

Near-surface Stokes drift in operational wave forecast

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Abstract

This preliminary draft is provided as 'display material' for EGU21¹.

We apply an operational model for the mean drift of small particles near and at the surface. The drift is a sum of the Stokes drift and the current. The Stokes drift is derived from a wave model and the current is calculated by means of a 3D hydrodynamic model.

In an operational wave model setup (based on WAVEWATCH-III version 7) a parametric tail is supplied as an extension to the prognostic wave spectrum. The parametrisation is based on the modelled spectra level and the first circular moment (the spectral spread) near the highest prognostic frequency, which is typically around 0.5 Hz in operational wave modelling. New source terms formulations has been introduced in wave modelling (e.g. WAVEWATCH-III with effect from 2019) that reproduces the spreading and spectral level well compared to many independent observations.

The Stokes drift is calculated at a discrete number of depths down to $z = 2/K_s$, where K_s is a wave number scale estimated from the prognostic spectrum. The calculation is integrated in the wave model output. The application is evaluated in complex weather situations between January 1 and January 6 2020 in the European North West Shelf region. A set of wave model setups are compared with a variation of values of the prognostic spectral cut-off.

It is demonstrated that a the near-surface drift profile is resolved well with an order of ten discrete depths. The parametric tail extension requires a low computational time.

¹Hansen, C.: Near-surface Stokes drift in operational wave forecast, EGU General Assembly 2021, online, 1930 Apr 2021, EGU21-9520, <https://doi.org/10.5194/egusphere-egu21-9520>, 2021

1 Introduction

As stated by van den Bremer and Breivik [2018] the Stokes drift velocity is the difference between the average Lagrangian flow velocity of a fluid parcel and the average Eulerian flow velocity of the fluid. In a relatively thin layer (compared to the wave length) below a wavy surface this results in a net drift velocity in the direction of wave propagation. Direct precise measurement of the Stokes drift in a random wind-generated sea has not been conducted so far (Tuomi et al. [2018]), although Smith [01 Jul. 2006] obtained important indirect estimates using a subsurface acoustic backscatter technique.

The profile very near the surface is strongly influenced by components in the wave spectrum of high frequencies on the order of more than 1 Hz (the corresponding high wave numbers are of order more than 4 m^{-1}). The influence of this high frequency spectral tail diminishes shortly below the surface because of the exponential decay of the Stokes drift profile. For a harmonic wave of amplitude a , wavenumber k , and angular frequency ω on water at depth D , the Stokes drift depends on the depth z below the surface as

$$(1) \quad U_S = c(ak)^2 \frac{\cosh(2k(D-z))}{2 \sinh^2(kD)} + O((ak)^6),$$

where $c = \omega/k$ is the phase speed. For deep water $kD \gg 1$, and the Stokes drift forms a profile

$$(2) \quad U_S = c(ak)^2 e^{-2kz}$$

The sensitivity to waves of very high frequency is limited in oil spill situations, where there is often an upper cut-off frequency for the wave spectrum. Browsing the Oil Fact Sheets published by NOAA Office of Response and Restoration [2020], it appears that any known type of oil that remains on or near the surface may form an extended slick is a light, bi-polar oil for which a slick spreads to a very thin film. Benetazzo et al. [2019] performed a wind/wave tank experiment and examined the resonance-type Marangoni damping of wind-forced wave spectra. We draw from their fig. 12 the conclusion that in a developed sea condition the damping inside a thin bi-polar type oil slick is insignificant except for frequencies higher than approximately 2 Hz. We suggest that this may be simply modelled as a truncation of the wave spectrum at 2 Hz.

In the high-frequency range we will assume the waves are free harmonics represented by wave components in a 2D wave spectrum (Peureux et al. [2018]). We further assume that the *mean* Stokes drift can be derived as an integral over a spectral version of Eq. 2 (Kenyon [1969]) because of only 'weak' non-linear interactions between spectral components.

In a third generation wave model like WAM or WAVEWATCH-III solves the evolution over discrete points in space and time of 2D wave spectra also defined on a discrete grid over frequencies and directions. There is an upper frequency for this model grid, typically on the order of 0.5 Hz in operational wave model setups. because the Stokes drift at and very near the surface is determined by higher frequencies, the *prognostic* spectrum must be extended with a *diagnostic tail* in order to achieve representative values at the very surface. The mean Stokes drift can be calculated as an integral over each of the two spectral ranges and adds up to

$$(3) \quad U_S = U_{S,prog} + U_{S,tail}$$

For operational use our aim is to reduce the model output. Therefore we will introduce a common formula for the vertical distribution of a few number of depths in the model output.

The techniques here are implemented and integrated in WAVEWATCH-III, currently as a *feature branch*, *fb_xsto*² to the WW3 develop version 7.

2 Extension of the model prognostic spectrum with a diagnostic tail

The wave model spectra are defined over a discrete set of angular frequencies $1..N_K$. The highest prognostic frequency bin is at $\omega[N_K]$. To reduce effects of the boundary conditions at $\omega[N_K]$ we will apply a parametric *diagnostic* tail for frequencies above a slightly lower frequency $\omega_t = \omega[N_K - 2]$ at bin number $N_K - 2$.

The contribution in a frequency bin $d\omega$ to the directionally integrated (or *mean*) Stokes drift dU_S at depth z below the surface is determined by a spectral version of Eq. 2 (Kenyon [1969])

$$(4) \quad dU_S(\omega, z) = 2\omega k e^{-2kz} m_1(\omega) S(\omega) d\omega$$

$k = |\vec{k}|$ is the wavenumber modulus and related to ω by the dispersion relation at water depth D , $c = \omega/k = \sqrt{g/k \tanh kD}$. Eq. 4 is valid for deep water, i.e. for $kD \gg 1$ where the dispersion relation is $\omega \simeq \sqrt{gk}$. $S(\omega)$ is the elevation variance spectrum integrated over all directions θ ,

$$(5) \quad S(\omega) = \int_{-\pi}^{\pi} S(\omega, \theta) d\theta$$

²https://github.com/CarstenHansen/WW3/tree/fb_xsto. Switch 'XSTO' and namelist group '&XSTO' must be set.

