

How to model enhanced firm densification due to strain softening

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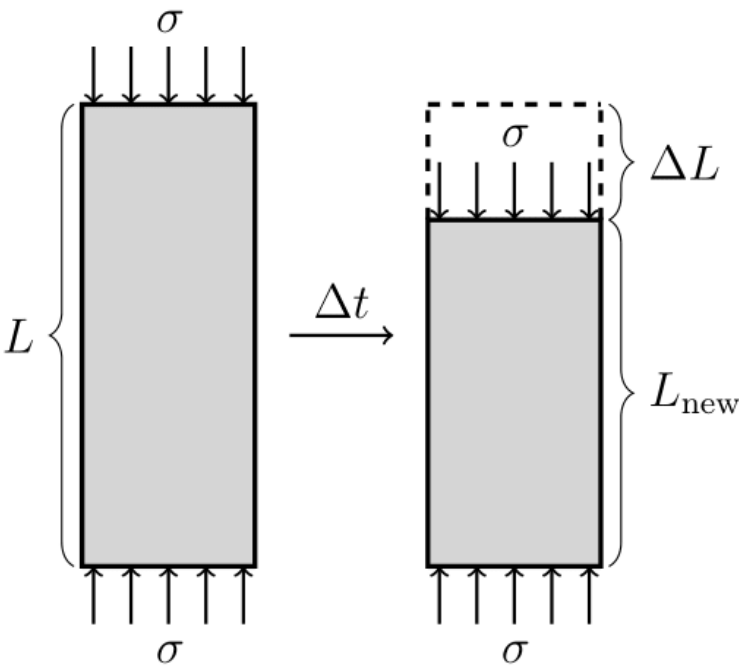
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Background

Strain rate



$$\dot{\epsilon} = \lim_{\Delta t \rightarrow 0} \frac{1}{\Delta t} \frac{\Delta L}{L}$$

Little America V
(Gow, 1968)

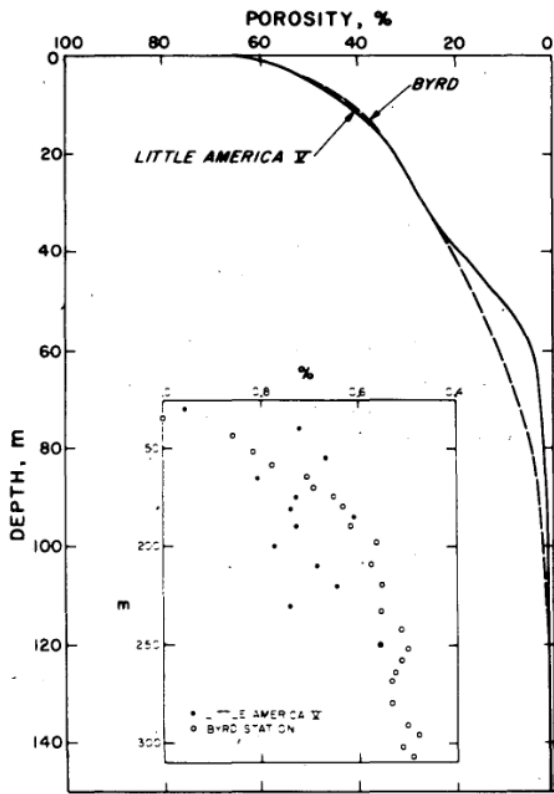


Figure 14. Depth-porosity curves for Byrd Station and Little America V, Antarctica. Individual values from below 130 m are included in the inset figure.

Upstream B
(Alley & Bentley, 1988)

