

Multi-frequency and multi-GNSS PPP-RTK for vehicle navigation in urban environments

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1. Introduction

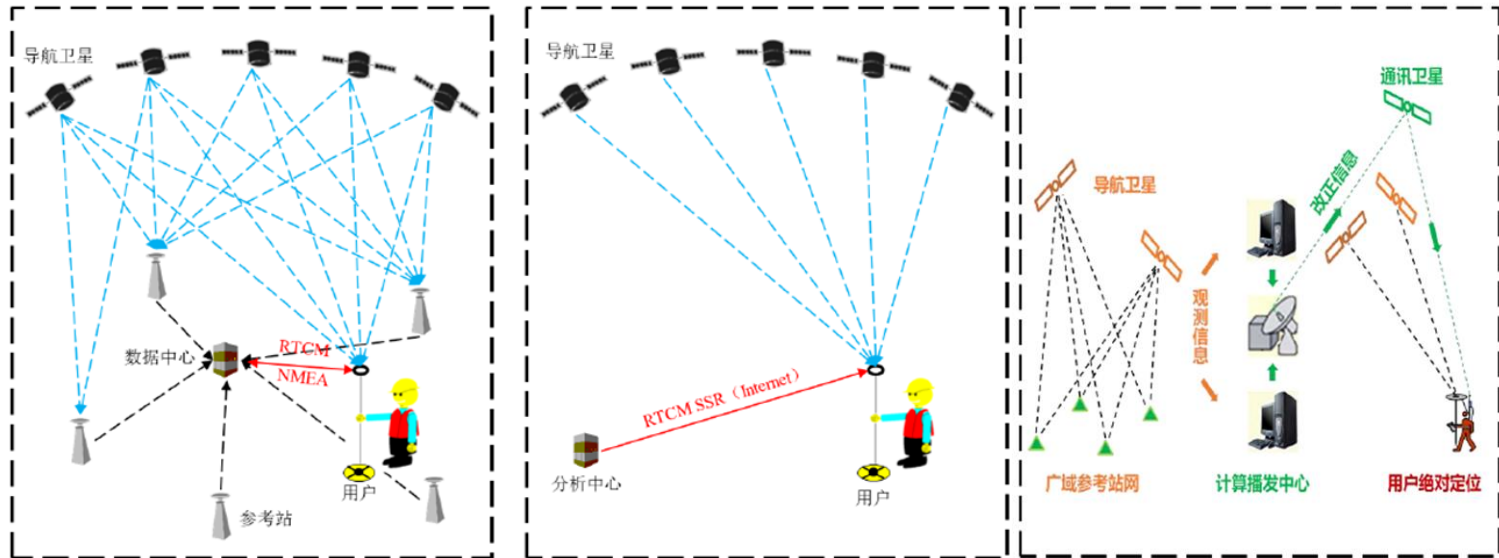


Fig.1 RTK, PPP and PPP-RTK methods.

- In order to integrate advantages of precise point positioning (PPP) and real-time kinematic (RTK), PPP-RTK method has been proposed to achieve centimeter-level positioning with rapid integer ambiguity resolution and implemented in some commercial systems such as Trimble RTX-Fast, NavCom StarFire etc.

1. Introduction



Table 1 Overview of multi-frequency and multi-GNSS satellites.

	GNSS	Sat. type	Navigation signals	NSat.
	GPS	Block IIR-A/B	L1 C/A, L1/L2 P(Y)	8
		Block IIR-M	+L2C	8
		Block IIF/III	+L5/ +L5, L1C	16
	GLONASS	GLONASS-M	L1/L2 C/A+P	24
		GLONASS-K	L1/L2 C/A+P, L3 (CDMA)	3
	BDS	BDS-2	B1, B2, B3	15
		BDS-3	B1, B3, B1C, B2a/b/a+b	30
	GALILEO	IOV	E1, E5a/b/a+b	4
		FOC	E1, E5a/b/a+b	22
	QZSS	IGSO	L1, L2, L5	4
	IRNSS/ NavIC	IGSO	L5, S	5
		GEO	L5, S	3

- As of March 2021, the GNSS constellation includes 140 available satellites in orbit. New-generation GNSS satellites such as GPS BlockIIF/III, GLONASS-K1, BDS, Galileo, and QZSS are capable of transmitting signals at three or more frequencies. Multi-GNSS and multi-frequency observations bring more possibilities for enhancing the performance of PPP-RTK.
- We developed a multi-frequency and multi-GNSS PPP-RTK system aimed at realizing precise positioning for vehicle navigation in urban environments.