In the diagnostic spectral range we assume that the mean direction is the same for all frequency bins, and refer $\theta = 0$ to that direction. $m_1(\omega)$ is the spectral 'first circular moment' introduced by Kuik et al. [01 Jul. 1988] and measured extensively by Ewans [1998].

$$(6) \quad m_1(\omega) = \int_{-\pi}^{\pi} \cos(\theta - \theta_m) S(\omega, \theta) d\theta / S(\omega)$$

where θ_m is the mean direction in a spectral band at frequency ω . The assumption of deep water is fulfilled for high values of the diagnostic frequencies. At a depth of 1 m a high value $kD \gtrsim 2$ is obtained for $k[N_K] \gtrsim 2 \text{ m}^{-1}$, or $\omega[N_K] \gtrsim 5 \text{ rad/s}$, or $f[N_K] \gtrsim 0.7 \text{ Hz}$. For wave model spectra defined in the prognostic frequency range $\omega \leq \omega[N_K]$ we replace Eq. 4 by a more general expression (see e.g. van den Bremer and Breivik [2018] or Kenyon [1969]) which is valid for all values of kD and for any directional distribution over the spectrum. This is implemented in WAVEWATCH-III as explained in the Users Manual (Tolmann et al. [2021], Sec. 2.7.2).

Based on data from the 'Maui' experiment for growing sea states, Ewans [1998] observed that the wave spectrum has a directional spread that increases with frequency, which means that $m_1(\omega)$ decreases monotonically. Ewans [1998] provided a parametric formulation for the spreading. Now, if we choose a frequency ω_h chosen at the high end of the prognostic frequency range, the implicitly $m_1(\omega)$ is a function of the first circular moment at this frequency $m_1(\omega) = F(\omega_h, m_1(\omega_h))$. A resulting spectrum for the mean Stokes drift is obtained by combining $m_1(\omega)$ with an assumed form of the omni-directional spectrum $S(\omega)$. We simply assume $S(\omega) \propto \omega^{-4}$. Further details are given in Appendix A.

3 Integral profile forming the 'total Stokes transport'

An estimate for the total Stokes transport, also known as the integral 'wave pseudo momentum' $\vec{M}$,

$$(7) \quad \vec{M} = \int_{z=0}^{\infty} \int_{\omega=0}^{\infty} \vec{d}U_S(\omega, z) dz$$

The term 'pseudo' refers to the fact (as explained by e.g. van den Bremer and Breivik [2018]) that 'there is zero vertically integrated momentum associated with the centre of a surface gravity wave group' and there is a deep Eulerian return flow because 'the Stokes transport flux is divergent on the scale of the group'. We ignore the presence of this return flow.

$\vec{M}$ is also derived from the representation of the wave *action* spectrum $A(\vec{k})$ in the wave model as an integral over all wavenumbers $\vec{k}$. It is a sum of this *prognostic* part and a *diagnostic* part from the extended tail.

$$\begin{aligned}\vec{M} &= \vec{M}_{prog} + \vec{M}_{tail} \\ (8) \quad \vec{M}_{prog} &= \int \int A(\vec{k}) \vec{k} dk_x dk_y\end{aligned}$$

M_{prog} is the output 2D field named *ust* of the wave model WAVEWATCH-III (Tolmann et al. [2021]). $\vec{M}_{prog} = ust$ may be used to determine a parametric extension of the Stokes drift profile to large depths.

4 Simulation case

As a case model simulation characteristic we choose a wave situation in the European North West Shelf area on January 5, 2020 00). The location 'B' in Fig. [1] is just before a front system between the Shetland Islands and Norway, where larger depth scales $zK_s > O(1)$ a swell from the north is dominating the Stokes drift at under a local moderate wind blowing from the south-west.

4.1 Illustration of the tail extension procedure

In the illustration Fig. 2 we compare two different model setups with high-frequency prognostic cut-off $f_h = 2.0$ Hz and $f_h = 0.6$ Hz, where a parametric tail has been added to the latter up to $2.0Hz$. $f_h = 0.6$ Hz is a practical, limited high frequency for operational forecast.

4.2 The two-dimensional Stokes profile

The two-dimensional Stokes profile $U_S(z)$ from the case model simulation is shown in Fig. [3].

5 Simulation case results and discussion

We have compared two different model setups with high-frequency prognostic cut-off $f_h = 2.0$ Hz and $f_h = 0.6$ Hz, where a parametric tail has been added to the latter up to $2.0Hz$. In the illustrative figure 2 the resulting 1-dimensional spectrum $\delta U_m(f)$ and profile $U_{Sm}(z)$ agree well. The difference is small between the upper figure (black lines) and lower figure (red lines).