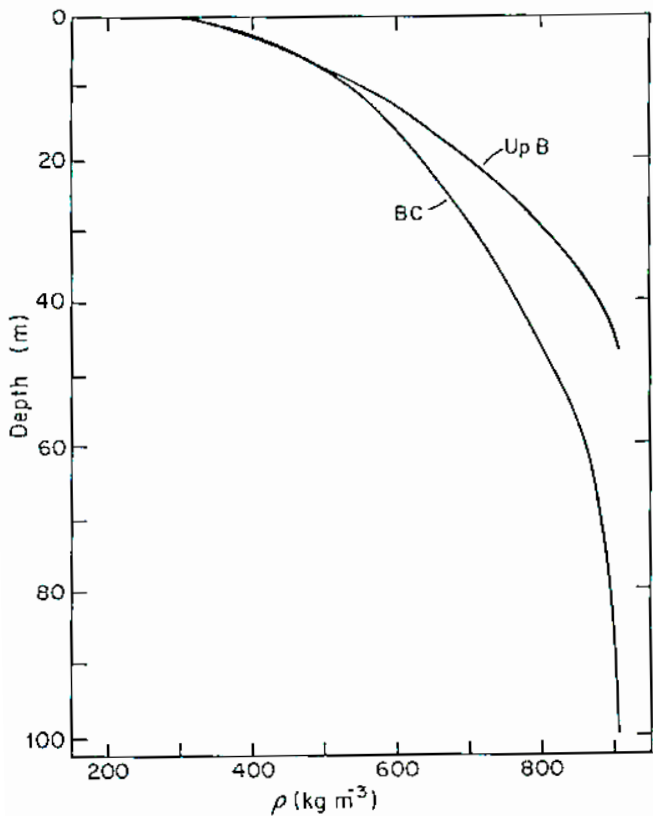


Fig. 6. Smoothed depth-density curves for UpB and BC.

Gow, A. J. 1968. Deep core studies of the accumulation and densification of snow at Byrd Station and Little America V, Antarctica. CRREL Res. Rep. 197.

Alley, R. B. and Bentley, C. R. 1988. Ice-core analysis on the Siple Coast of West Antarctica, Ann. Glaciol., 11, 1-7.



Background

A firn densification rate corresponds to a vertical strain rate

$$\dot{\epsilon}_{zz} = -\frac{1}{\rho} \frac{D\rho}{Dt}$$

Classical firn model

$$\dot{\epsilon} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \dot{\epsilon}_{zz,c} \end{pmatrix}$$

With horizontal strain rates

$$\dot{\epsilon} = \begin{pmatrix} \dot{\epsilon}_{xx} & \dot{\epsilon}_{xy} & 0 \\ \dot{\epsilon}_{xy} & \dot{\epsilon}_{yy} & 0 \\ 0 & 0 & \dot{\epsilon}_{zz} \end{pmatrix}$$

Horizontal strain rates decrease the firn viscosity and thereby enhance the vertical strain rate

$$\dot{\epsilon}_{zz,c} \neq \dot{\epsilon}_{zz}$$

Densification rate can be corrected according to

$$\frac{D\rho}{Dt} = \frac{\dot{\epsilon}_{zz}}{\dot{\epsilon}_{zz,c}} \left(\frac{D\rho}{Dt} \right)_c$$

The scale factor

Power-law creep:

$$\dot{\epsilon}_{zz} = A\sigma_{zz}^n$$

From $\frac{\dot{\epsilon}_{zz}}{\dot{\epsilon}_{zz,c}} = \frac{\eta_c}{\eta} = \frac{\dot{\epsilon}_E^m}{\dot{\epsilon}_{E,c}^m}$ it follows

Generalized power-law creep

$$\dot{\epsilon}_{zz} = \frac{1}{2\eta} \sigma_{zz},$$

with the effective viscosity

$$\eta = \frac{k}{\dot{\epsilon}_E^m},$$

Exponents: $n = 3$ and $m \equiv 1 - \frac{1}{n}$.

$$\frac{\dot{\epsilon}_{zz}}{\dot{\epsilon}_{zz,c}} = \left(\frac{\dot{\epsilon}_{xx}^2 + \dot{\epsilon}_{yy}^2 + 2\dot{\epsilon}_{xy}^2 + \dot{\epsilon}_{zz}^2}{\dot{\epsilon}_{zz,c}^2} \right)^{\frac{m}{2}}$$

