A combined Reduced Order-Bayesian scheme to drastically accelerate stochastic inversions

Sergio Zlotnik, O. Ortega, P. Díez and J.C. Afonso

sergio.zlotnik@upc.edu

Laboratori de Càlcul Numèric (LaCàN)
Universitat Politècnica de Catalunya - BarcelonaTech (Spain)
www.lacan.upc.edu

UNIVERSITAT POLITÈCNICA DE CATALUNYA
LABORATORI DE CÀLCUL NUMÈRIC
LaCàN

Inverse problem (parameter identification)

Deterministic approach

- Find the values of the parameters minimizing a discrepancy functional

$$m^\star = \arg\min_m \underbrace{\|\text{observables} - \text{model}(m)\|}_{=:\text{discrepancy}(m)}$$

Bayesian approach

- Parameters considered random variables

$$\pi_{\text{post}}(m) \propto \pi_{\text{prior}}(m) L(m)$$

- **Markov chain Monte Carlo** method. **Metropolis-Hastings**

UNIVERSITAT POLITÈCNICA DE CATALUNYA
LABORATORI DE CÀLCUL NUMÈRIC

Monte Carlo method

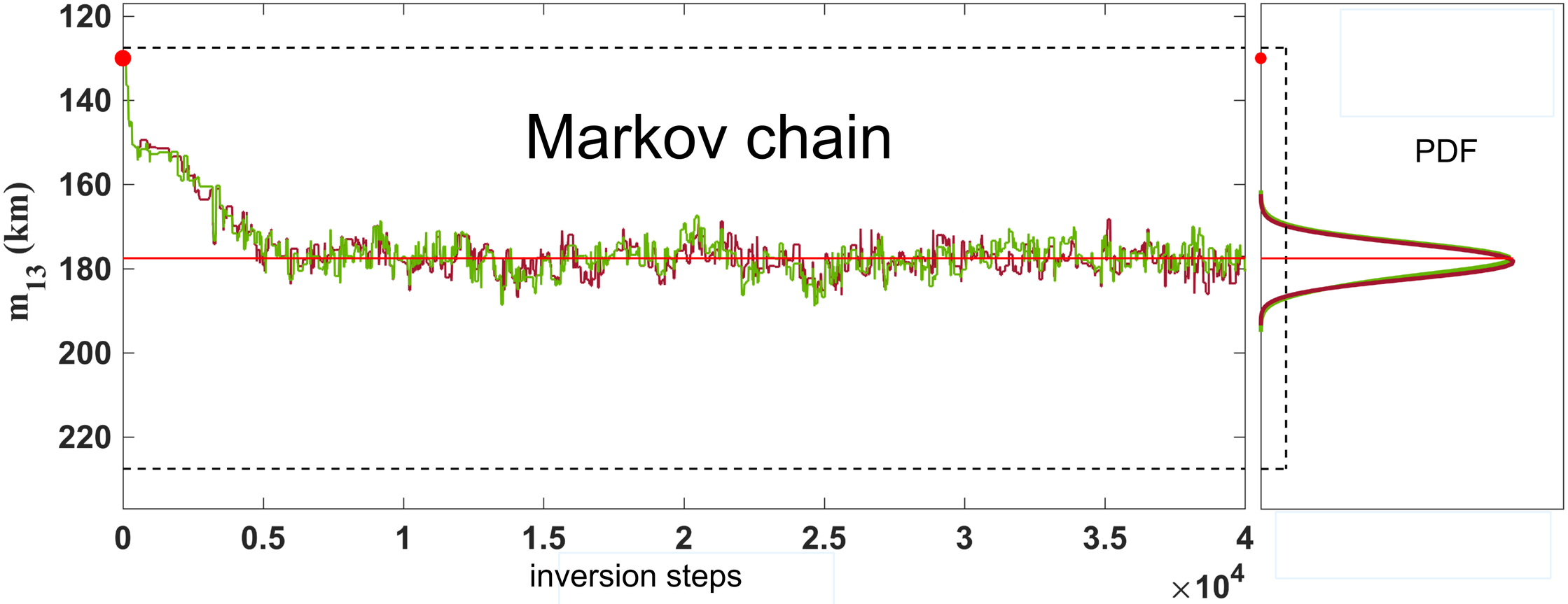
⇓

many queries to the forward problem
(counted in millions)