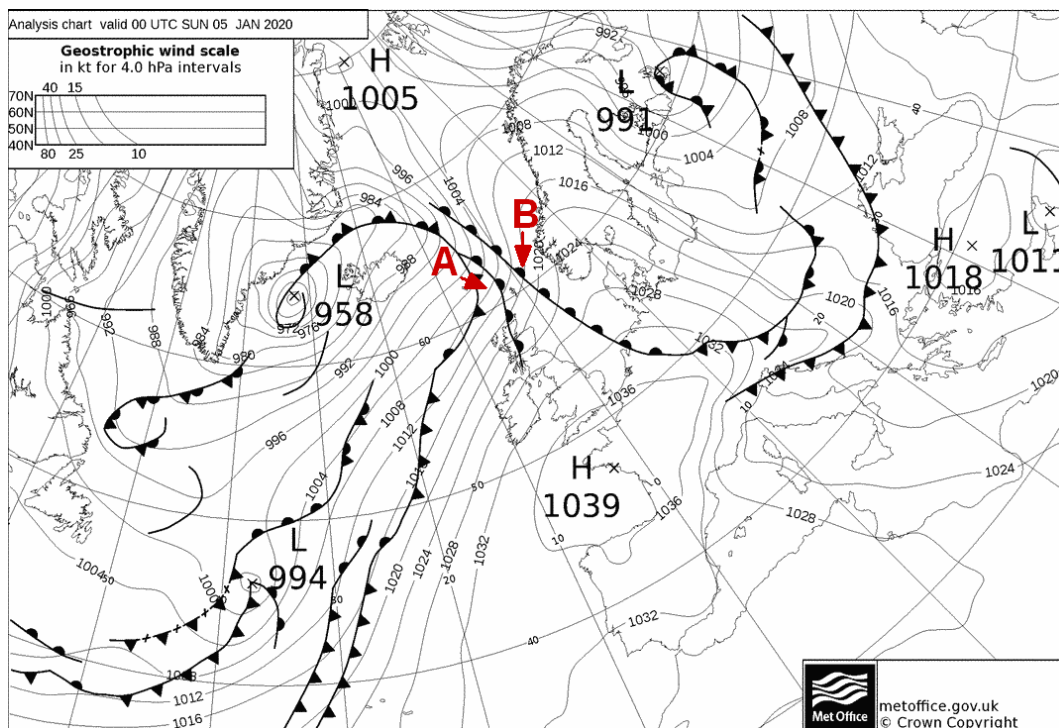


Figure 1: The example location (**B**) shown on a UKMET analysis chart⁴ at January 5 00Z. (The other location **A** is analyzed in Hansen and Mattsson [2020]).

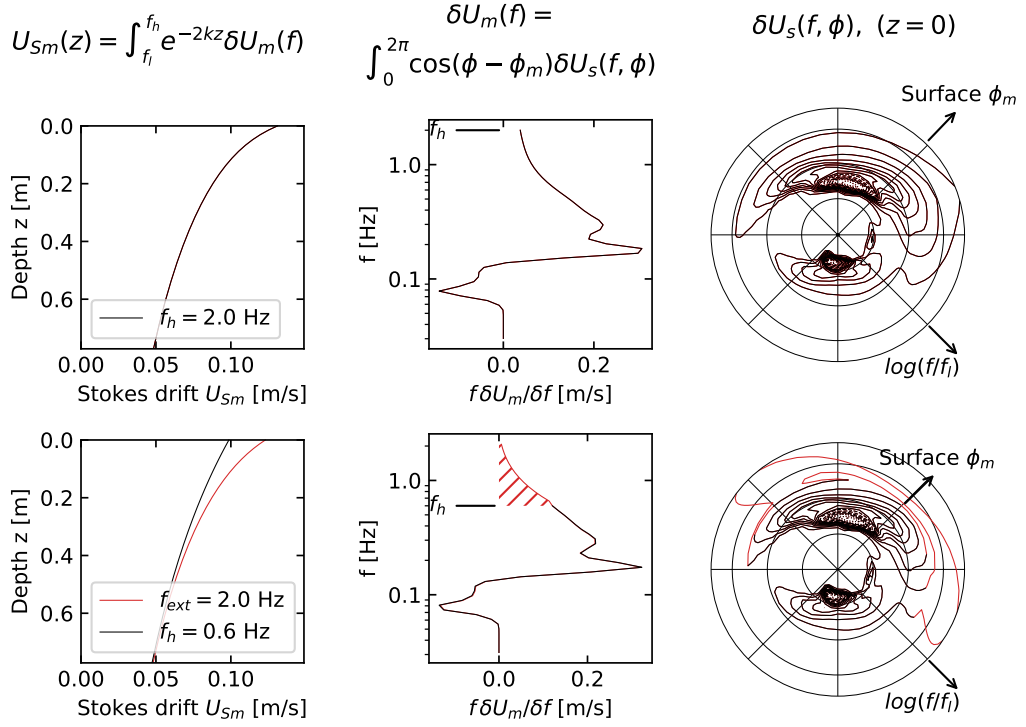


Figure 2: Two different model setups with high-frequency prognostic cut-off $f_h = 2.0$ Hz (upper three figures) and $f_h = 0.6$ Hz (lower three figures). Rightmost: $\delta U_s(f, \phi)$ is the 2D spectrum of the Stokes drift. Middle: $\delta U_m(f)$ is the spectrum projected on the surface direction ϕ_m . Left: The vertical profile from integration over all directions and projected on the direction ϕ_m .