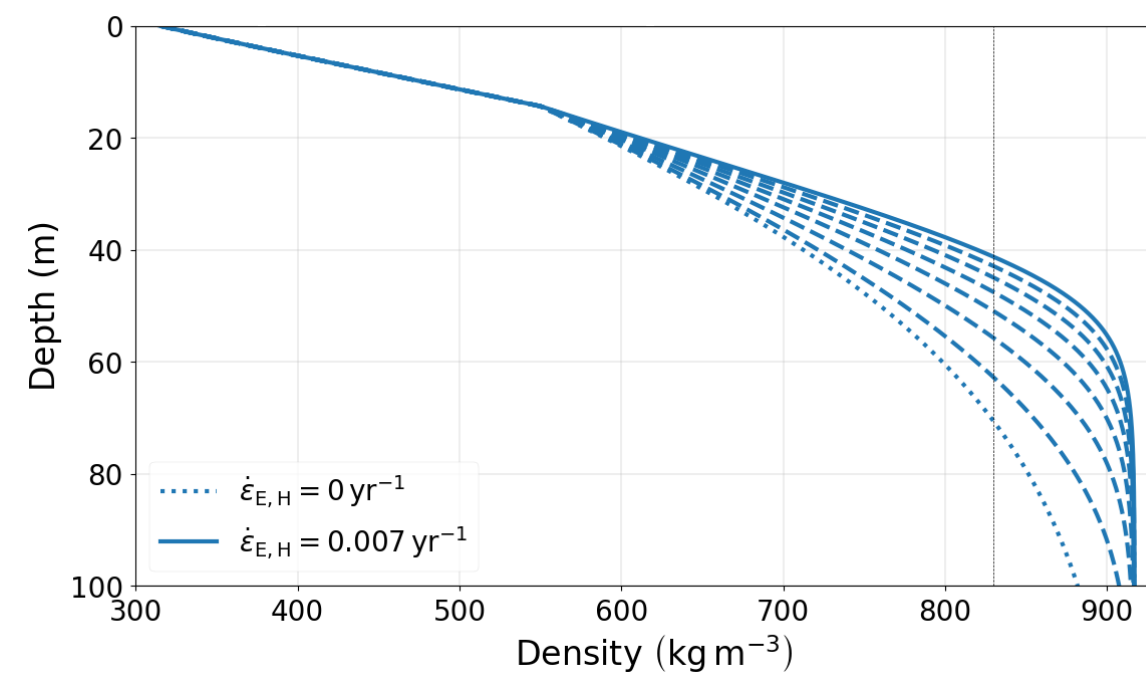
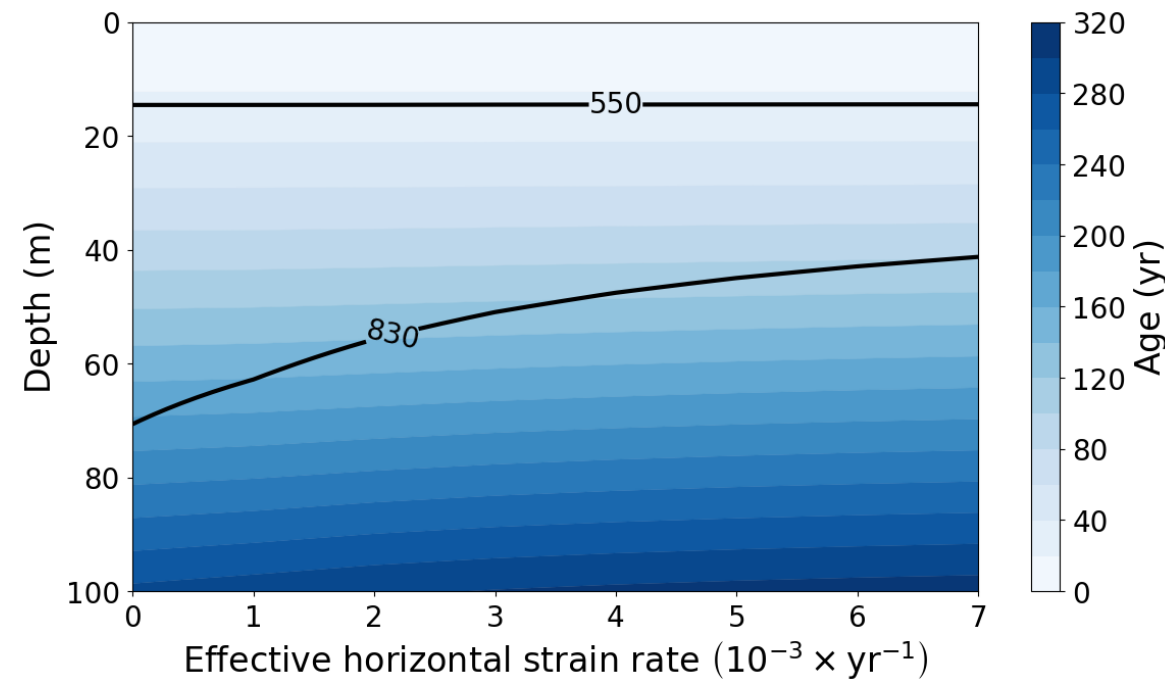
$\dot{\epsilon}_{zz,c}$ is known from the classical model.