2. Methods



Estimating EWL/WL/NL UPD

- Hatch-Melbourne-Wübbena (HMW) (Hatch 1982; Melbourne 1985; Wübbena 1985).

$$\begin{cases} \hat{N}_{r,ewl25}^s = \left(\frac{L_{r,2}^s}{\lambda_2} - \frac{L_{r,5}^s}{\lambda_4} - \frac{(1-\gamma)P_{r,1}^s + \eta P_{r,2}^s}{\lambda_{ewl25}} \right) = N_{r,ewl25}^s + d_{r,ewl25} - d_{ewl25}^s \\ \hat{N}_{r,ewl24}^s = \left(\frac{L_{r,2}^s}{\lambda_2} - \frac{L_{r,4}^s}{\lambda_4} - \frac{(1-\beta)P_{r,1}^s + \eta P_{r,2}^s}{\lambda_{ewl24}} \right) = N_{r,ewl24}^s + d_{r,ewl24} - d_{ewl24}^s \\ \hat{N}_{r,ewl}^s = \left(\frac{L_{r,2}^s}{\lambda_2} - \frac{L_{r,3}^s}{\lambda_3} - \frac{(1-\alpha)P_{r,1}^s + \eta P_{r,2}^s}{\lambda_{ewl}} \right) = N_{r,ewl}^s + d_{r,ewl} - d_{ewl}^s \\ \hat{N}_{r,wl}^s = \left(\frac{L_{r,1}^s}{\lambda_1} - \frac{L_{r,2}^s}{\lambda_2} - \frac{\lambda_2 P_{r,1}^s + \lambda_1 P_{r,2}^s}{(\lambda_2 + \lambda_1)\lambda_{wl}} \right) = N_{r,wl}^s + d_{r,wl} - d_{wl}^s \\ \hat{N}_{r,nl}^s = \frac{\hat{N}_{r,12}^s}{\lambda_{nl}} - \frac{\lambda_1 \cdot N_{r,wl}^s}{\lambda_2 - \lambda_1} = N_{r,nl}^s + d_{r,nl} - d_{nl}^s \end{cases}, \begin{cases} \alpha = \frac{f_1^2 f_2 - f_2^2 f_3}{f_3(f_1^2 - f_2^2)} \\ \beta = \frac{f_1^2 f_2 - f_2^2 f_4}{f_4(f_1^2 - f_2^2)} \\ \gamma = \frac{f_1^2 f_2 - f_2^2 f_5}{f_5(f_1^2 - f_2^2)} \end{cases}$$

- Uncalibrated phase delay (UPD) equation based on PPP solution.

- Extra wide lane (EWL), wide lane (WL) and narrow lane (NL) can be estimated by the least square method (Li et al., 2018; Li et al., 2019).

$$\begin{cases} \hat{N}_r^G = N_r^G + d_G - d^{s,G} \\ \hat{N}_r^R = N_r^R + d_R - d^{s,R} \\ \hat{N}_r^E = N_r^E + d_E - d^{s,E} \\ \hat{N}_r^C = N_r^C + d_C - d^{s,C} \end{cases}$$

$$\begin{bmatrix} \hat{N}_1^1 - N_1^1 \\ \vdots \\ \hat{N}_1^m - N_1^m \\ \hat{N}_2^1 - N_2^1 \\ \vdots \\ \hat{N}_2^m - N_2^m \\ \hat{N}_n^1 - N_n^1 \\ \vdots \\ \hat{N}_n^m - N_n^m \end{bmatrix} = \begin{bmatrix} R_{1G} & R_{1C} & R_{1R} & R_{1E} & S_1 \\ R_{2G} & R_{2C} & R_{2R} & R_{2E} & S_2 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ R_{nG} & R_{nC} & R_{nR} & R_{nE} & S_n \end{bmatrix} \begin{bmatrix} d_G \\ d_C \\ d_R \\ d_E \\ d^s \end{bmatrix}$$



2. Methods

□ Extracting precise atmospheric delays

- Multi-GNSS undifferenced and uncombined (UDUC) PPP model

$$\begin{cases} p_{r,n}^{s,G} = \mu_r^{s,G} \cdot \chi + \hat{t}_r - \bar{t}_{ij}^{s,sys} + \gamma_n^G \cdot \hat{I}_{r,1}^{s,G} + m_{r,w}^{s,G} \cdot Z_{r,w} + \kappa \cdot IFB_n^G + e_{r,n}^{s,G} \\ p_{r,n}^{s,E} = \mu_r^{s,E} \cdot \chi + \hat{t}_r - \bar{t}_{ij}^{s,sys} + \gamma_n^E \cdot \hat{I}_{r,1}^{s,E} + m_{r,w}^{s,E} \cdot Z_{r,w} + ISB_{E-G} + e_{r,n}^{s,E} \\ p_{r,n}^{s,C} = \mu_r^{s,C} \cdot \chi + \hat{t}_r - \bar{t}_{ij}^{s,sys} + \gamma_n^C \cdot \hat{I}_{r,1}^{s,C} + m_{r,w}^{s,C} \cdot Z_{r,w} + ISB_{C-G} + e_{r,n}^{s,C} \end{cases}$$

