Joint Distribution of Rainfall Characteristics: Intensity, Total Depth, Spatial and Temporal Moments

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STUDY CONTEXT AND GOALS

EVENT SELECTION PROCEDURE

SPATIAL AND TEMPORAL MOMENTS

MARGINAL DISTRIBUTIONS THROUGH L-MOMENTS

JOINT DISTRIBUTION THROUGH VINE COPULA

GOODNESS-OF-FIT COMPARISON

University of Bristol

UKRI Engineering and Physical Sciences Research Council

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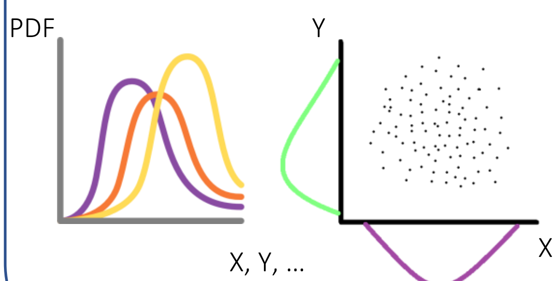
Project overview

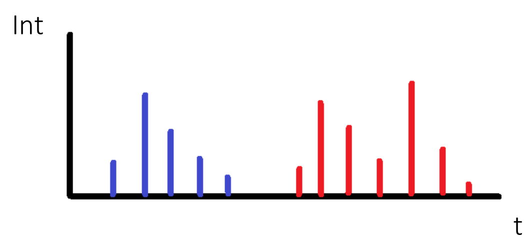
This project aims to establish a method to fit joint distribution function of several rainfall event characteristics.

Marginal distributions are analysed by **L-moment ratios**, while dependence between variables is represented through **Vine Copula.**

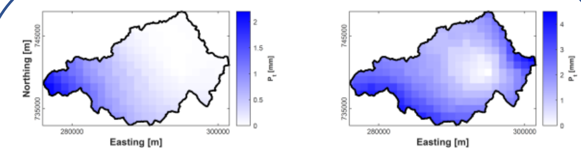


Rainfall events are defined by event-based variables such as
Intensity and **Total Depth**.

Their internal variability is described temporally on the hyetograph through **temporal moments.**

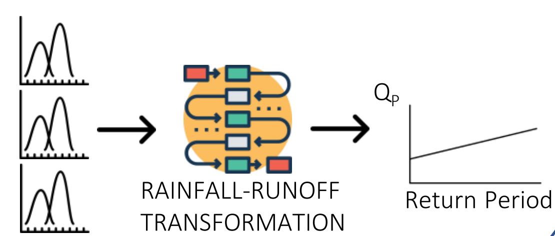


They aim to describe features like the rainfall center of mass or whether events are unimodal/bimodal.



The spatial internal variability is also taken into account by considering **spatial moments**, computed about time-averaged rainfall intensity.

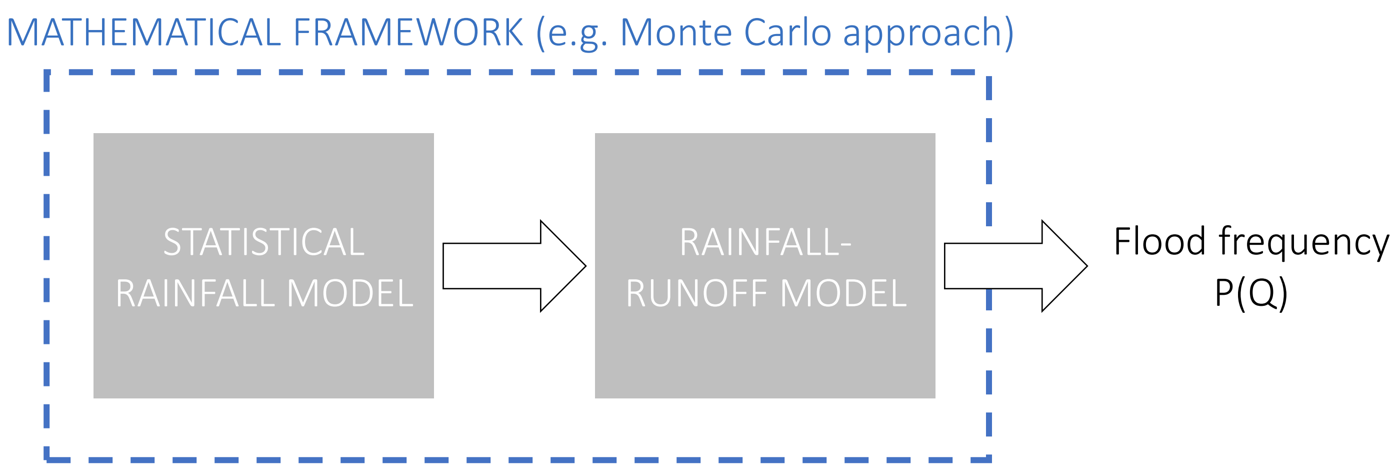
Eventually this information would be used to link flood frequency with those associated to rainfall events.



Project context

Flood distribution function can be studied and estimated by **derived flood frequency method**, firstly described by *Eagleson* (1972) and then adopted, among many, by *Sivapalan* et al. (2005).