$\dot{\epsilon}_{xx}^2$, $\dot{\epsilon}_{yy}^2$ and $\dot{\epsilon}_{xy}^2$ are known from velocity maps.

Rewrite the relation and solve for a :

$$a = \sqrt[3]{b^2 + a^2}$$

Firn sensitivity to strain softening

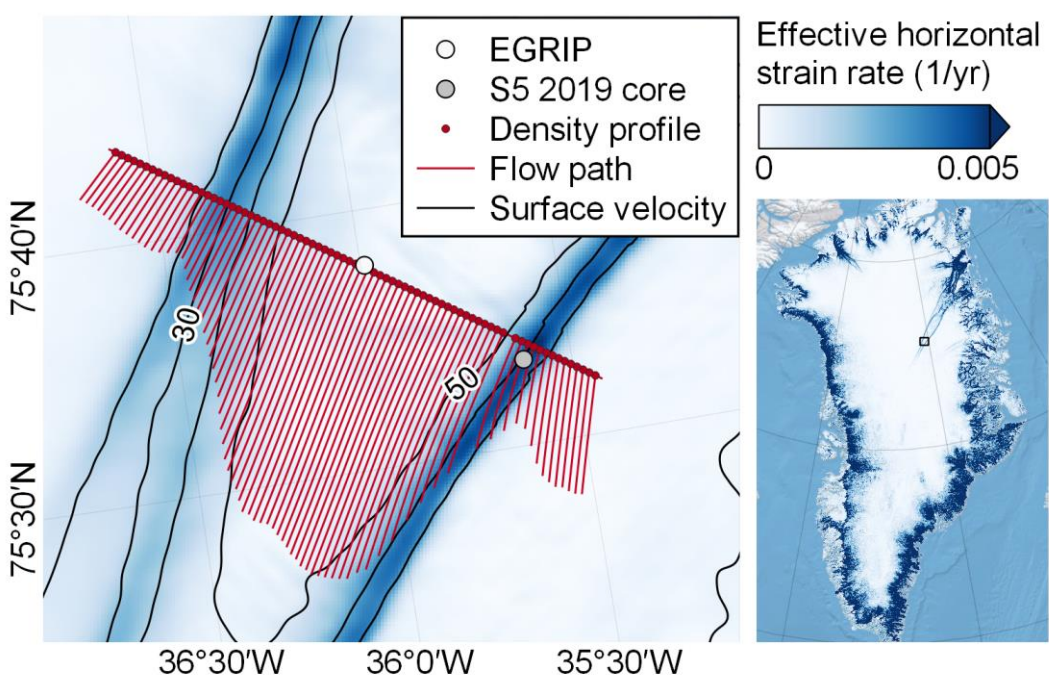
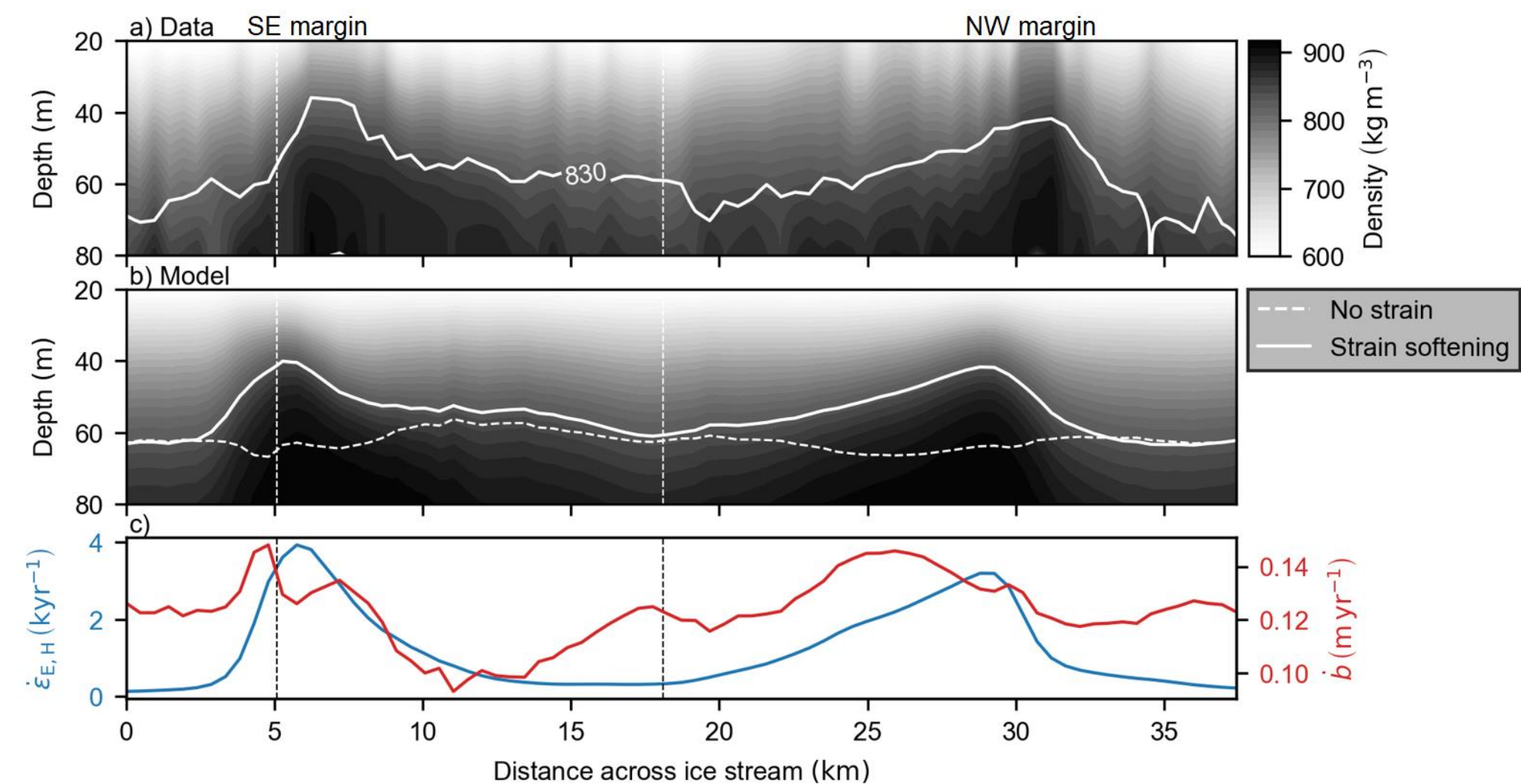