$$\begin{cases} l_{r,n}^{s,G} = \mu_r^{s,G} \cdot \chi + \hat{t}_r - \bar{t}_{ij}^{s,sys} - \gamma_n^G \cdot \hat{I}_{r,1}^{s,G} + m_{r,w}^{s,G} \cdot Z_{r,w} + \hat{N}_{r,n}^{s,G} + \varepsilon_{r,n}^{s,G} \\ l_{r,n}^{s,E} = \mu_r^{s,E} \cdot \chi + \hat{t}_r - \bar{t}_{ij}^{s,sys} - \gamma_n^E \cdot \hat{I}_{r,1}^{s,E} + m_{r,w}^{s,E} \cdot Z_{r,w} + \hat{N}_{r,n}^{s,E} + \varepsilon_{r,n}^{s,E} \\ l_{r,n}^{s,C} = \mu_r^{s,C} \cdot \chi + \hat{t}_r - \bar{t}_{ij}^{s,sys} - \gamma_n^C \cdot \hat{I}_{r,1}^{s,C} + m_{r,w}^{s,C} \cdot Z_{r,w} + \hat{N}_{r,n}^{s,C} + \varepsilon_{r,n}^{s,C} \end{cases}$$

- By conducting multi-GNSS PPP AR with UPDs, we can get **precise zenith tropospheric delay** from the parameter $Z_{r,w}$ that is obtained by the least square method.

- **Geometry-free** model.

$$\begin{aligned} \Phi_{r,4}^s &= -(\gamma_{i_1} - \gamma_{i_2}) \cdot I + (N_{r,i_1} - N_{r,i_2}) \\ &= -(\gamma_{i_1} - \gamma_{i_2}) \cdot I + \lambda_{i_1} N_{r,i_1} - \lambda_{i_2} N_{r,i_2} + \lambda_{i_1} (b_{r,i_1} - b_{i_1}^s) - \lambda_{i_2} (b_{r,i_2} - b_{i_2}^s) \end{aligned}$$

- Precise **ionospheric delays** can be extracted when the geometry-free observations and undifferenced ambiguities are known.
- Then **modified linear combination method (MLCM)** is employed to interpolate the precise tropospheric delays and ionospheric delays at user station (Li et al. 2011).

2. Methods

Multi-frequency and multi-GNSS PPP-RTK

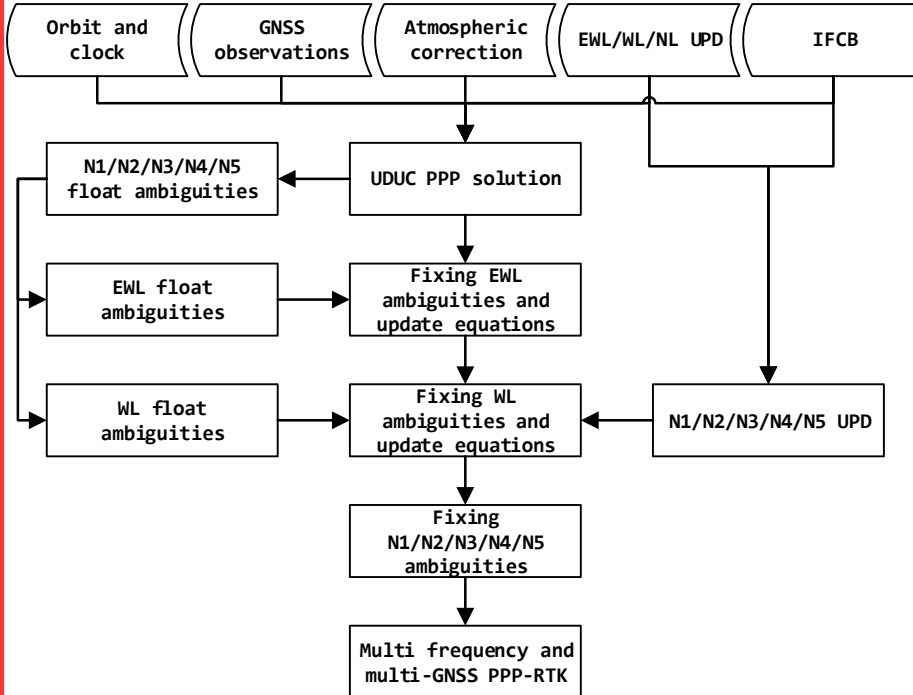


Fig.2 Flow chart of multi-frequency and multi-GNSS PPP-RTK in client (EWL ambiguity consists of observations at 2/3, 2/4 or 2/5 frequencies)

