
An open source Matlab[®] package for solving Fokker-Plank Equation and validation of Integral Fluctuation Theorem

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IRIG-DSBT, UMR CEA-Grenoble University, CEA Grenoble, 17 rue des Martyrs, 38054 Grenoble, France



Products Solutions Academia Support Community

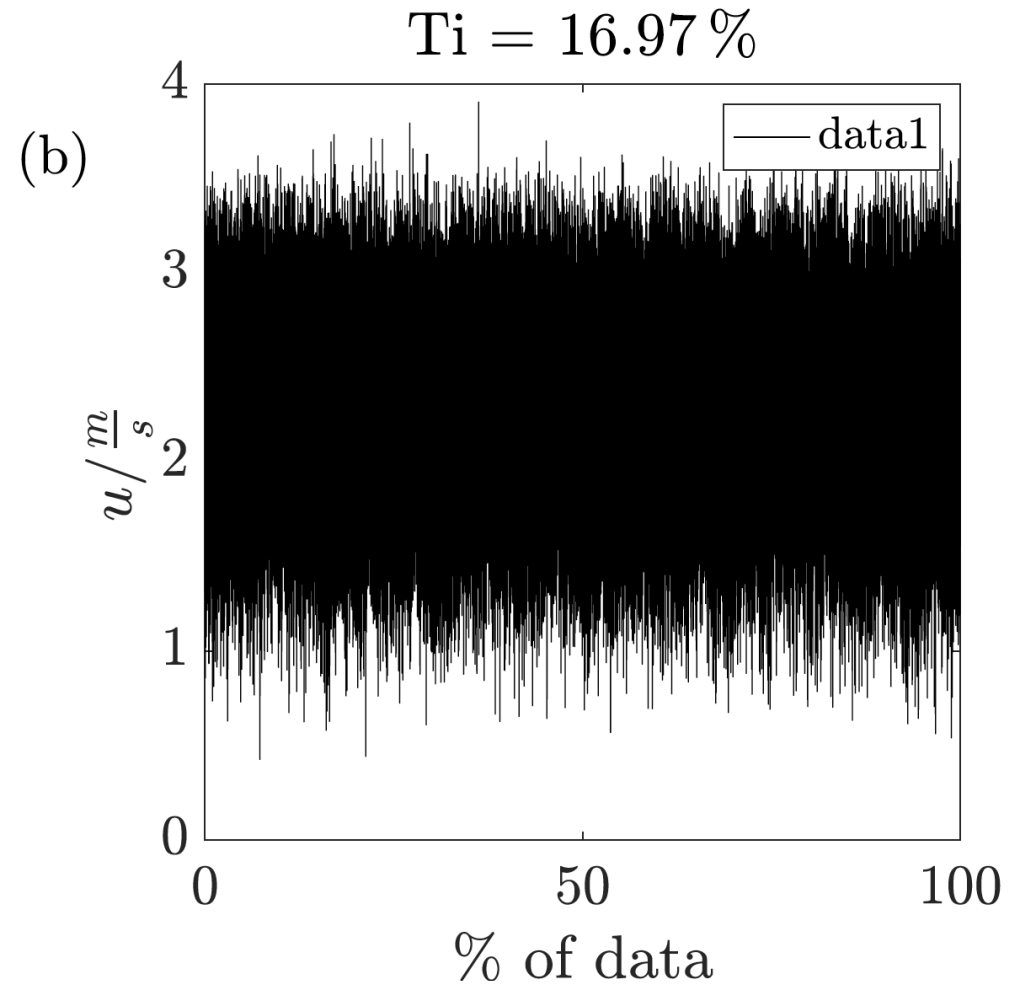
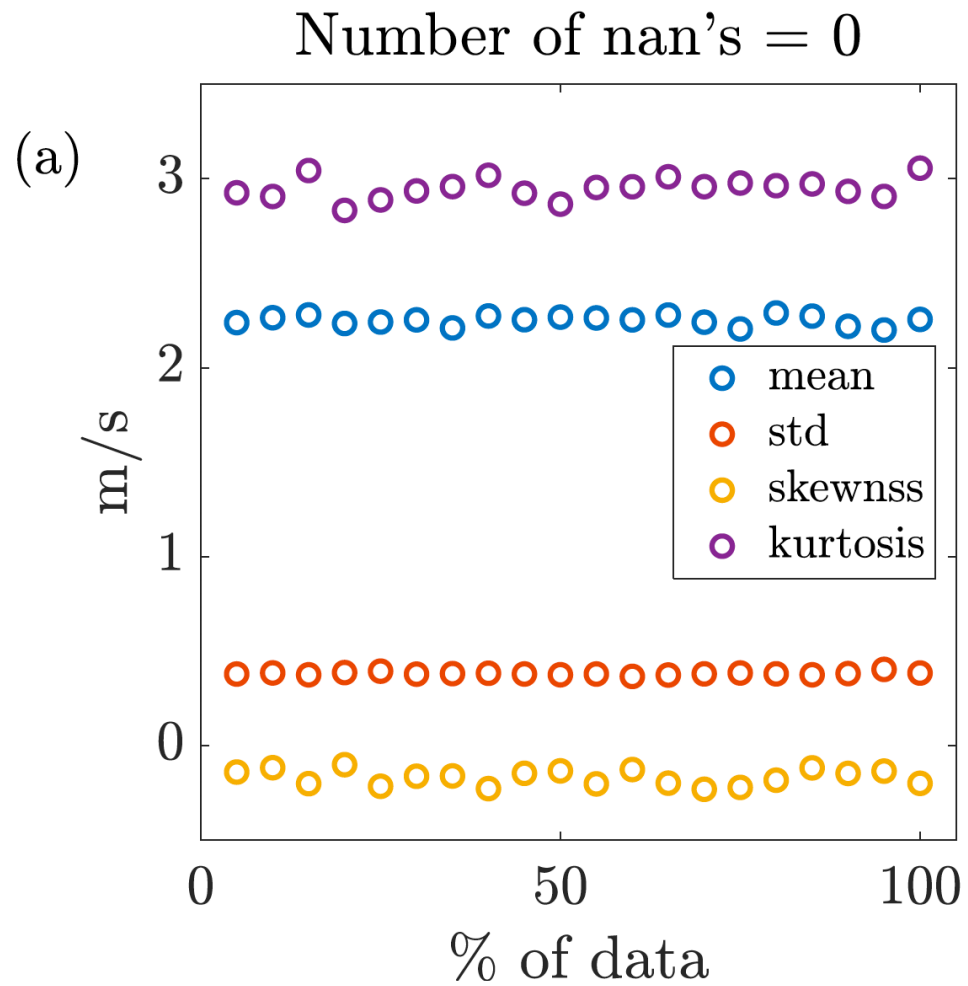
File Exchange