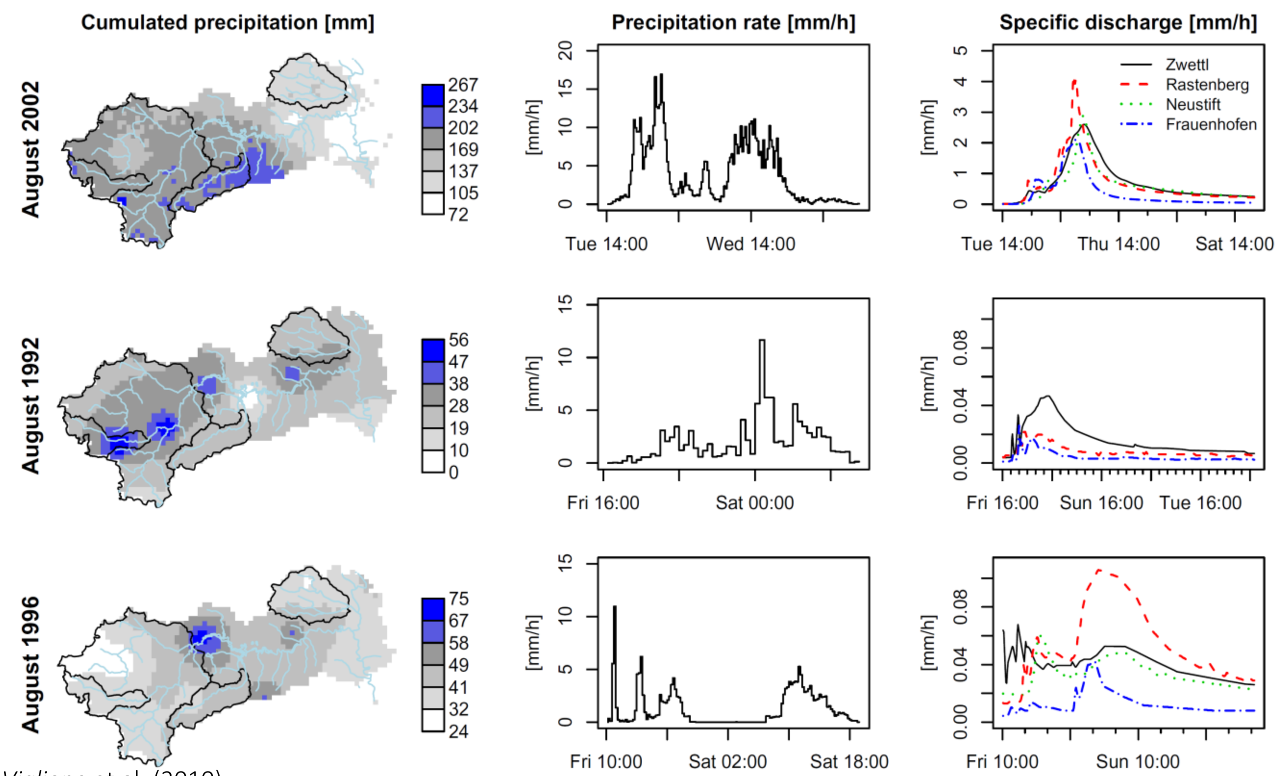
This approach is based on derived distribution theory and relates peak streamflow statistics with those of climate and catchment antecedent conditions. The main advantage introduced by this method is to understand which processes are dominant to flood frequency behaviour. Its application requires knowledge of both rainfall distribution and rainfall-runoff transformation.



Derived Flood Frequency approach,
as described by *Sivapalan* et al. (2005)

Project context

Peak flow estimation methods usually define rainfall events by prespecified ad-hoc temporal and spatial patterns, along with relevant intensity-duration-frequency (IDF) curves. Some example of these patterns are represented by *Chicago* hyetograph and areal reduction factor usage. We complement the standard precipitation metrics of **intensity**, **duration** or **total depth** with **spatial and temporal moments**, which give indication of rainfall event spatial and temporal internal variability. These properties are associated to streamflow response and its timing (*Viglione* et al., 2010).



Viglione et al. (2010)

Project goals and datasets

The aim of this project is to determine a methodology to fit **multivariate joint cumulative distribution** of rainfall events, which in turn are defined as sets of characteristics such intensity, total depth, spatial and temporal moments. This task would require also the definition of their **marginal distributions**. The statistical approach developed within this project is intended to be generally applicable on any other set of catchments.

Datasets:

- 15-min streamflow data series were provided by *Environmental Agency*.
- Hourly rainfall gridded data were interpolated and distributed by *Lewis* et al. (2018).