➤ Multi-frequency and multi-GNSS **UDUC PPP** model is constrained by the precise ionospheric and tropospheric delays.

$$\begin{cases} p_{r,n}^{s,\text{sys}} = \mu_r^{s,\text{sys}} \cdot x + \hat{t}_r^{\text{sys}} - \bar{t}_{IF}^{s,\text{sys}} + \gamma_{ik}^{\text{sys}} \hat{I}_{r,i}^{s,\text{sys}} + m_{r,w}^{s,\text{sys}} Z_{r,w} + \kappa \cdot IFB^{\text{sys}} + e_{r,n}^{s,\text{sys}} \\ l_{r,n}^{s,\text{sys}} = \mu_r^{s,\text{sys}} \cdot x + \hat{t}_r^{\text{sys}} - \bar{t}_{IF}^{s,\text{sys}} - \gamma_{ik}^{\text{sys}} \cdot \hat{I}_{r,i}^{s,\text{sys}} + m_{r,w}^{s,\text{sys}} Z_{r,w} + \hat{N}_{r,n}^{s,\text{sys}} + \varepsilon_{r,n}^{s,\text{sys}} \\ ll_i^{s,\text{sys}} = \hat{I}_{r,n}^{s,\text{sys}} - \tilde{I}_{r_1,r_2 \dots r_n}^{s,\text{sys}} = w_I, \quad w_I \sim N(0, \sigma_{w_I}^2) \\ ll_z^{s,\text{sys}} = Z_{r,w} - \tilde{Z}_{r_1,r_2 \dots r_n} = w_Z, \quad w_Z \sim N(0, \sigma_{w_Z}^2) \end{cases}$$

$$\Sigma_{UC} = \text{diag}(\sigma_p^2, \sigma_I^2, \sigma_{w_I}^2, \sigma_{w_Z}^2)$$

➤ EWL-WL-NL ambiguity resolution.

$$\begin{cases} p_{r,n}^s = \mu_r^s \cdot x + \hat{t}_r^s - \bar{t}_{IF}^s + \gamma_{ik}^s \hat{I}_{r,i}^s + m_{r,w}^s Z_{r,w} + \kappa \cdot IFB + e_{r,n}^s \\ l_{r,n}^s = \mu_r^s \cdot x + \hat{t}_r^s - \bar{t}_{IF}^s - \gamma_{ik}^s \cdot \hat{I}_{r,i}^s + m_{r,w}^s Z_{r,w} + \hat{N}_{r,n}^s + \varepsilon_{r,n}^s \\ ll_1 = \Delta \hat{N}_{r,ewl_jk}^{s_1 s_2} = N_{r,ewl_jk}^{s_1 s_2} + d_{ewl_jk}^{s_1 s_2} \\ ll_2 = \Delta \hat{N}_{r,wl_jk}^{s_1 s_2} = N_{r,wl_jk}^{s_1 s_2} + d_{wl_jk}^{s_1 s_2} \\ ll_3 = \Delta \hat{N}_{r,nl_jk}^{s_1 s_2} = N_{r,nl_jk}^{s_1 s_2} + d_{nl_jk}^{s_1 s_2} \end{cases}$$

$$\Sigma_{UC} = \text{diag}(\sigma_p^2, \sigma_I^2, \sigma_{ll_1}^2, \sigma_{ll_2}^2, \sigma_{ll_3}^2)$$