(In practice a forward problem needs be solved in less that about 2 seconds)

Markov chain

For each parameter…



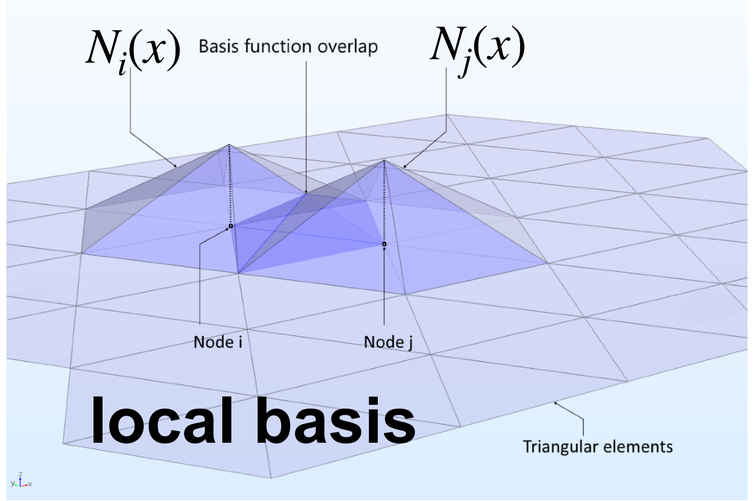
How can we solve all the forward problems
required for the inversion
in an efficient way?

Reduced Basis
(Tailored to the inversion)

Reduced Basis

- Finite Elements (FE)

$$u(x) \approx u^h(x) = \sum_{i=1}^{n} N_i(x) u_i = \mathbf{N}\mathbf{u}$$



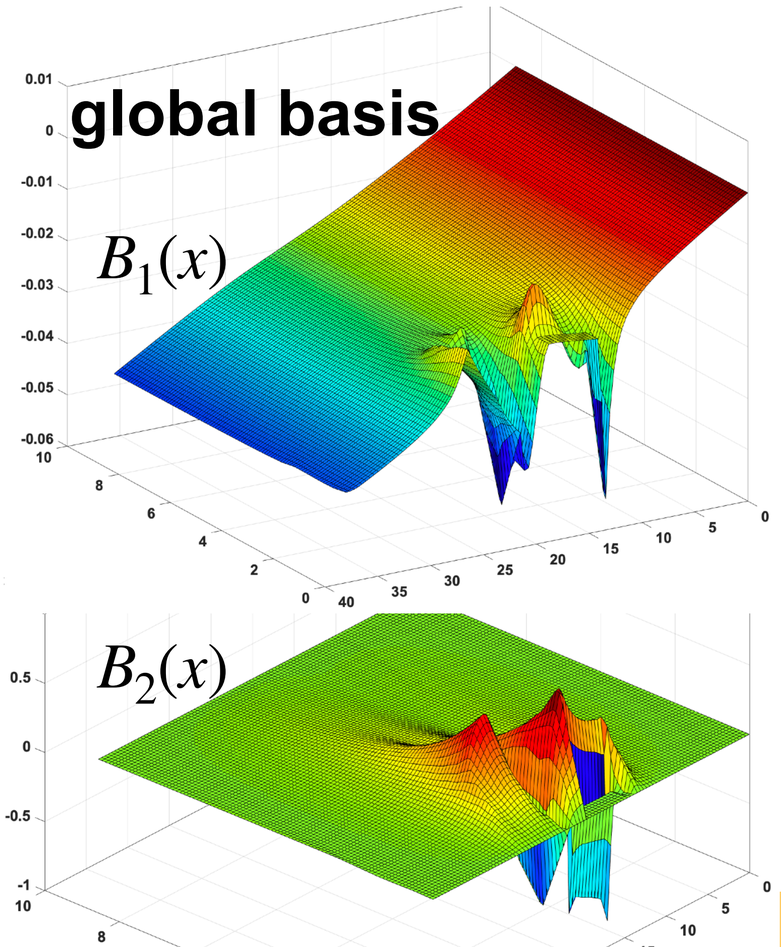
LSE with matrix

$$\mathbf{K} \in \mathbb{R}^{n \times n}$$

n is ~ to number of nodes (large)