https://de.mathworks.com/matlabcentral/fileexchange/80551-open_fpe_ift

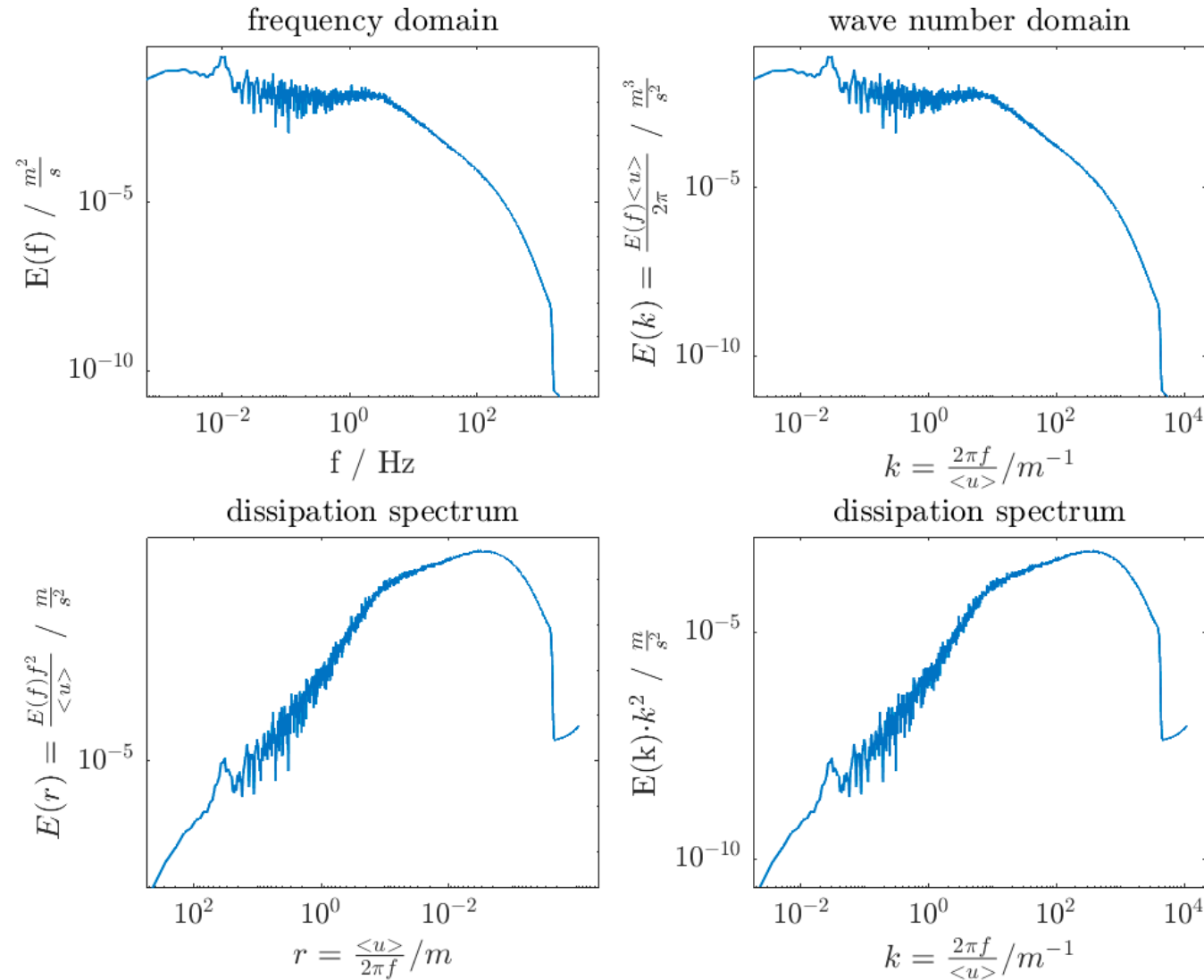


https://github.com/andre-fuchs-uni-oldenburg/OPEN_FPE_IFT

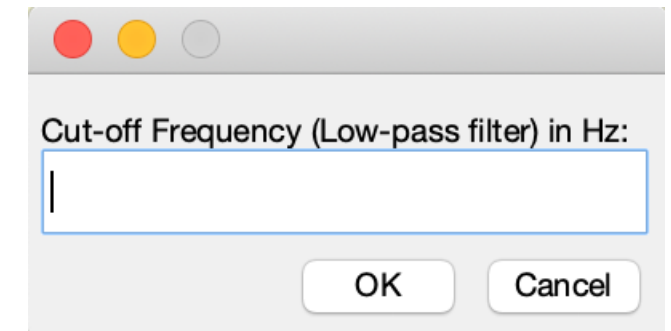
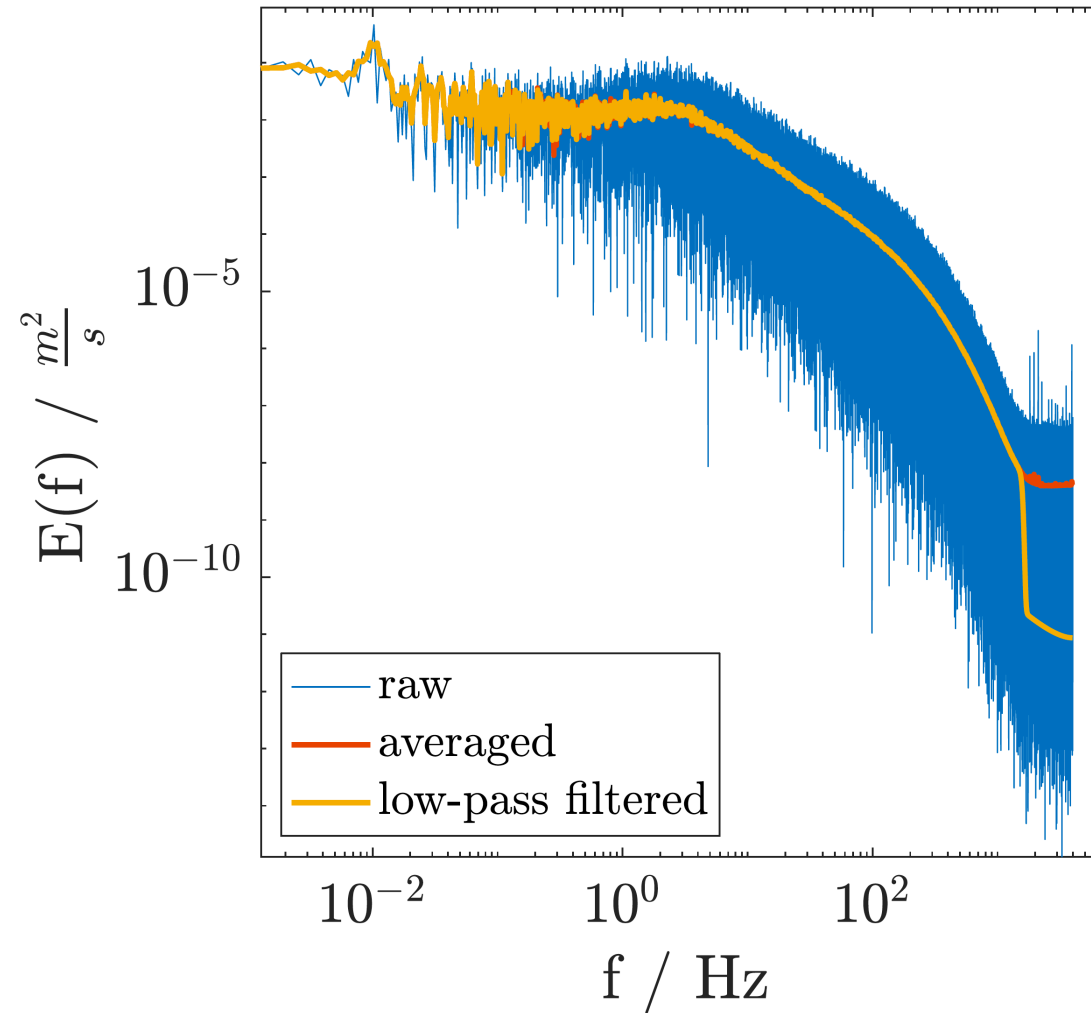
Detecting stationarity in time series data