3. Observations



Server-side product generation

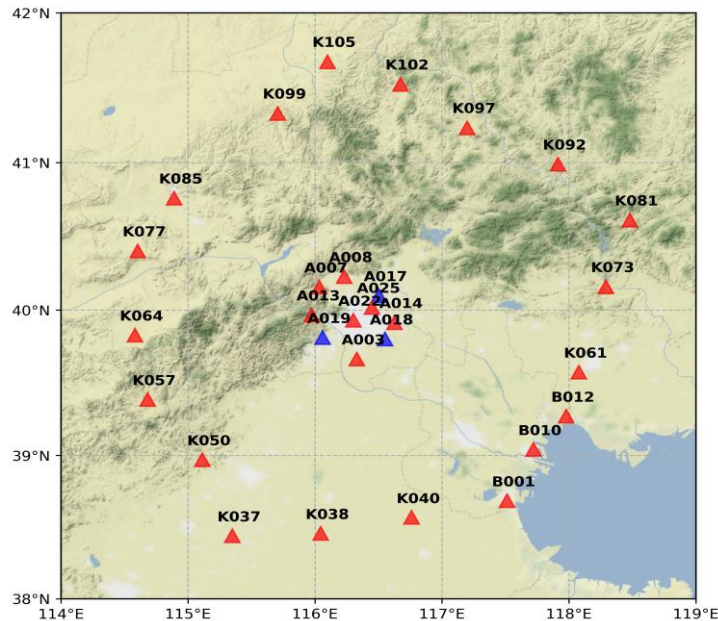


Fig.3 Distribution of the Beijing reference network

- GPST: 18/03/2021
- UPD estimation: 29 Beijing stations
(all triangles in Fig.3)
- Atmospheric delays estimation:
A017/A018/A019 stations in Beijing
(blue triangles in Fig.3)
- Observations: GPS L1/L2/L5, Galileo
E1/E5a/E5b/E5/E6, BDS B1I/B2I/B3I

3. Observations



Client-side vehicle experiment

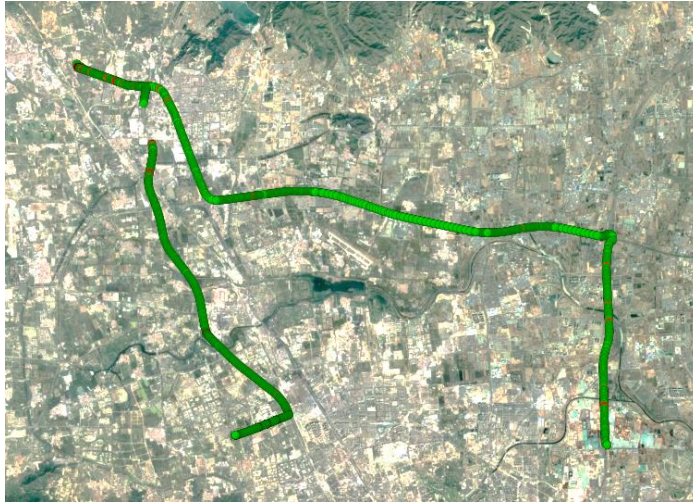


Fig.4 PPP-RTK vehicle experiment route



Fig.5 PPP-RTK experiment device

- GPST: 4:30 – 5:30, 18/03/2021
- Location: Changping, Beijing
- Device: GNSS antenna(Trimble AT1675-540TS), GNSS receiver(NovAtel OEM729), SBG IMU system(Apogee-D)
- Observation: GPS L1/L2/L5, Galileo E1/E5a/E5b/E5/E6, BDS B1I/B2I/B3I
- The result processed by PPK/INS is regarded as the true value

4. SSR Products Evaluation



GPS Galileo BDS narrow-lane UPD result

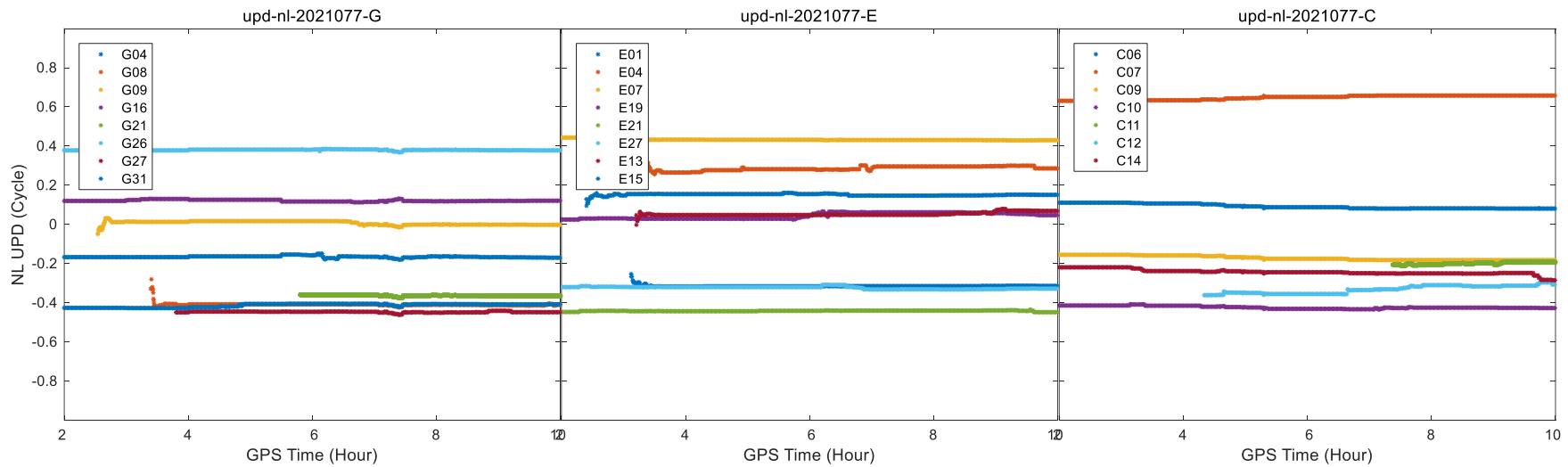


Fig.6 Typical satellites' UPDs of three GNSS (from left to right : GPS Galileo and BDS)

Table.2 The averaged STDs of GPS Galileo and BDS UPDs (cycles)

System	NL-STD
GPS	0.019
Galileo	0.044
BDS	0.013

- The NL UPDs are **stable** over whole time of GPS, Galileo and BDS.
- The STD of UPD at three systems are **less than 0.1 cycle**, which verifies the stability of UPDs.