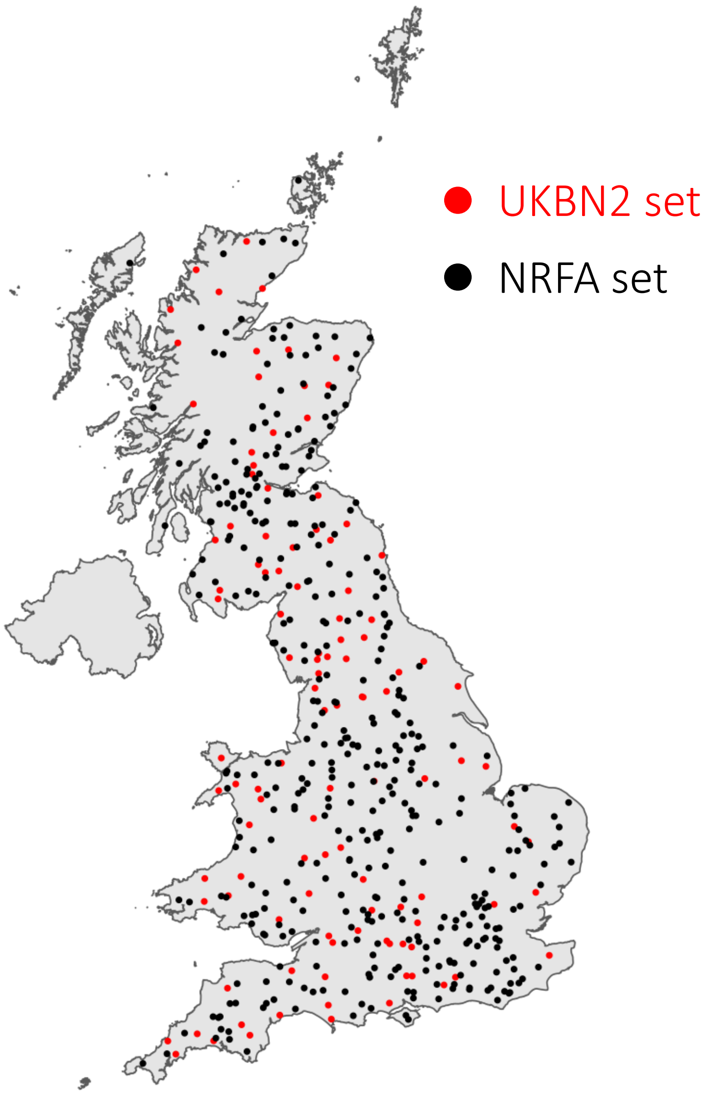
Lewis et al. (2018)

Case studies

The analysis is undertaken over 523 United Kingdom (UK) catchments with an area ranging up to 1505 km^2, during 25-year period from 1990 to 2014.

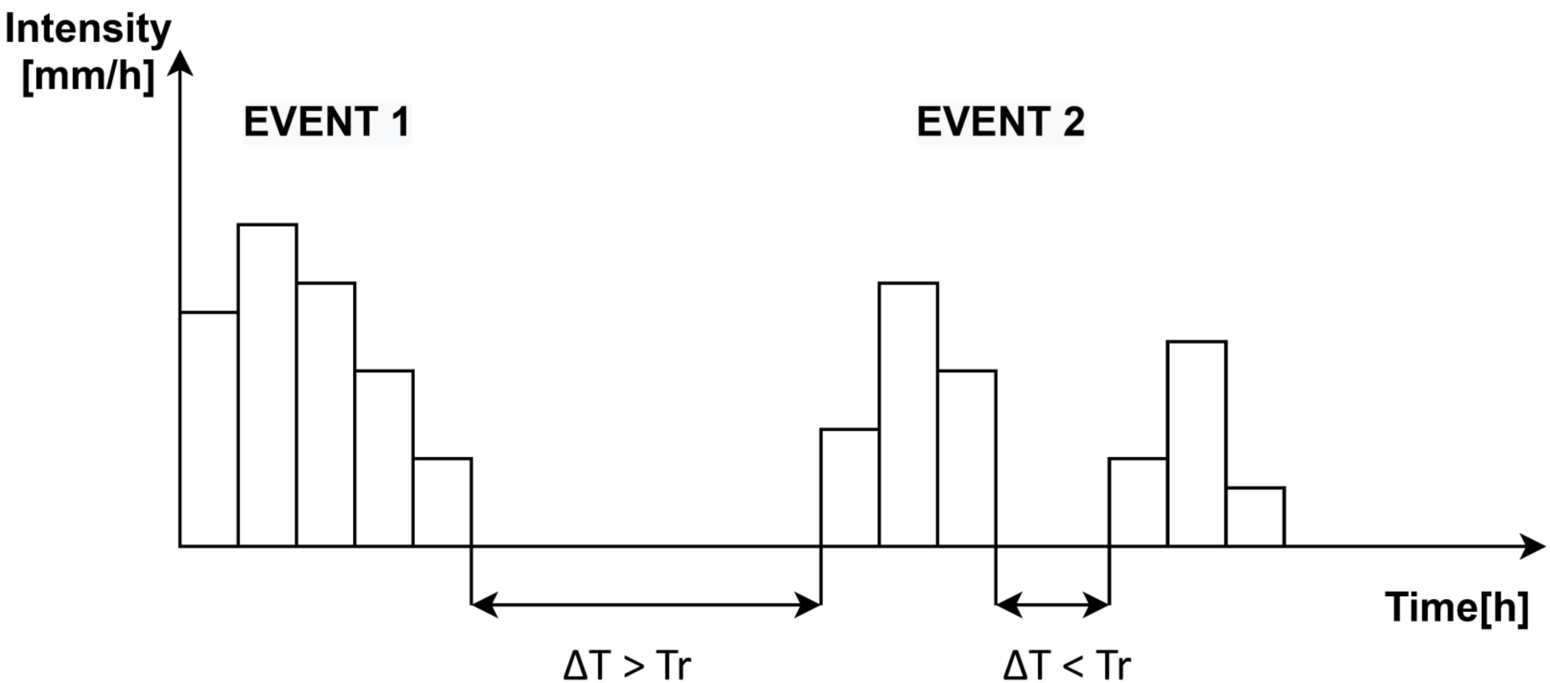
The distribution analysis was undertaken only on 112 out of 523 UK catchments (UKBN2 set), chosen from the UK Benchmark network 2 (UKBN2) (*Harrigan* et al., 2018), which includes stations most suited for long-term hydrological analysis on changes and variability.

