The physical meaning of transport properties evaluated from BTC in karst conduit networks using transfer functions

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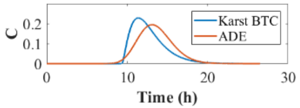
1. Introduction

Advection Dispersion Equation

Governing equation:

$$\frac{\partial C_{ADE}(x,t)}{\partial t} + u\frac{\partial C_{ADE}(x,t)}{\partial x} - D\frac{\partial^2 C_{ADE}(x,t)}{\partial x^2} = 0$$

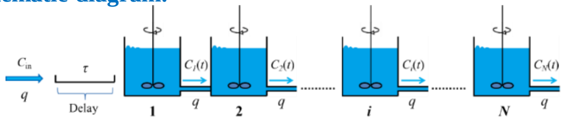
Disadvantage:



Unable to capture the tailing effect.

Transfer Function Approach

Schematic diagram:



Governing equation (Laplace domain):

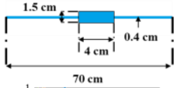
$$\hat{C}_{TFA}(s) = \left(\frac{1}{1+As}\right)^N \exp(-\tau s)$$

s: Laplace complex variable

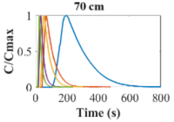
Advantage: Parameter N represents the intrinsic property about the aquifer. (Proven by its applications to lab and field experiments)

Lab Experiment and TFA fitting the BTCs

Lab model:



5 tracing tests:



Fitted parameters

Tracer tests	Flow rate, (cm³/s)	Fitted TFA parameters A (s)	N	τ (s)
1	0.049	69.68	2.10	119.38
2	0.126	26.02	2.10	41.07
3	0.161	20.05	2.10	33.25
4	0.270	11.19	2.10	20.42
5	0.355	8.69	2.10	14.95

Limit: The TFA parameters don't have direct physical meaning related to the classical transport parameters (u, D).

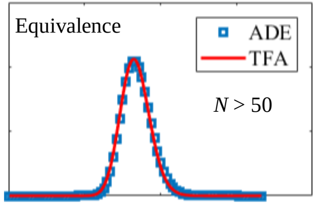
2. Study target and method

TFA / ADE → Laplace transform, Taylor series expansion. → Make two solutions comparable → Assumes them equivalent → A, N, τ ⟷ u, D

3. Equivalence between ADE and TFA parameters

$$u = \frac{L}{AN+\tau}$$

$$D = \frac{A^2 N L^2}{2(AN+\tau)^3}$$



4. Conclusion and Perspective

1. Parameter N represents the intrinsic property about the aquifer.
2. The relationship between the TFA parameters and ADE parameters is obtained (N>50).
3. We plan to study the equivalence with the TFA and the Mobile-Immobile model for small N values.

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