4. SSR Products Evaluation

□ Evaluation of atmospheric delays accuracy

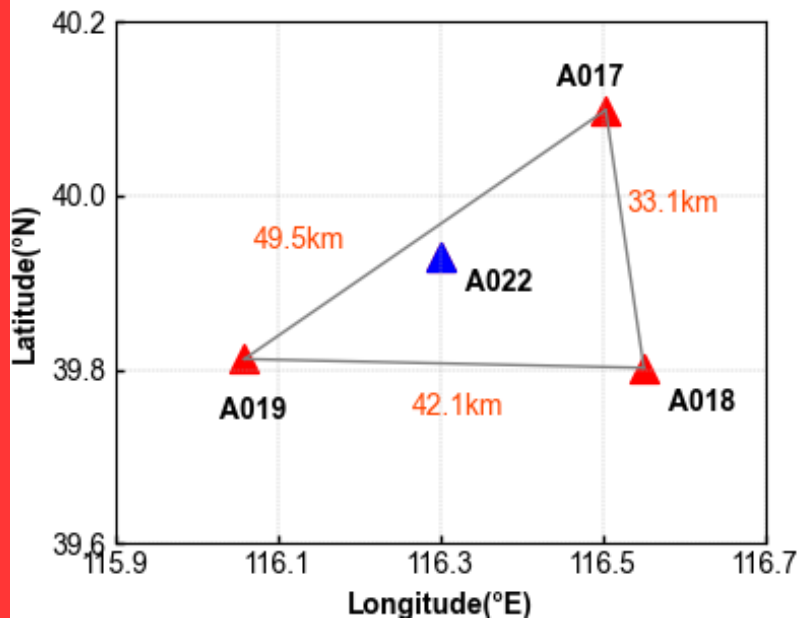


Fig.7 Distribution of four stations in evaluation of atmosphere delays, reference stations marked in red, rover station in blue

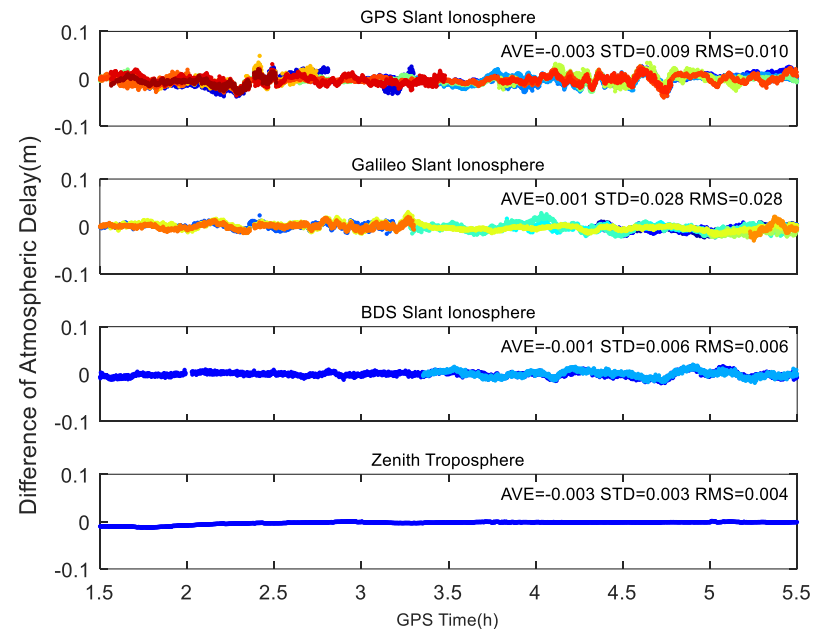


Fig.8 Differences between the interpolated and estimated atmospheric delays

- The accuracy on RMS of tropospheric delays is **0.4 cm**.
- RMS accuracy of GPS, Galileo and BDS satellites ionosphere delays is **1.0, 2.8 and 0.5 cm**.
- The results of interpolated atmospheric delays are accurate enough for PPP-RTK.