The remainding 411 basins (NRFA set) represent stations from National Flow River Archive (NRFA) as well and were used to test distribution model goodness-of-fit.



Event selection procedure

Rainfall events are identified using a Minimum Inter-event Time (MIT), the minimum number of hours of zero rainfall to separate rainfall events. We set the MIT equal to **catchment response time T_r**, computed following the methodology by *Giani* et al. (2021). We often see when taking this approach that, if the inter-event time is shorter than the response time, the two rainfall contributions will be part of the same hydrograph; if instead the same two rainfall contributions have an inter-event time that is longer than the catchment response time, we will likely find two separate peaks in the hydrograph.

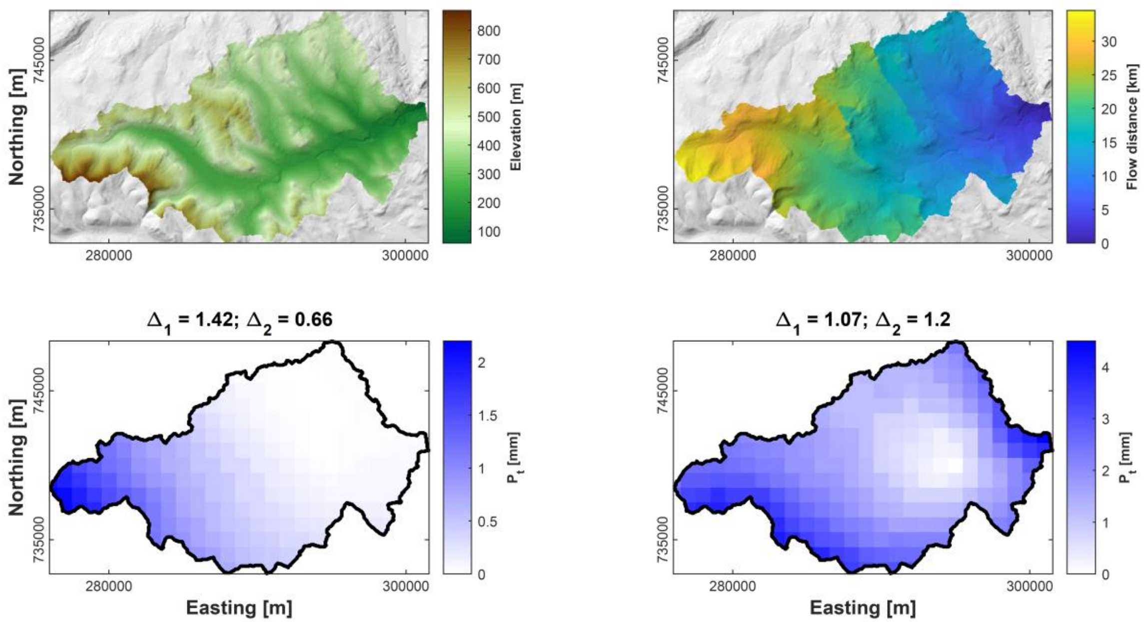


Spatial moments of catchment rainfall

They were firstly defined by *Zoccatelli* et al. (2011) and aim to represent spatial rainfall event distribution measured along the flow path. They are both dimensionless and can assume only positive values.

First spatial moment $\Delta_1 > 1$ indicates storm is mainly **located towards the headwater**, while $\Delta_1 < 1$ when the event is spatially **closer to the outlet**. When second spatial moment $\Delta_2 > 1$ rainfall event is **bimodal**, having two top intensity values, vice versa $\Delta_2 < 1$ indicates a **unimodal** trend with only one spatial peak. Spatially uniform events are characterised by $\Delta_1 \approx 1$ and $\Delta_2 \approx 1$.

Pictures on the right show elevation and flow distance maps of Scottish catchment (NRFA ID. 15023), with time-average rainfall intensity of two selected events reported below, along with their moment values.



