute of Theoretical Geophysics,
DAMTP, University of Cambridge
²Department of Earth Sciences,
Bullard Laboratories, University of
Cambridge
³BP Institute, University of
Cambridge
⁴Department of Mathematics,
University College London

KASIA WARBURTON¹, JEROME A. NEUFELD^{1,2,3}, DUNCAN HEWITT⁴

✉ klpw3@cam.ac.uk
@KasiaWarburton

Governing equations

Following granular experiments, use a plastic flow law where the friction coefficient depends on the dimensionless shear rate (inertial number) $I_\nu = \frac{\eta\dot{\gamma}}{N}$

$$\tau = \mu N$$

$$\mu = \mu_1 + M\sqrt{I_\nu}$$

Equilibrium porosity also depends on the local inertial number

$$\phi_{eq} = \frac{\phi_m}{1 + \sqrt{I_\nu}}$$

Shear drives the till to compact or dilate towards this equilibrium value

$$\frac{\nabla \cdot \mathbf{u}_s}{\dot{\gamma}} = \beta = K(\phi - \phi_{eq})$$

$$\frac{\partial \phi}{\partial t} + \frac{\partial}{\partial z}(v_s \phi) = 0$$

Changes in porosity induce a flow of water into the newly created pore space, by conservation of mass.

$$v_f = -\frac{\phi}{1 - \phi}v_s$$

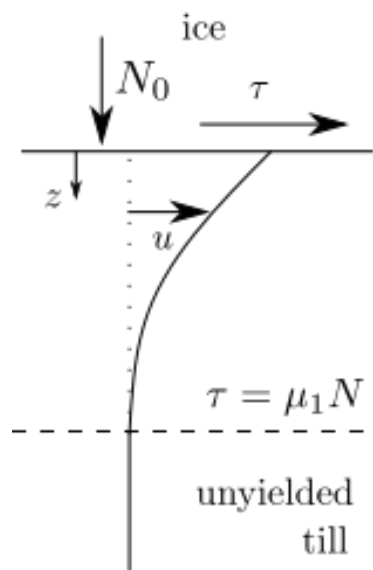
Any flow of water requires a pore-water pressure gradient to drive it

$$-\frac{k}{\eta(1 - \phi)} \left(\frac{\partial p_w}{\partial z} - \rho_f g \right) = (v_f - v_s)$$

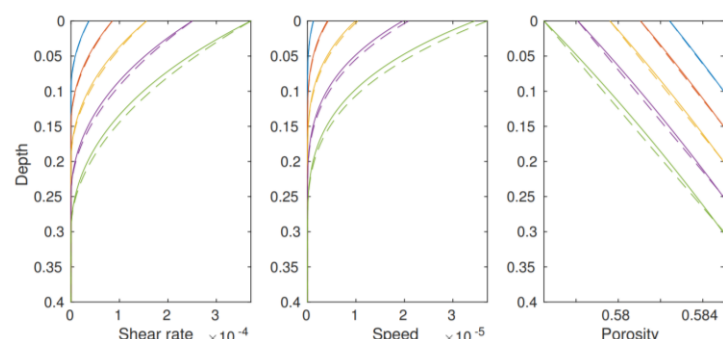
That pore water pressure in turn modifies the effective pressure felt by the grains

$$\frac{\partial p_w}{\partial z} - \frac{\partial \sigma_{zz}^e}{\partial z} = \rho_f g + \Delta \rho g \phi$$

altering the yield strength and shear rate of the till at depth.



Steady state



Depth of deforming region

$$\frac{\tau - \mu_1 N_0}{\mu_1 \Delta \rho g \phi_m}$$

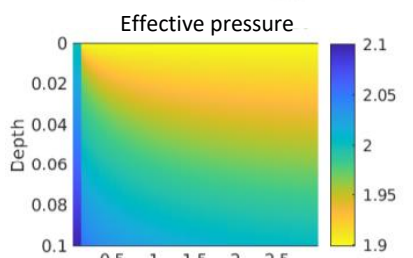
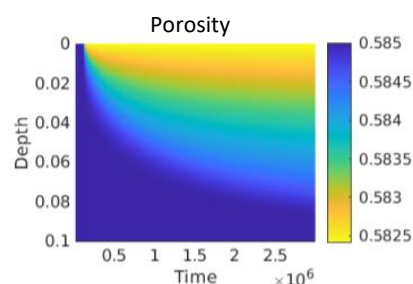
Sliding law

$$u_b = u_s(0) = \begin{cases} 0 & \tau < \mu_1 N_0 \\ \frac{1}{3\eta\mu_1\Delta\rho\phi_m g M^2} \frac{(\tau - \mu_1 N_0)^3}{N_0} & \tau \geq \mu_1 N_0, \end{cases}$$

By rearranging the steady state sliding law we get a measure of the effective friction coefficient in the transient case – $\mu_{eff} = \frac{\tau - (Au_b N_0)^{1/3}}{N_0}$

Higher values indicate the till is stronger and deforming less than the steady state predicts.

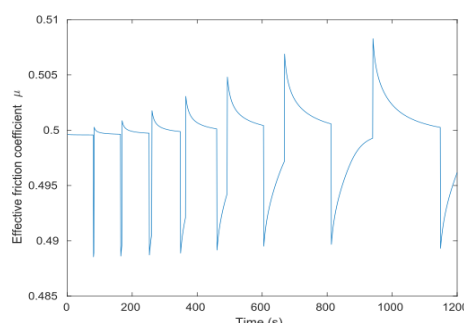
Dilatant strengthening



Timescale to re-equilibrate

(from hours to years - depends strongly on the till permeability k)

For episodic shear, longer periods of compaction are associated with a stronger till when shearing rate resumes.



Comparable to the predictions of a rate-and-state friction model, with porosity playing the role of a state variable, and shear driving the porosity towards its new steady state.

After a drop in effective pressure at the ice-till interface, water is sucked in to the increasing pore space as the till dilates. This requires a decreased water pressure at depth, buffering most of the till from the surface changes.

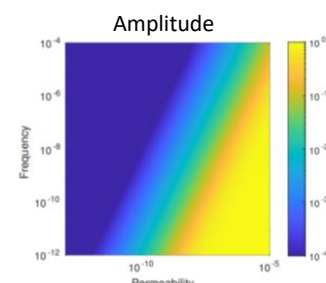
$$T \sim \frac{\eta(\tau - \mu_1 N_0)^2}{\tau k M (\Delta \rho g \phi_m)^2}$$

Periodic response

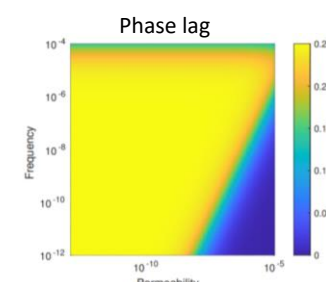
We can use the governing equations to write down a simple time and frequency dependent sliding law, modifying the steady state expression

$$u_b = u_b(\bar{N}) + A \frac{du_b}{dN} \Delta N \sin(\omega t - \theta)$$

The amplitude of the sliding speed response to changes in effective pressure is buffered by induced pressure changes at depth.



The time to re-equilibrate introduces a phase lag between the forcing and the response. Slow forcing has time to equilibrate.



Fast frequencies get filtered out as the till does not have time to adjust, and the initial dilatant strengthening at depth is strong enough that the ice sliding speed does not respond to the changes in pressure forcing.