Sensitivity of firn age and thickness to the effective horizontal strain rate is the highest at low strain rates.

Small strain rates can already have a significant impact.

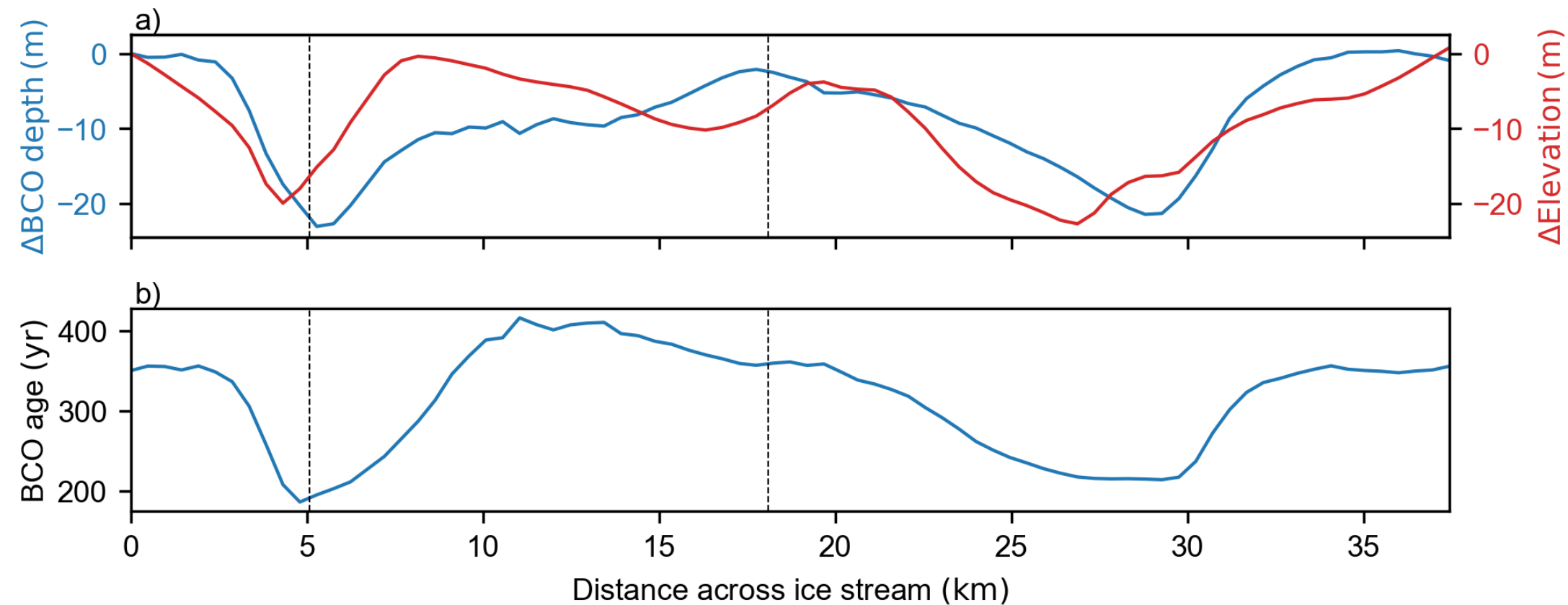
Validation with NEGIS density data

Accuracy of the model is validated by firn density profile along a cross section of NEGIS (Riverman et al. 2019).



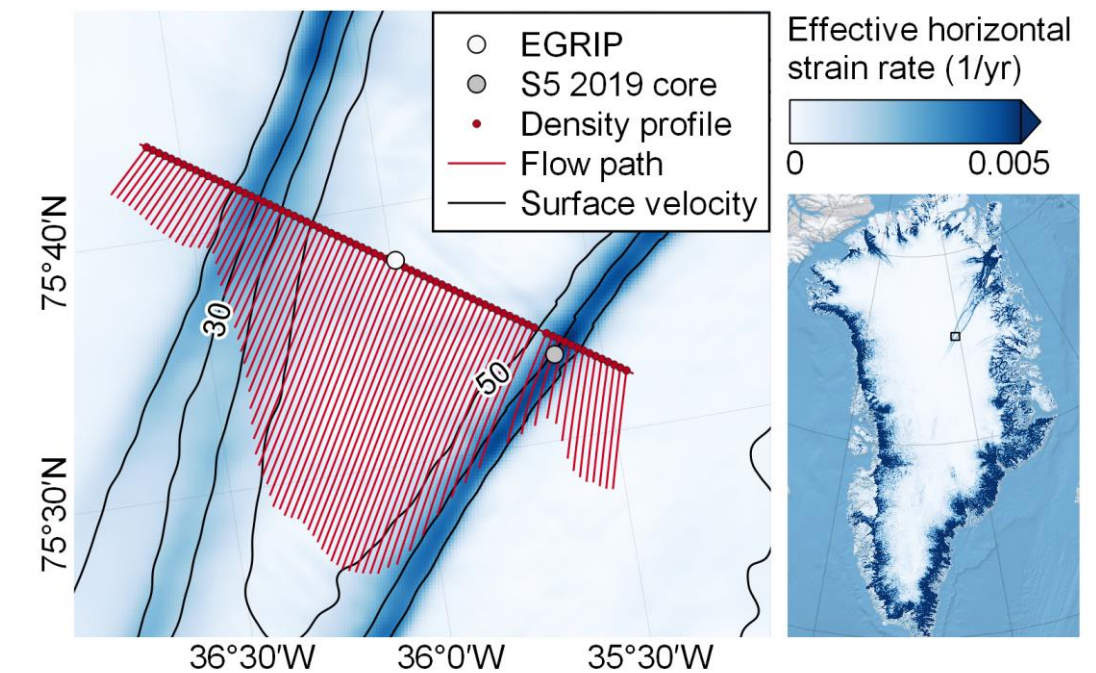
Riverman, K. L. et al. 2019. Enhanced firn densification in high-accumulation shear margins of the NE Greenland Ice Stream. *J. Geophys. Res. Earth Surf.*, 124, 365–382. 6

Impact on firn properties

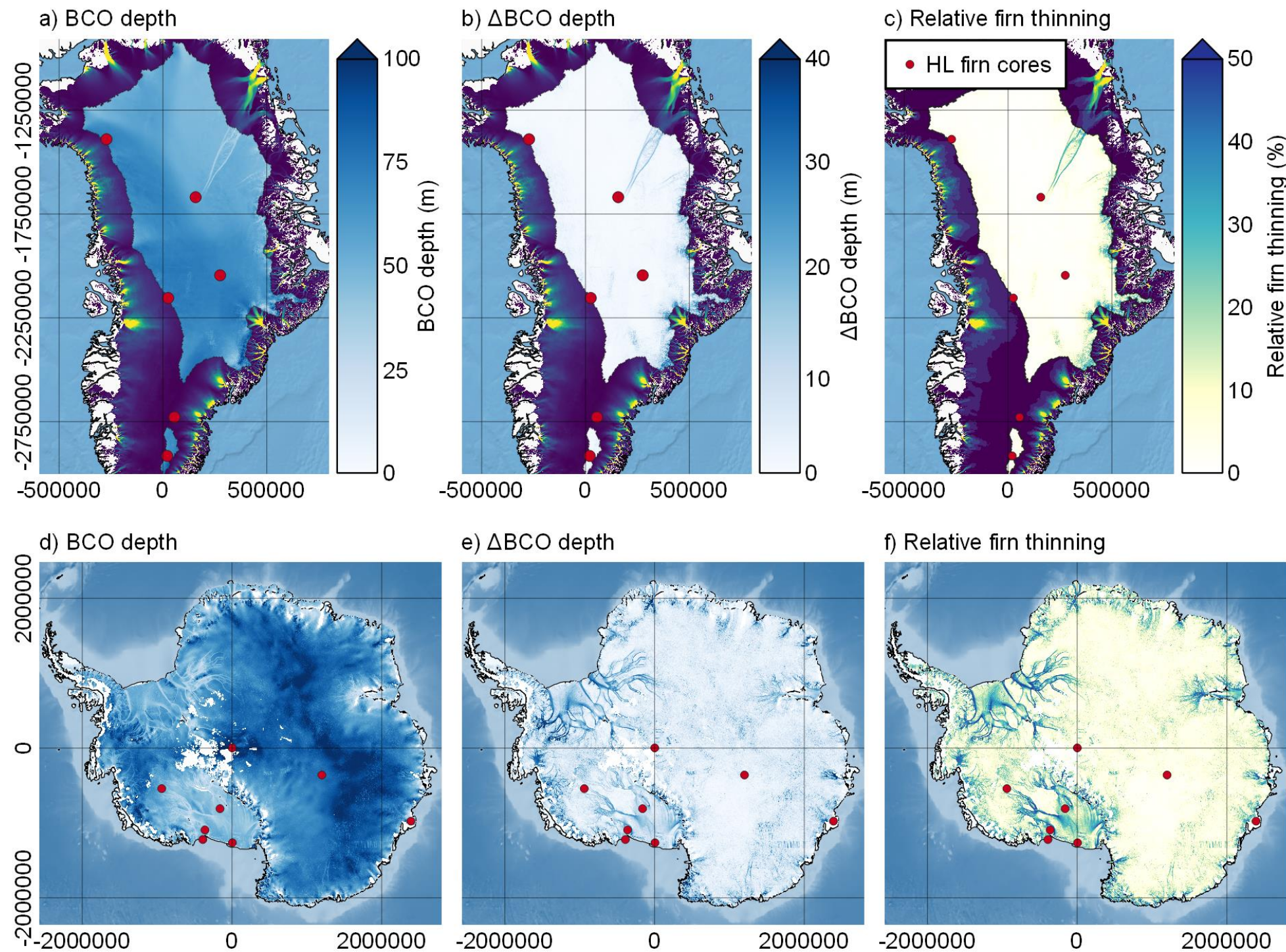


Firn thinning is on the same scale as surface elevation reduction in the shear margins
→ Shear margin troughs caused by strain softening?

BCO age reduced by 50% over small distances with little climatic variability
→ Exploit the ice stream shear margins as a natural laboratory for firn air studies?



Strain softening in classical firn models



Moderate effective horizontal strain rates are implicitly considered in empirically tuned parameters of classical firn models.

“The very high densification rate at Little America V [...] has been attributed to local areas of high stress (Gow, 1968)” Herron & Langway (1980)

Herron, M. M. and Langway, C. C. 1980. Firn densification: an empirical model, *J. Glaciol.*, 25, 373–385.

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Conclusion

- Horizontal strain rates can significantly impact firn densification. Temperature, accumulation and horizontal strain should be considered in empirical tuning of firn models.
- Further investigate the role of strain softening for shear margin trough formation
- Exploit ice streams as a natural laboratory for firn air processes.

Code availability:

Please contact me!