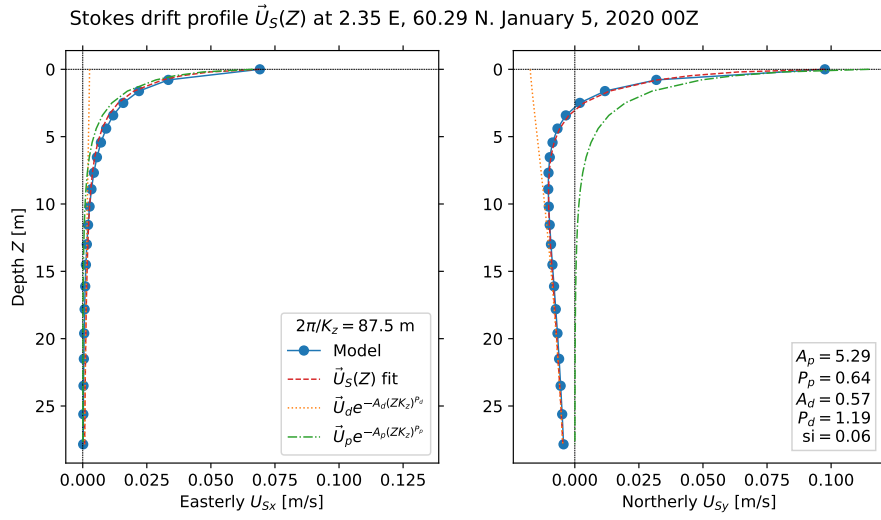


Figure 3: Wave model Stokes drift profiles at a characteristic situation on January 5 2020 00Z in the northern North Sea. Left and right side show the Easterly (x) and Northerly (y) vector components. Blue dots: wave model output. (The colored lines are construction of a parametric fit described in Hansen and Mattsson [2020] and to be input to the oil tracking system SeaTrackWeb. The green line is a 'primary' part, the orange a 'secondary' part, and the red dashed line is the resulting vector sum.)

Scatterplot comparisons (Figs. 4 to 7) demonstrate that this agreement holds over the whole model domain (more than three grid cells inside the water boundaries).

From the example (Fig. 2) it is clear that the directional width of WAVE-WATCH III is narrower than Ewans [1998]. We infer that simulation of an oil spill head may be realistic, but simulation of the directional spreading requires further experiments.

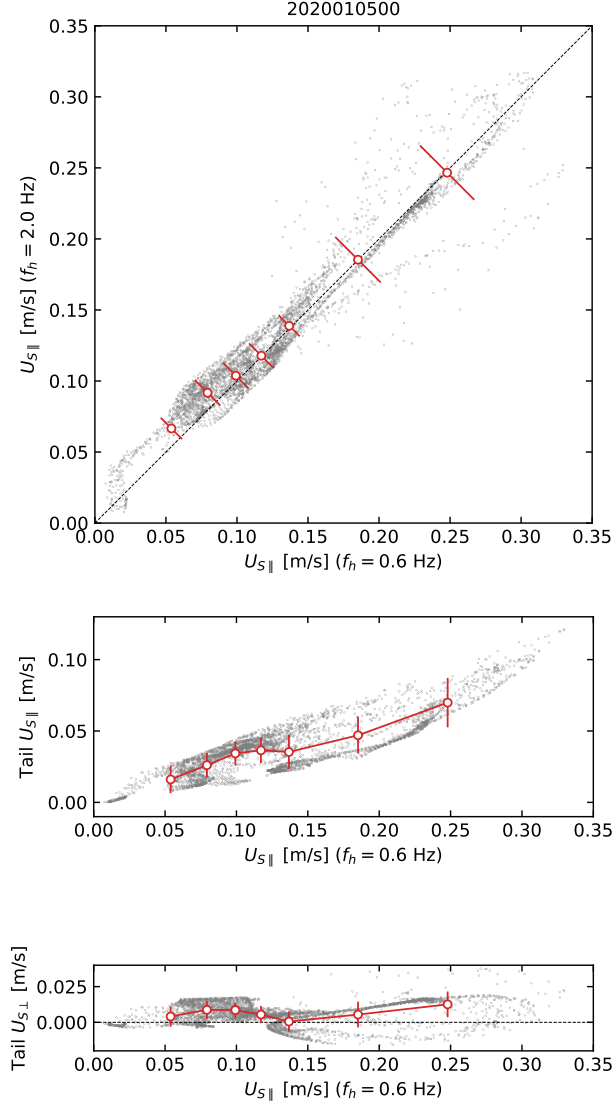


Figure 4: Top graph: Scatterplot comparison of the surface Stokes drift simulated with two different high-frequency prognostic cut-off $f_h = 0.6$ Hz and $f_h = 2.0$ Hz. Shown is the component of the latter drift vector parallel to the former. The red circles and bars are the mean and standard deviation of $U_{S,2Hz} - U_{S,0.6Hz}$ of the points divided evenly into 7 sections. All grid points at distance greater than 4 grid cells from the coast and outer boundary are included. Middle graph: The part due to the tail extension from 0.6 Hz to 2.0 Hz (parallel component). Lower graph: The part due to the tail extension from 0.6 Hz to 2.0 Hz (perpendicular component). The date is 2020-01-05 00Z.