- Reduced Order (RO)

$$u(x) \approx u^{RB}(x) = \sum_{j=1}^{m} B_j \alpha_j = \mathbf{B}\alpha$$



$$\tilde{\mathbf{K}} \in \mathbb{R}^{m \times m}$$

$$m \ll n$$

UNIVERSITAT POLITÈCNICA DE CATALUNYA
LABORATORI DE CÀLCUL NUMÈRIC

Answer in

JGR Solid Earth

RESEARCH ARTICLE
10.1029/2019JB018314

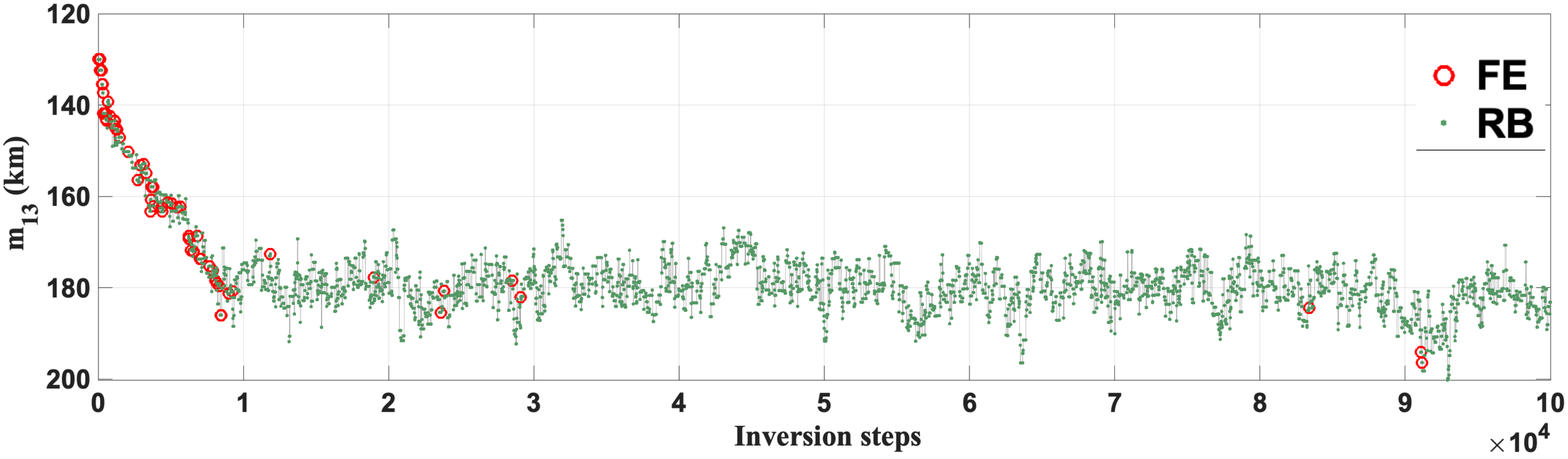
Key Points:
- An efficient method for producing ultrafast solutions of the 3-D Stokes problem for mantle convection is presented

Fast Stokes Flow Simulations for Geophysical-Geodynamic Inverse Problems and Sensitivity Analyses Based On Reduced Order Modeling