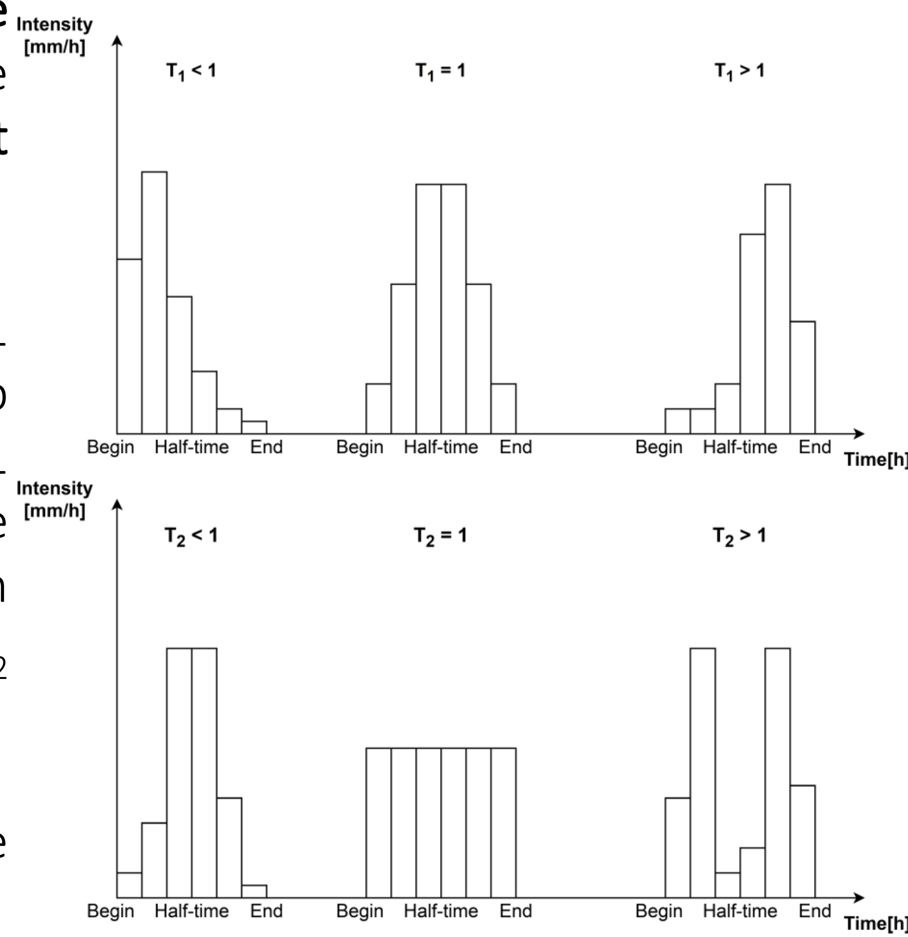
Temporal moments of catchment rainfall

Analogously to those introduced before, these moments aim to represent temporal rainfall event distribution measured along the hyetograph. They are both dimensionless and can assume only positive values.

First temporal moment $T_1 > 1$ indicates storm is mainly distributed **after the event half-time**, while $T_1 < 1$ when the event is occurring mainly **within the first half**.

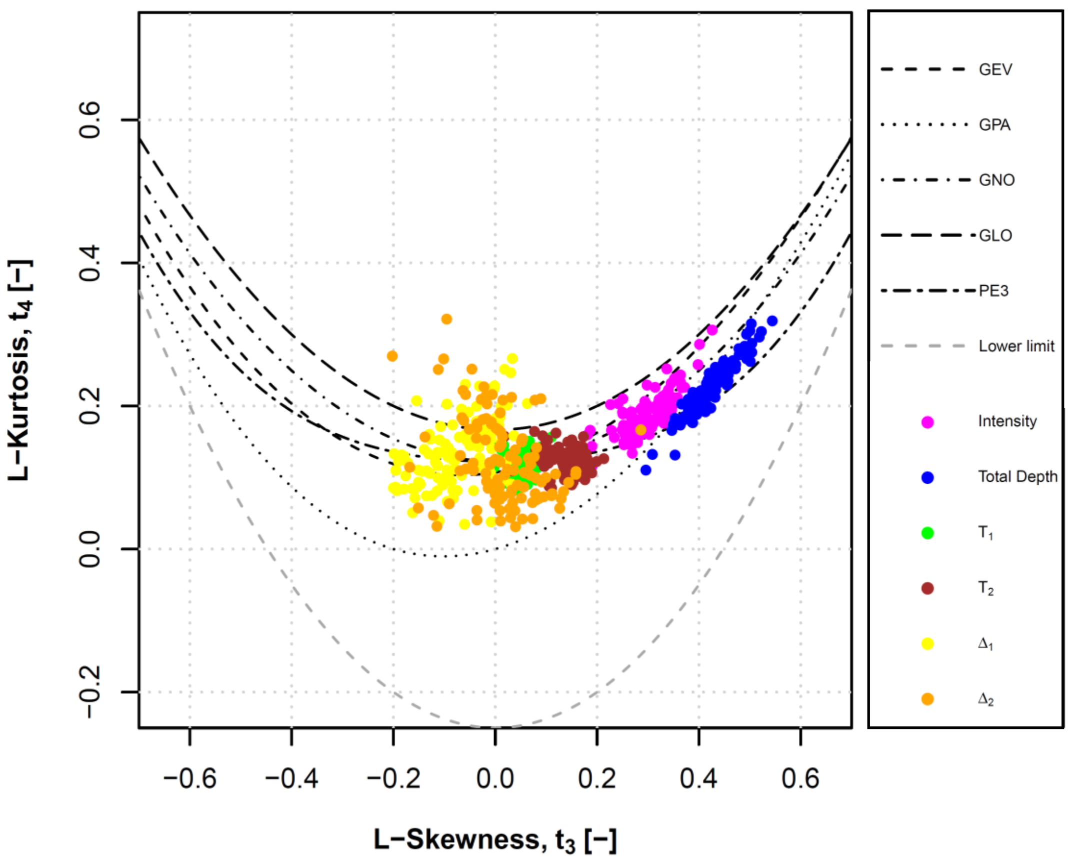
When second temporal moment $T_2 > 1$ rainfall event is **bimodal**, having two top intensity values, vice versa $T_2 < 1$ indicates a **unimodal** trend with only one hyetograph peak. Temporally uniform events are characterised by $T_1 \approx 1$ and $T_2 \approx 1$.