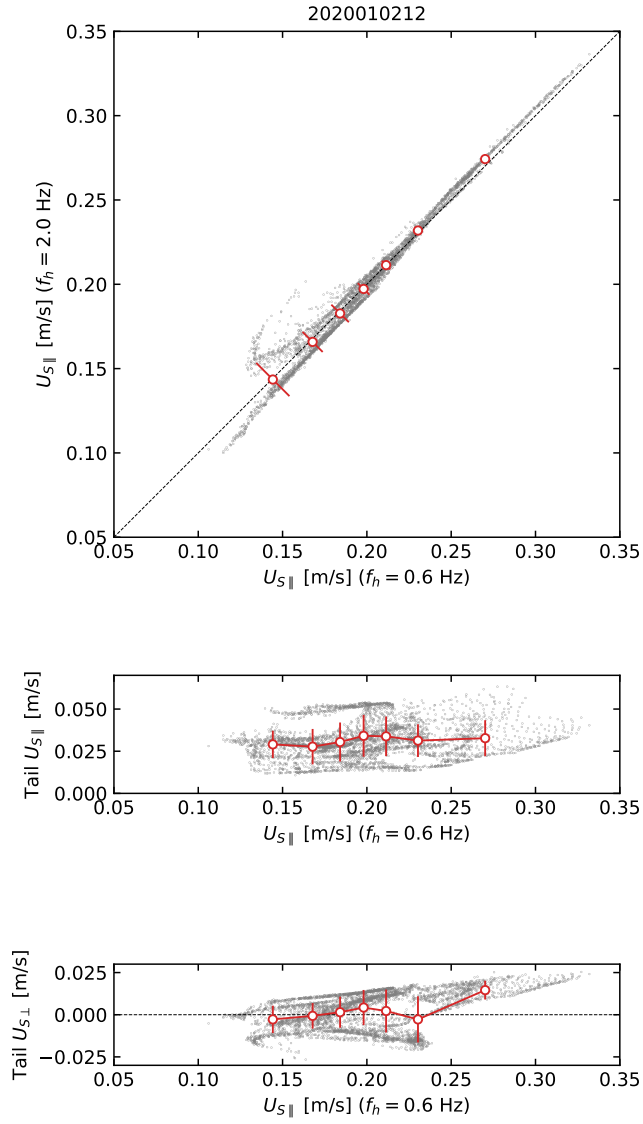


Figure 5: As Fig. 4 for simulation date 2020-01-02 12Z

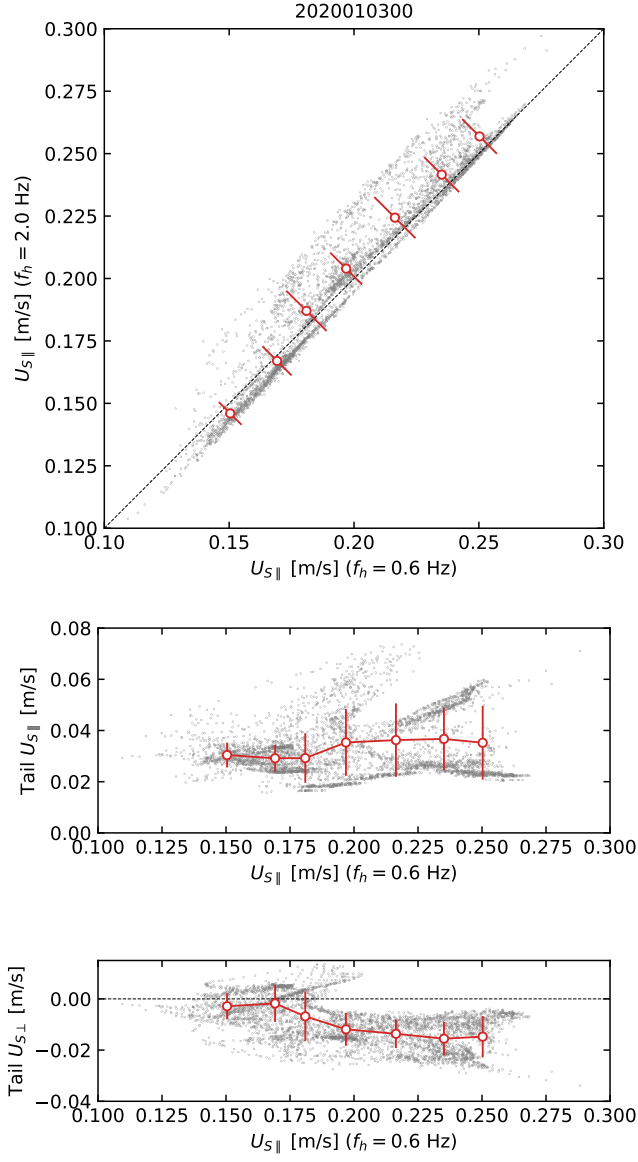


Figure 6: As Fig. 4 for simulation date 2020-01-03 00Z

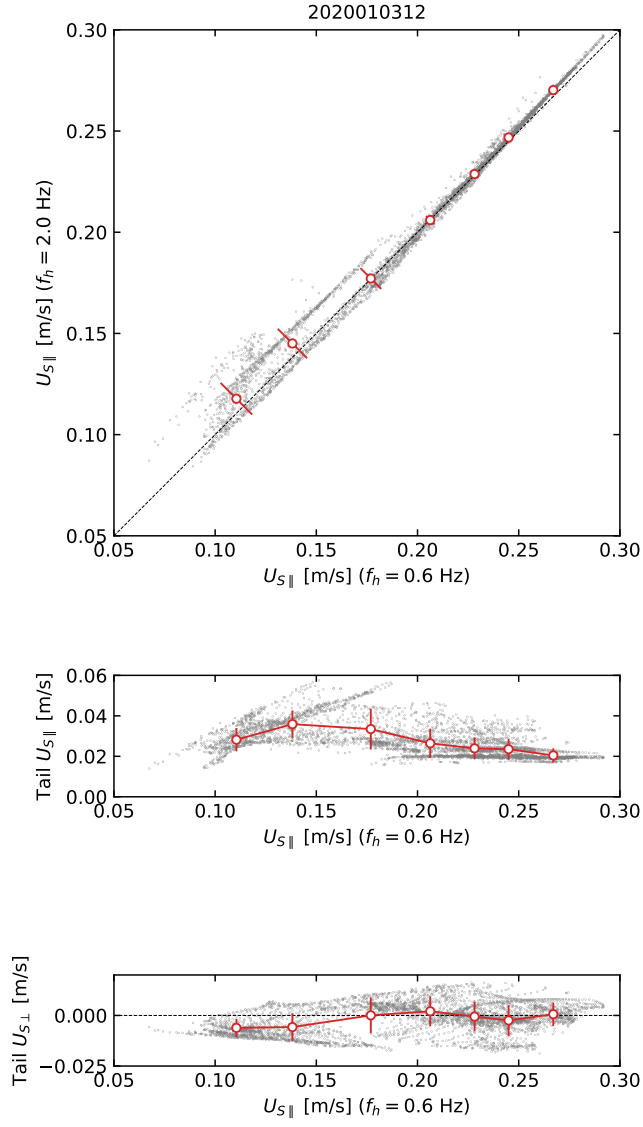


Figure 7: As Fig. 4 for simulation date 2020-01-03 12Z

APPENDIX

A Mathematical expression for the spectral tail extension

Ewans [1998] suggests a mathematical fit to their observation data, which results in the following expression:

$$(9) \quad m_1(\omega) \simeq m_{1E}(\hat{\omega}) = \cos(\theta_E(\hat{\omega})) e^{-\sigma^2(\hat{\omega})/2},$$

where $\hat{\omega} = \omega/\omega_P$ is the ratio of local spectral frequency over the peak frequency ω_P of the growing wind sea. $\theta_E(\hat{\omega})$ is the deviation from the mean direction of the centre of each of two 'binodal' directional peaks, and $\sigma(\hat{\omega})$ is the standard deviation of Gaussian-shaped distributions around each of the two peaks:

$$\begin{aligned} \theta_E(\hat{\omega}) &= t_a \exp(t_b/\hat{\omega}) \cdot \pi/180 \\ \sigma(\hat{\omega}) &= s_a - s_b/\hat{\omega}^2 \cdot \pi/180 \\ t_a &= 0.5 \exp(5.453) \text{ [deg]} \\ t_b &= -2.750 \\ s_a &= 32.13 \text{ [deg]} \\ s_b &= 15.393 \text{ [deg]} \end{aligned}$$

A formula for the spectral tail is assumed in the self-similar form of Donelan et al. [1985],

$$(10) \quad S(\omega) = \alpha W_{10} g \omega^{-4} \text{ for } \omega \gg \omega_P$$

where W_{10} is the 10 m wind speed. By applying equation 10 it is assumed that all components in the spectral tail are deep-water waves with $kD \gg 1$ for which $k = \omega^2/g$. The factor α may depend on integral wave properties, among which a dependance on the wave age $(W_{10}\omega_P/g) \cos(\theta_{mean} - \theta_W)$ was described by Donelan et al. [1985]. In the present usage, $\alpha W_{10}g$ is estimated from each model spectrum based on a few of the highest frequency bins of the prognostic spectrum. We note that the resulting estimates of α is sensitive to the choice of formulation of spectral forcing terms, i.e. the wind input, energy dissipation and weak non-linear wave-wave interactions. We originally applied this as a module in WAVEWATCH-III, originally with the Tolman-Chalikov forcing terms formulation (WW3 switch '/ST2'). In the present version it is applied with the '/ST4-Romero' terms introduced in WAVEWATCH-III version 7 (Romero [2019] and Tolmann et al. [2021]). For

the high-frequency spectral tail the '/ST4' forcing shares properties with the '/ST6' forcing (Liu et al. [01 Feb. 2019]) available as of WW3 version 6.

A self-similar form of Eq. 4 is obtained by inserting the spectral form of Eq. 10. The estimate for the differential Stokes drift becomes

$$(11) \quad dU_S(\omega, z) = 2\alpha W_{10} e^{-2kz} m_1(\omega) \omega^{-1} d\omega$$

With the spectrum (Eq. 10) of Donelan et al. [1985] and the expression of Ewans' (Eq. 9) for $m_1(\omega)$, the integrated Stokes drift $\int_{\omega_t}^{\omega_c} dU_S(\omega, z)$ is *not* designed to converge to a finite value as $\omega_c \rightarrow \infty$. However, it is realistic to assume that at values of $\hat{\omega} = \omega/\omega_P$ in the range 3 to 6 there exist a further transition to a spectrum of saturated form of Phillips [1958], going asymptotically as $S(\omega) \rightarrow \omega^{-5}$. This is also a property of the '/ST4-Romero' forcing terms in WAVEWATCH-III (Romero [2019]). Such a spectral tail works as a low-pass filter, and the exact shape of $m_1(\omega)$ beyond frequencies of order $\omega/\omega_P \approx 6$ does only influence the integrated Stokes drift by a small amount. In the present usage we approximate Ewans' expression by a function $m_{1t}(\omega/\omega_P) \simeq m_1(\omega)$,

$$(12) \quad m_{1t}(\hat{\omega}) = m_{1E}(\hat{\omega} = 1) \exp(a \sinh[\phi_h(\hat{\omega} - 1)])$$

where $\hat{\omega} = \omega/\omega_P$, and the constants are

$$\begin{aligned} \phi_h &= -0.187 \\ a &= 1.25 \end{aligned}$$

See Figure 8. ϕ_h represents the directional spread which increases with $\hat{\omega}$. The constants were determined by examining a variation of two parameters $\hat{\omega}$ and a in a construction

$$\phi_h = \frac{d}{d(\hat{\omega})} m_1(\hat{\omega}) / (a m_1(\hat{\omega}))$$

The value $\phi_h = -0.187$ is obtained with $a = 1.25$ and $\hat{\omega} = 2.0$

The approximation of Eq. 12, $m_{1t}(\omega)$, provides a low-pass filter at a frequency scale equivalent to the transition to an ω^{-5} diagnostic spectral tail. Thus we choose not to apply this transition, but keeps the Donelan tail ω^{-4} at high frequencies.