O. Ortega-Gelabert[1], S. Zlotnik[1], J. C. Afonso[2,3], and P. Díez[1]

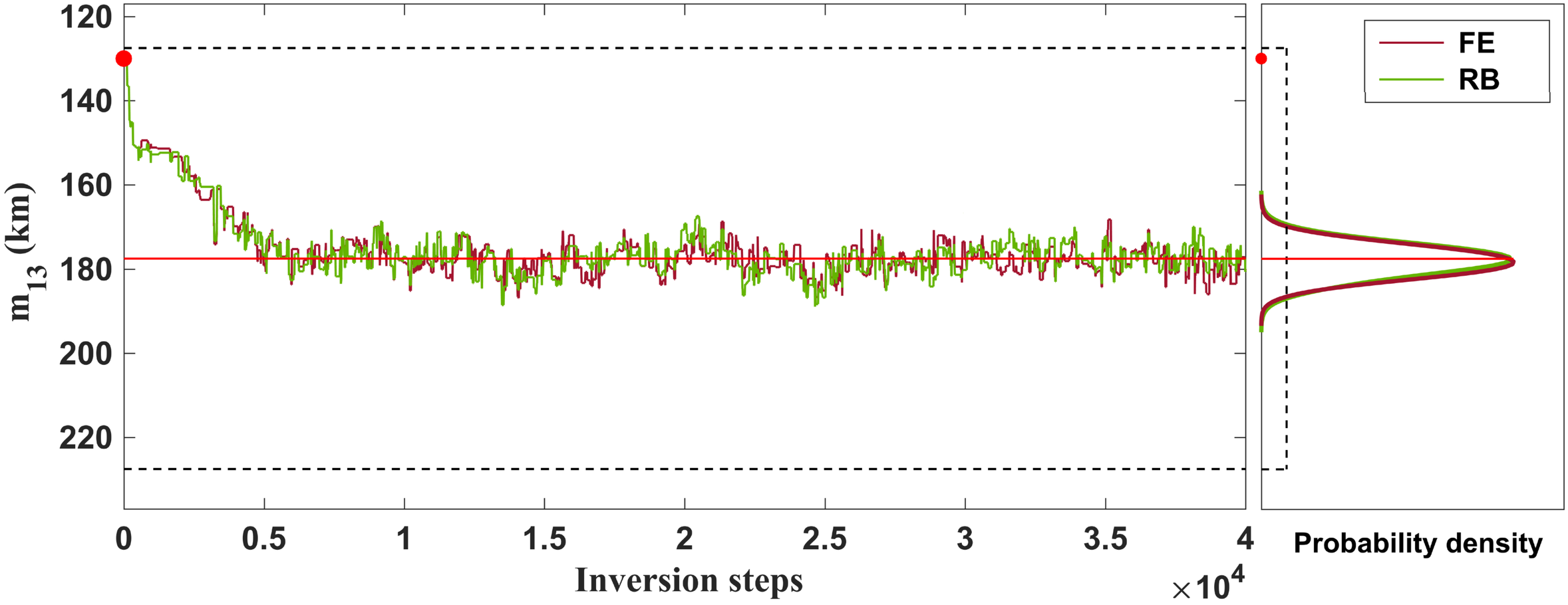
We proposed a greedy approach that learns from the inversion and creates a basis B_j.

Use of FE during the inversion



Number of FE solves: 115 (0.11%, one every 1000 inversion steps)
Number of RB solves: 99885 (99.89%)

Comparison of RB and FE inversions



LABORATORI DE CÀLCUL NUMÈRIC UNIVERSITAT POLITÈCNICA DE CATALUNYA

- Greedy Reduced Basis with Goal-Oriented error control provides an interesting framework to drastically reduce the computational cost associated with (linear) forward problems within an stochastic inversion.