5. Performance of multi-GNSS PPP-RTK



□ GEC PPP & PPP-RTK result

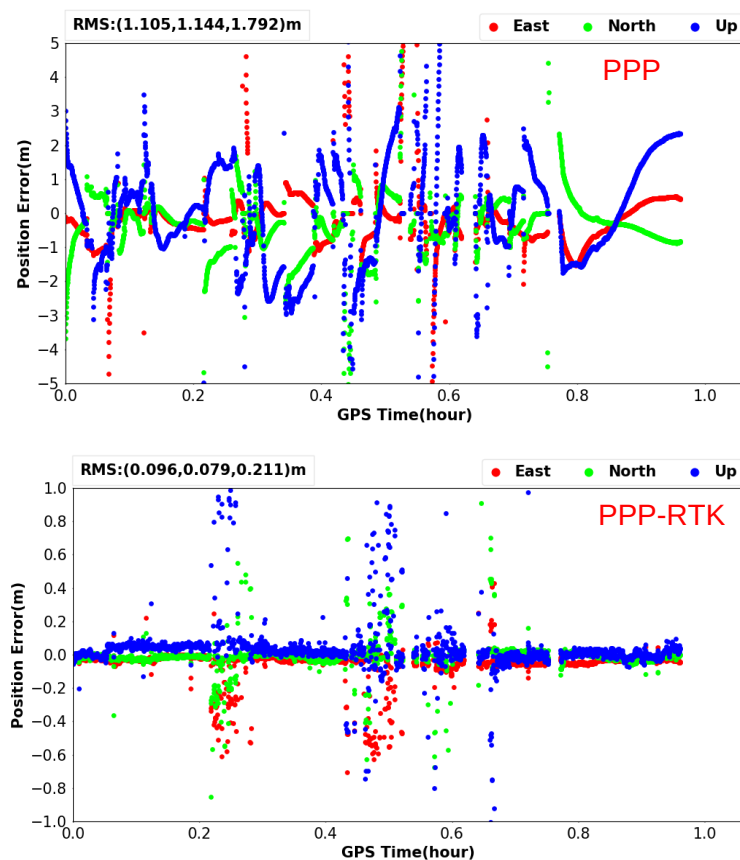


Table.3 Convergence performance and fixing percentage of GEC PPP and GEC PPP-RTK.

GEC	Horizontal		Vertical		Fixing percentage	Convergence time
	<5cm	<10cm	<10cm	<20cm		
PPP	0.83%	3.10%	4.09%	7.93%	-	410s
PPP-RTK	82.69%	84.87%	84.57%	86.78%	88.18%	9s

- PPP-RTK solution significantly improves positioning accuracy and shortens convergence time compared to PPP solution.

Fig.9 Positioning series for PPP and PPP-RTK of GPS Galileo and BDS.

5. Performance of multi-GNSS PPP-RTK



□ G & GEC dual-frequency PPP-RTK result

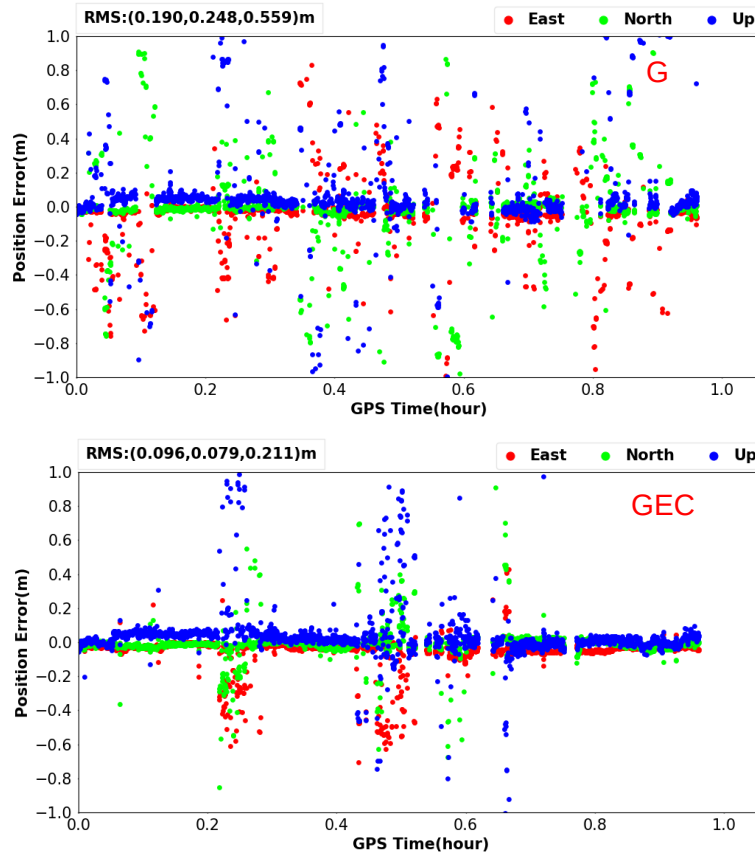


Table.4 Convergence performance and fixing percentage of G and GEC dual frequency PPP-RTK.