On the right all the six possible events are shown with their related moment values.



Marginal distributions analysis

After having extracted 2966436 **spatially uniform rainfall events** over 112 UKBN2 basins data availability, their intensity, total depth, temporal and spatial moments were selected for each catchment. Those characteristic marginal distributions were studied through **L-moments** and represented on **L-moments ratio diagram** (*Hosking & Wallis*, 1997).

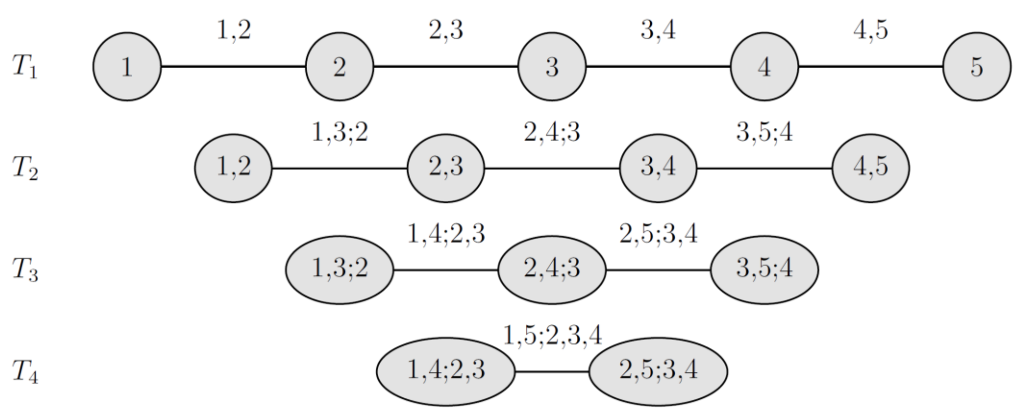


Best parametric models were chosen accordingly best RMSE among all the tested families. Results suggest intensity is represented by **Generalised Normal (GNO),** total depth by **Generalised Pareto (GPA)**, spatial and temporal moments by **Pearson type 3 model (PE3).**

Multivariate joint distribution analysis

Joint distribution aims to describe dependence between a set of random variables. **Vine copula** is a very flexible and promising tool for multivariate dependence analysis, given its capability to describe high number of variables (*Hao & Singh*, 2016). It can specify a whole degree of dependence with one **multiblock structure** made by bivariate building blocks, in turn specified by **simpler copulas** with their set of parameters (*Czado*, 2019).

A graphical example of one possible Vine is reported on the right. This is an example of **D-Vine** fitted for 5 random variables(1, 2, 3, 4 and 5). It can be noticed how Vines are always characterised by n - 1 trees (T_1, T_2, T_3 and T_4), with n = number of random variables. Edges represent bivariate copula distributions while nodes are variables. Edges of j-th tree become nodes in (j + 1)-th tree.