JGR Solid Earth

RESEARCH ARTICLE
10.1029/2019JB018314

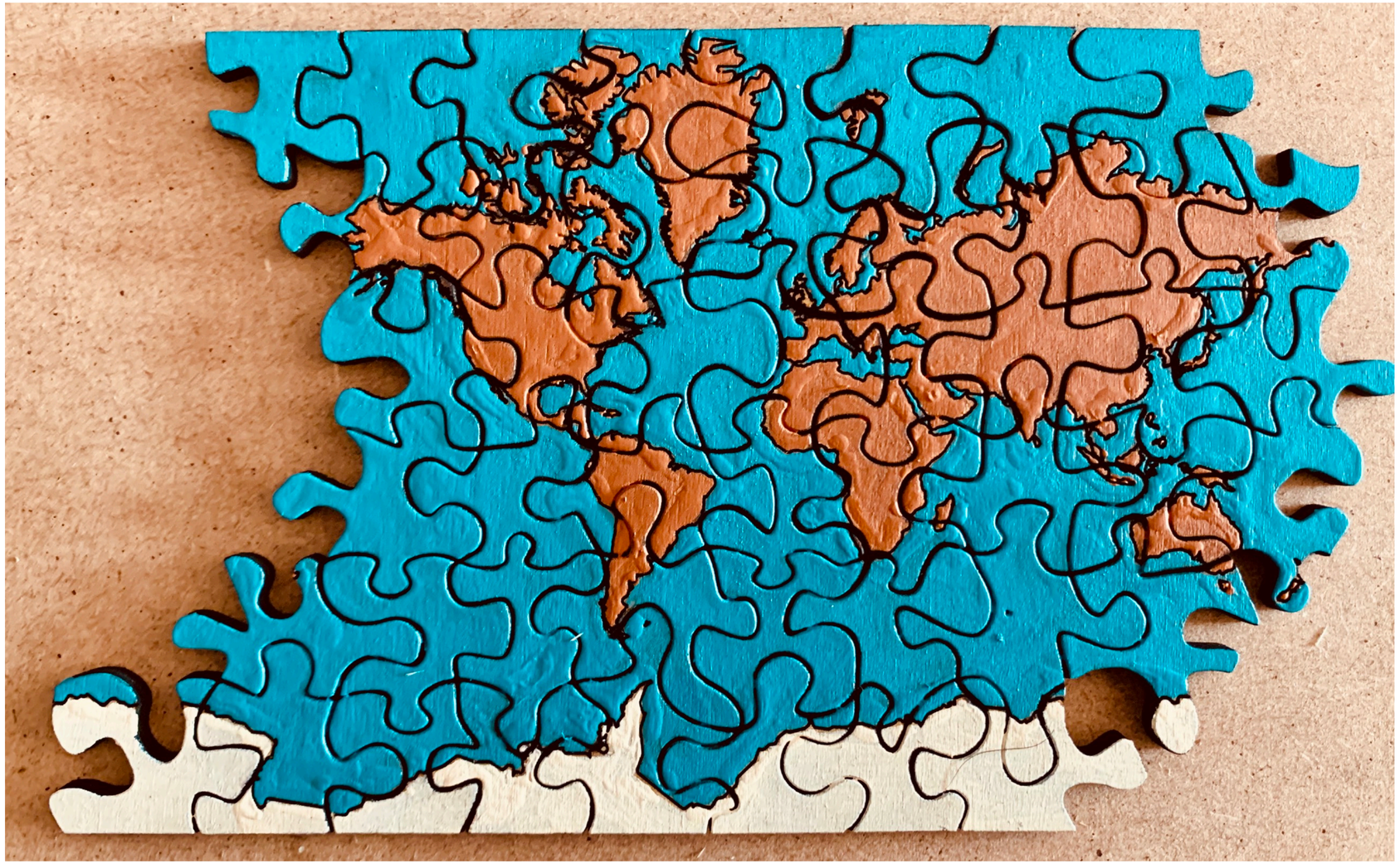
Key Points:
- An efficient method for producing ultrafast solutions of the 3-D Stokes problem for mantle convection is presented

Fast Stokes Flow Simulations for Geophysical-Geodynamic Inverse Problems and Sensitivity Analyses Based On Reduced Order Modeling

O. Ortega-Gelabert[1], S. Zlotnik[1], J. C. Afonso[2,3], and P. Díez[1]

A reduced order approach for probabilistic inversions of 3–D magnetotelluric data I: general formulation

M C Manassero ✉, J C Afonso, F Zyserman, S Zlotnik, I Fomin

Geophysical Journal International, Volume 223, Issue 3, December 2020, Pages 1837–1863, https://doi-org.recursos.biblioteca.upc.edu/10.1093/gji/ggaa415
Published: 01 September 2020 **Article history ▾**

Thank you!