PPP-RTK	Horizontal		Vertical		Fixing percentage	TTFF
	<5cm	<10cm	<10cm	<20cm		
G	42.39%	52.52%	52.91%	56.74%	69.80%	10s
GEC	82.69%	84.87%	84.57%	86.78%	88.18%	7s

- Compared to GPS only solution, GEC solution positioning accuracy can be improved by 49.47%, 68.15% and 62.25% in the east, north and up direction, and the fix percentage can be improved by 18.38%.

Fig.10 Positioning series of G and GEC dual frequency PPP-RTK.

5. Performance of multi-GNSS PPP-RTK



□ GEC dual-frequency & multi-frequency PPP-RTK result

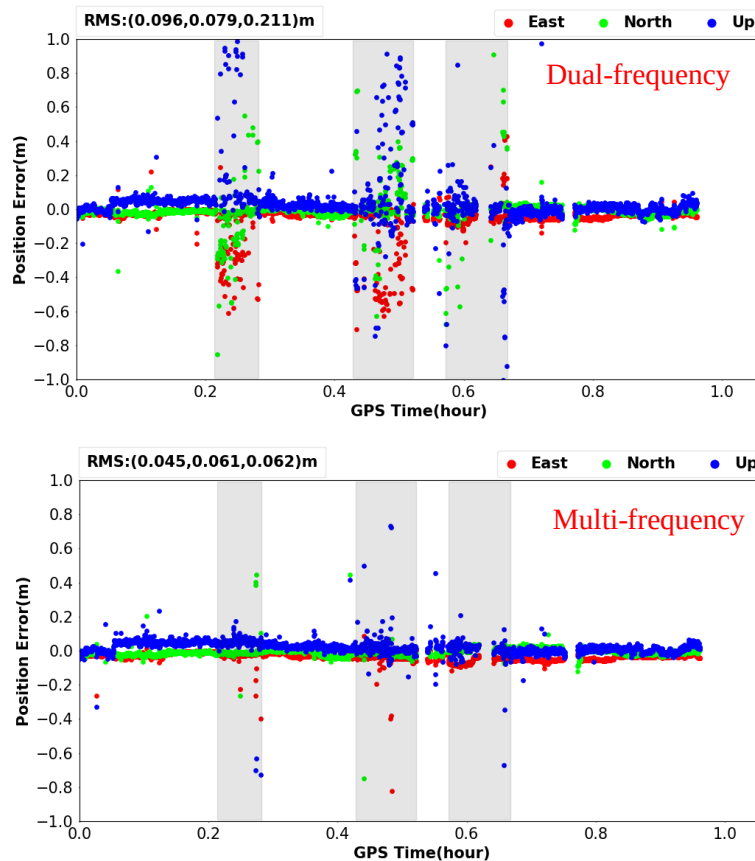


Fig.11 Positioning series of GEC dual-frequency and multi-frequency PPP-RTK.

Table.5 Convergence performance and fixing percentage of GEC dual-frequency and multi-frequency PPP-RTK.

GEC PPP-RTK	Horizontal		Vertical		Fixing percentage	TTFF
	<5cm	<10cm	<10cm	<20cm		
Dual-frequency	82.69%	84.87%	84.57%	86.78%	88.18%	7s
Multi-frequency	91.74%	97.46%	96.98%	97.76%	95.93%	6s

- Five-frequency solution performs best and achieves 5~6 cm positioning accuracy with the time to first fix (TTFF) of 6 s. The fixing percentage can be 95.93%.
- Compared to dual-frequency solution, five-frequency solution is more precise and reliable with more ambiguities are correctly fixed.



6. Conclusion & Outlook

□ Conclusion

- PPP-RTK solution significantly improves positioning accuracy and shortens convergence time compared to PPP solution.
- Multi-GNSS PPP-RTK solution positioning accuracy can be improved by 50% and the fix percentage can be improved by 17% compared to GPS only solution.
- Multi-frequency and multi-GNSS PPP-RTK can achieve 5~6 cm positioning accuracy with the TTFF of 6 s, and the fixing percentage can be 95%.
- Multi-frequency solution is more precise and reliable with more ambiguities are correctly fixed compared to dual-frequency solution.

□ Outlook

- BDS-3 multi-frequency PPP-RTK.
- PPP-RTK performance in large scale reference networks.
- PPP-RTK for low-cost receivers.

Thank You For Your Attention!

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