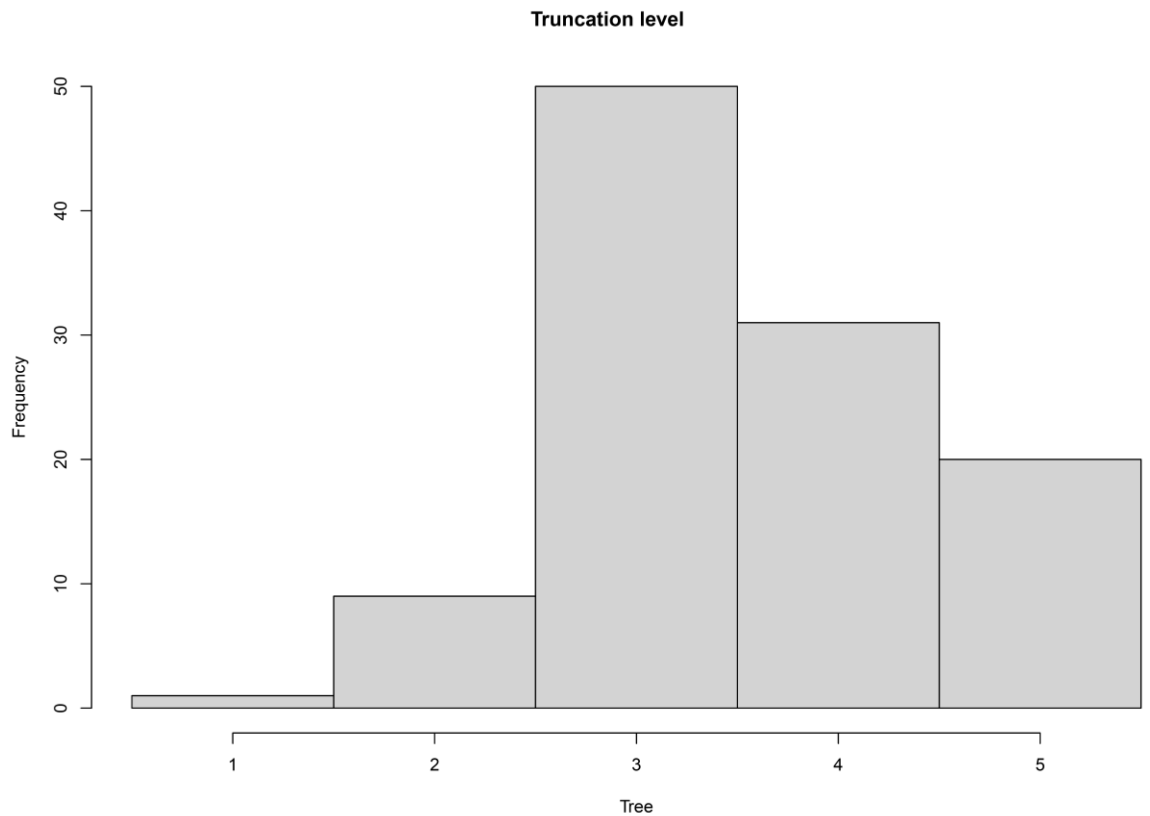


Czado (2019)

Multivariate joint distribution analysis

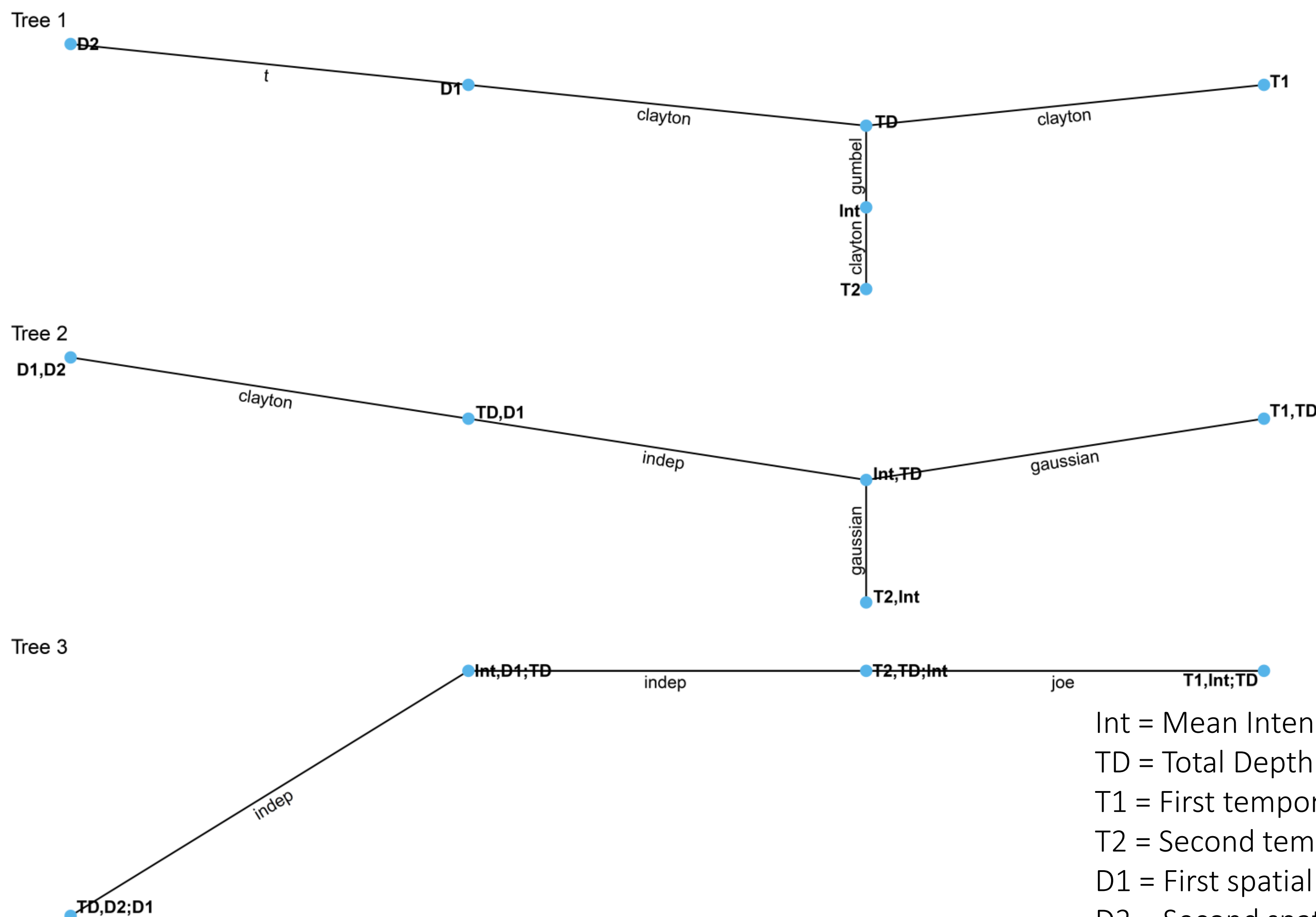
First step was to define a **truncation level** for the whole distribution. This is specifically the j-th tree beyond which all edges would be represented by independence copulas, which are parameter-free. This is done in favour of parsimony (less parameters) but also because most dependencies are usually represented already within lower order trees (*Brechmann* et al., 2012).