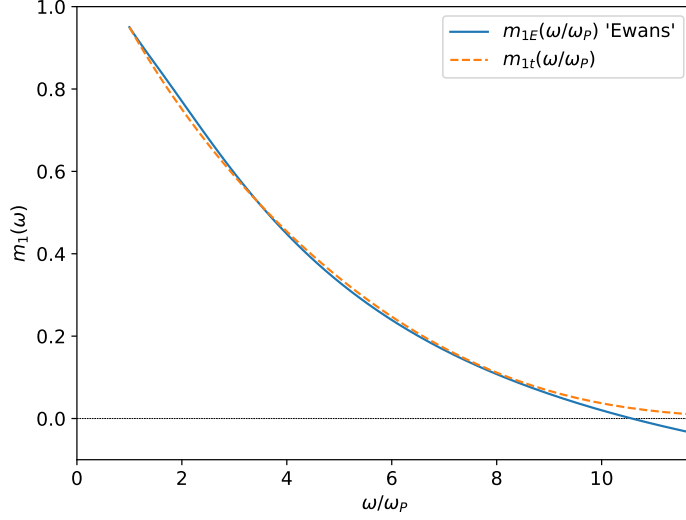


Figure 8: Approximation $m_{1t}(\hat{\omega})$ to Ewans' first circular moment $m_{1E}(\hat{\omega})$. m_{1E} will have to be truncated to a value of zero for the highest frequencies $\hat{\omega} \gtrsim 10$.

B A common 2D look-up array for the diagnostic tail

Selection of discrete depths

A set of profile depths Z_S are pre-described in *dimensionless* form as an array $K_S Z_S(I_z)$ of discrete dimensionless depths to be applied for all model grid cells and output time steps. We have chosen the following construction

$$(13) \quad K_S Z_S(I_z) = X_z^{(I_z-1)^b} - 1. \text{ for } I_z = 1..N_z$$

where b is a user-defined power that determines the progression of intervals between the discrete depths, and X_z is determined by the specified max dimensionless depth $K_S Z_S(N_z)$ as

$$X_z = (1. + K_S Z_S(N_z))^{(N_z-1.)^{-b}}$$

In the present version we have considered a default values of $b = 2$. Our experience is (e.g. the case simulation of Sec. 4) that a fairly high vertical resolution of the profile in most model situations is obtained with $N_z = 11$ and $K_S Z_S(N_z) = 2$.

High-frequency spectrum of normalized Stokes drift

The relative increment of spectral frequencies from bin to bin has a value of $X_F = 1.1$. This increment factor is standard in WW3 and other wave models using the DIA scheme of Hasselmann et al. [1995] for weak non-linear wave-wave interactions. The wavenumber relative increment from bin to bin is $X_K = X_F^2$.

We will define the extending tail based on a general array over indices $I_d = 0..N_d - 1$. The start index $I_d = 0$ represents an abstract *equivalent* 'peak' frequency ω_{Pd} which is simply defined as $\omega_{Pd} = \omega_t/\hat{\omega}$, where $\hat{\omega}$ is the parameter for which the expression $m_1(\hat{\omega})$ (e.g. Eq. 9) equals the first circular moment estimated from the model spectrum at the upper prognostic frequency ω_t . This is explained further in Appendix C.

Each bin in the tail array is defined as the closest representation of

$$\frac{\omega}{\omega_{Pd}} \simeq X_F^{I_d}$$

Thus $I_d = \text{round}(P_d)$ where $X_F^{P_d} = \frac{\omega}{\omega_{Pd}}$ or $P_d = \log(\frac{\omega}{\omega_{Pd}})/\log(X_F)$.

The differential Stokes drift over the tail bins is given from Eq. 11 and normalized as

$$\begin{aligned} (14) \quad \delta\tilde{U}_S(\omega) &= \delta U_S(\omega, z)/(\alpha W_{10} e^{-2kz}) \\ &= 2m_1(\omega)\omega^{-1}\delta\omega \end{aligned}$$

The first circular moment m_1 may be represented by Ewans' formula (Eq. 9), $m_1 = \max(m_{1E}(\omega/\omega_{Pd}), 0)$. However to avoid the truncation to zero when $m_{1E} < 0$ and provide a more smooth result we apply the approximation of Eq. 12, $m_1 = m_{1t}(\omega/\omega_{Pd})$.

Combination into a common 2D look-up array

Let N_d be a maximum value for the number of frequencies in the diagnostic spectrum to be applied as an extension to the prognostic spectrum.

We also define a discrete array $R_K(I_d)$ representing the squared ratio of frequencies $(\frac{\omega}{\omega_{Pd}})^2$:

$$R_K(I_d) = X_K^{I_d} \text{ for } I_d = 0..N_d - 1$$

Based on Eq. 14 we can calculate a common 2D *look-up* array

$$\begin{aligned} (15) \quad \delta S_d(I_z, I_d) &= \delta\tilde{U}_S(\omega) \exp(-2R_K(I_d)K_S Z_S(I_z)) \\ &\text{for } I_z = 1..N_z; I_d = 1..N_d \end{aligned}$$

The array $\delta S_d(I_z, I_d)$ is calculated in the initial stage of the model run and applies to the spectrum of every water point in the model at all time steps.