Following the approach of *Brechmann* et al. (2012), truncation level was indicated as Tree 3 to be the best for most of the case studies. Trees structure selection was performed by means of *Dißmann*'s algorithm, with BIC set as selection criterion.



Multivariate joint distribution analysis

Eventually it was chosen the joint distribution configuration which was more frequently fitted over the whole set of basins, in terms of both established relations (edges) and selected bivariate parametric copula families.

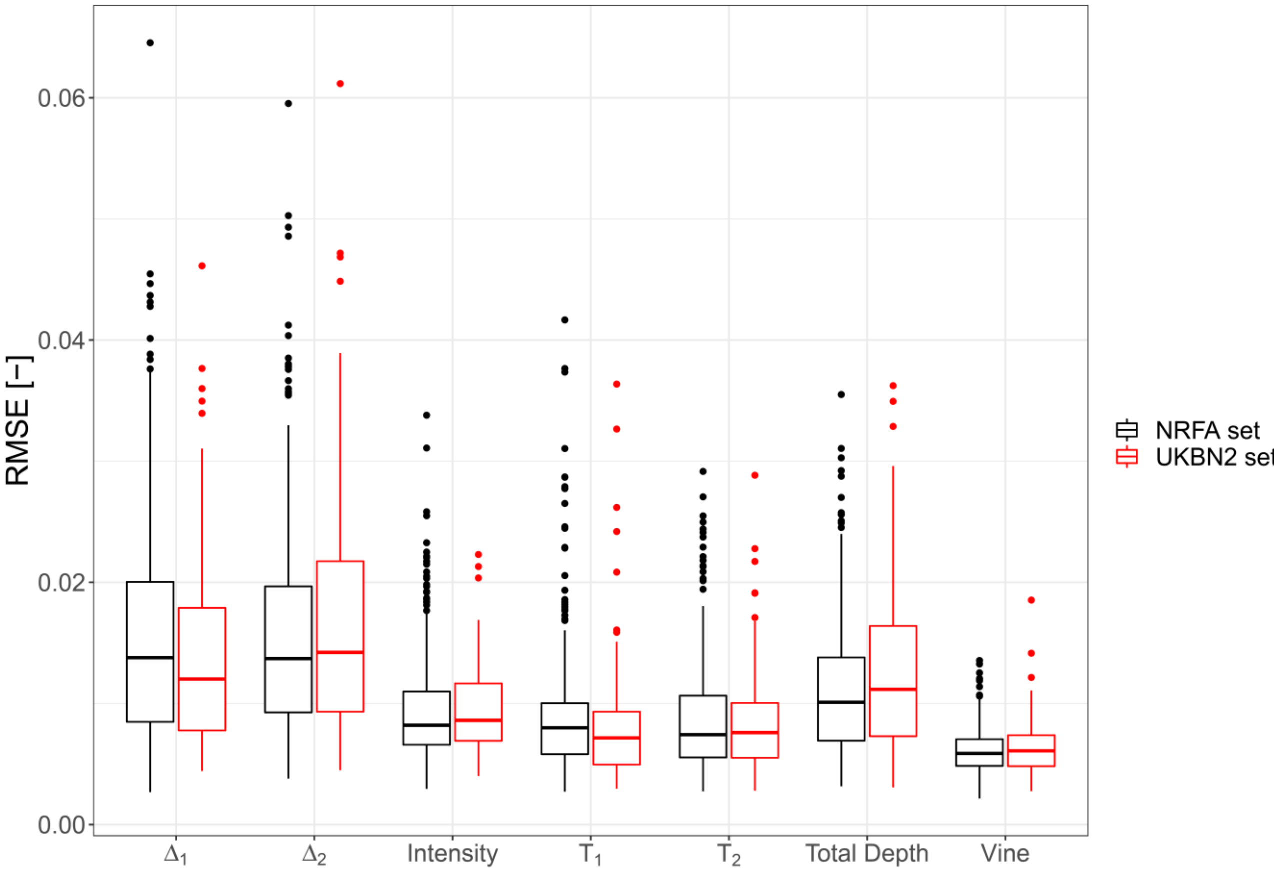


Int = Mean Intensity
TD = Total Depth
T1 = First temporal moment
T2 = Second temporal moment
D1 = First spatial moment
D2 = Second spatial moment

Goodness-of-fit comparison

The outlined model was tested in terms of its **goodness-of-fit** on both 112 basins selected from UKBN2(UKBN2 set) and remainder 411 catchments(NRFA set). The goodness-of-fit was computed as RMSE for every rainfall characteristics and the final Vine structure. The fitted models were compared against their empirical cumulative distribution function for marginals, while Vine was compared against its empirical copula.

The results show how goodness-of-fit on both sets of catchments is very similar. Overall, that suggests models inferenced by the analysis have the potential of representing rainfall characteristic distributions and their dependences.



Conclusions and future works

This project has highlighted:

1. A methodology to fit joint distribution function for rainfall event features, along with their marginal distribution functions as well.
2. A joint distribution model valid for UK case of study, which appears to be suitable for spatially uniform rainfall event characteristics and smaller catchments.

These tools can be exploited within the derived flood frequency method (see Woods & Sivapalan, 1999), in order to randomly generate hydrologically-relevant rainfall events.

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