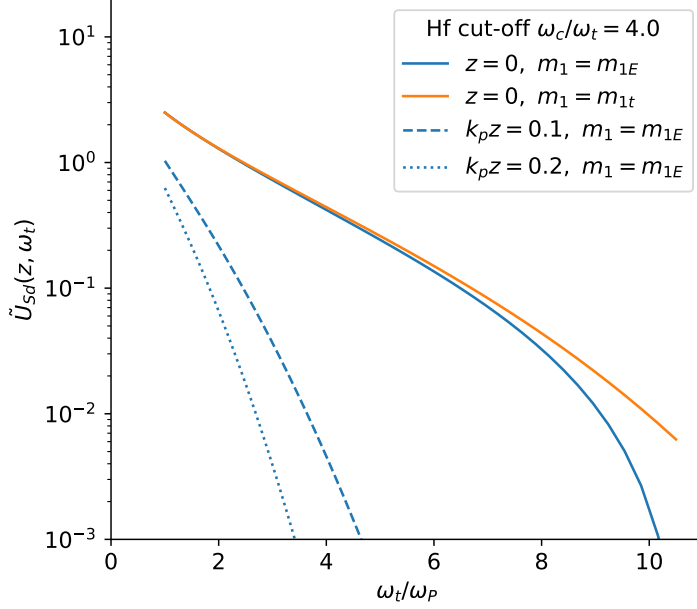


Figure 9: Normalized Stokes drift $\tilde{U}_{Sd}(z, \omega_t)$ of the diagnostic part of the spectrum as a function of ω_t/ω_P for three values of $k_P z = \omega_P^2/gz = 0$, $k_P z = 0.1$, and $k_P z = 0.2$ respectively. The surface drift is plotted both with Ewans' first circular moment m_{1E} and the approximation m_{1t} . As in Fig. 8 the tail region is $[\omega_t; \omega_c]$ with a high frequency cut-off at $\omega_c = 4\omega_t$

C Procedure for calculating the diagnostic part of the Stokes profile

At each output time step of the model run, the diagnostic contribution $\vec{U}_{Sd}(z)$ to the discrete Stokes profile is calculated as follows.

First, we estimate an *equivalent* value of the 'peak' frequency ω_{Pd} in the formula for m_{1E} in Eq. 9. We estimate this for four spectral bin frequencies just below ω_t , that is for the frequencies ω_{N_K-5} to $\omega_t = \omega_{N_K-2}$ where N_K is the highest prognostic frequency bin of the spectral model.

Directly from this frequency range of the model spectrum we calculate the three parameters: Mean wave direction dir_t , first circular moment m_1 , and spectral level αW_{10} . The equivalent value of ω_{Pd} is derived as a root of $m_{1E}(\omega_t/\omega_{Pd}) - m_1 = 0$ applying the Newton-Raphson method (Press et al. [1993], § 9.4). Then the lower index $I_d = I_t$ in the *look-up* array (Eq. 15) is obtained as the integer $I_t = \text{round}(P_t)$ where $\omega_t/\omega_{Pd} = X_F^{P_t}$.

The diagnostic contribution is calculated first in normalized form from the general look-up array (Eq. 15)

$$(16) \quad \tilde{U}_{Sd}(z, \omega_t) = \sum_{I=I_t}^{I_c} \delta S_d(I_z, I)$$

where I_z is the discrete index of depth z and the index for the upper diagnostic cut-off frequency ω_c is $I_c = \text{round}(P_c)$ where $\omega_c/\omega_P = X_F^{P_c}$. The default diagnostic cut-off is at 2 Hz, $\omega_c = 2\pi \cdot 2$ rad/s. Figure 9 shows plots of $\tilde{U}_{Sd}(z, \omega_t)$ for the case of $\omega_c/\omega_t = 4.0$. For a diagnostic cut-off is at 2 Hz this would be the case if the upper prognostic model frequency is $\omega_t = 2\pi \cdot 0.5$ Hz.

Then multiply with the spectral level to obtain the diagnostic part of the Stokes drift magnitude

$$|\vec{U}_{Sd}(z)| = \tilde{U}_{Sd}(z, \omega_t) \alpha W_{10}$$

The drift vector $\vec{U}_{Sd}(z)$ is obtained as a vector product with a unit vector in the direction dir_t at ω_t .

D Technical discussion

In the present version we have considered a default values of the parameter $b = 2$ in a specific construction (Eq. 13) of the vertical profile levels. This has been applied with a a profile array length of $N_z = 11$ and a dimensionless depth scale of $K_S Z_S(N_z) = 2$. Higher values of b may be applied for realistic wave model simulations, e.g. $b = 1$ for $K_S Z_S(N_z) = 1$.

Alternative constructions of the vertical profile levels are allowed, as long as overall values of an array $K_S Z_S(I_z)$ is specified for Eq. 15. The scale wavenumber K_S is determined for each spectrum individually during the model run, and is a model output field over space and time. The array $K_S Z_S(I_z)$ is also provided as output from the model run and allows the user to re-construct the actual depth values $Z_S(I_z)$. Constructions different from Eq. 13 may be implemented in the future.

In the present version, K_S is defined from the wave zero-upcrossing period, calculated from the second moment of the full prognostic model spectrum. It would make sense to estimate a scale $K_S = K_3$ from the magnitude of a third directional spectral moment as

$$(17) \quad K_3 = \frac{1}{g} \left(\frac{|\vec{m}_3|}{m_0} \right)^{2/3}$$

where the third moment is a vector $\vec{m}_3$

$$(18) \quad \vec{m}_3 = \int_{\omega_0}^{\omega_t} \int_{-\pi}^{\pi} g \vec{k} \omega S(\omega, \theta) d\theta d\omega$$

and m_0 is the integrated wave variance

$$(19) \quad m_0 = \int_{\omega_0}^{\omega_t} \int_{-\pi}^{\pi} \omega S(\omega, \theta) d\theta d\omega$$

There seems no absolute need to apply the approximation of m_{1t} rather than Ewans' original form m_{1E} truncated to zero for high frequencies. An approximation slightly different from m_{1t} was introduced in earlier versions of the present procedure, before the common 2D array of Eq. 15 was introduced. In that form the array of m_1 was partly re-calculated for each model spectrum at each output time step.

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