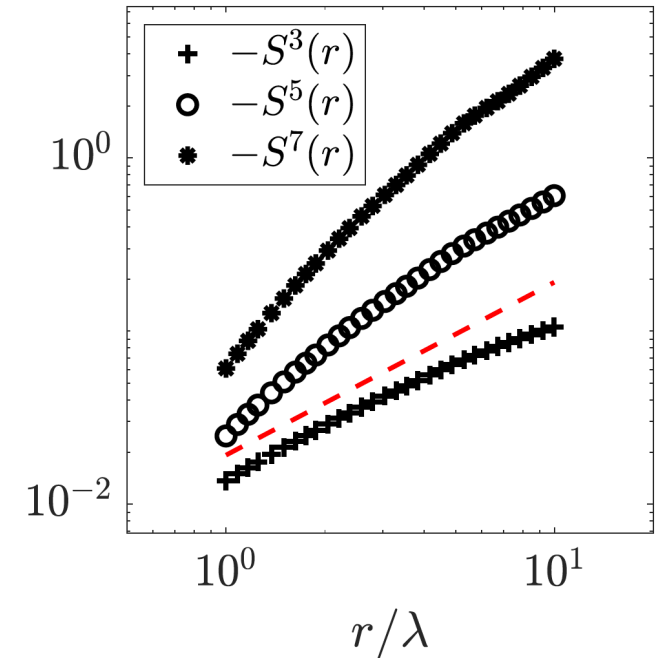
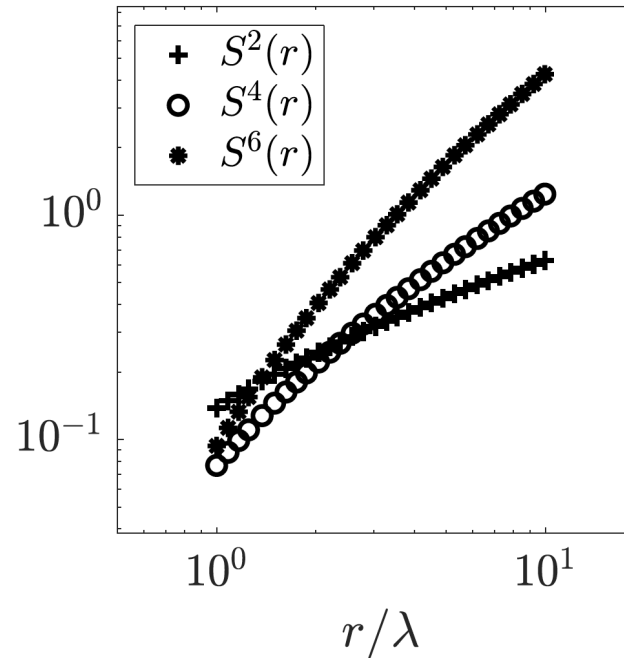
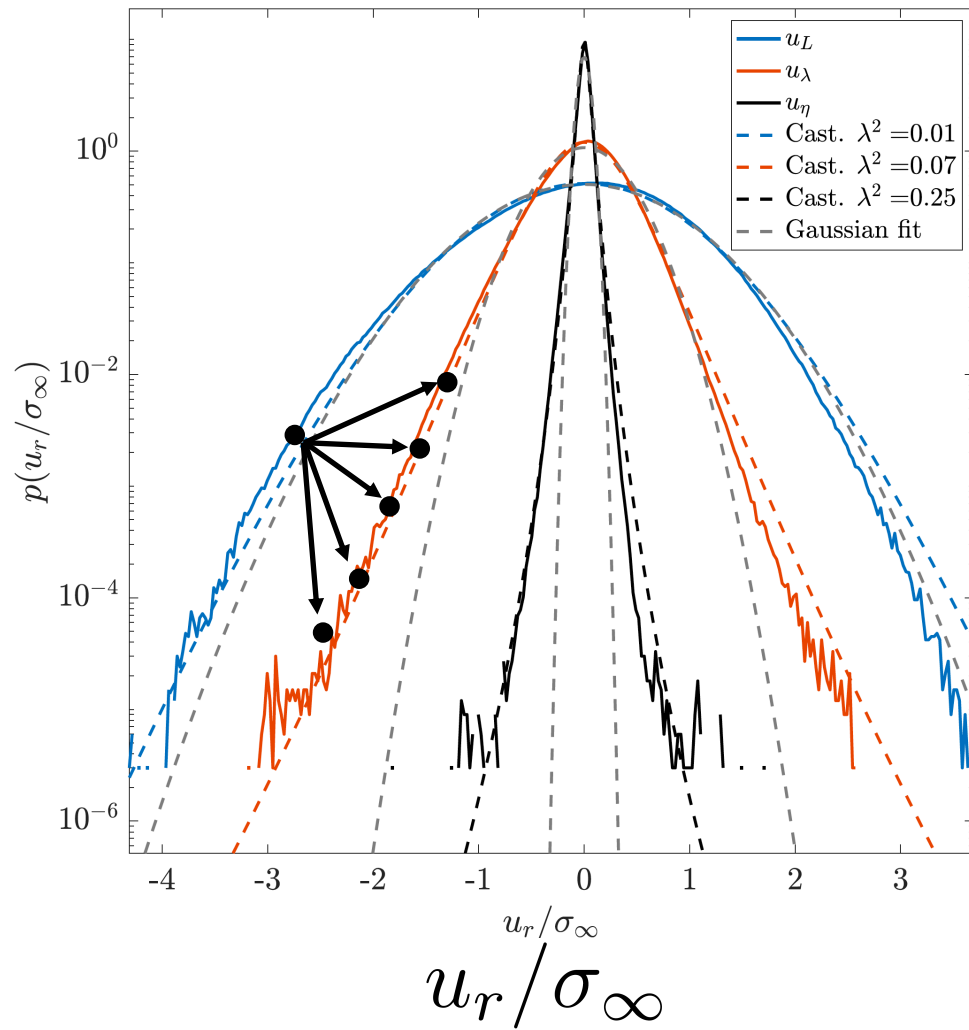
Energy spectral density



Low-pass filter: Energy spectrum density



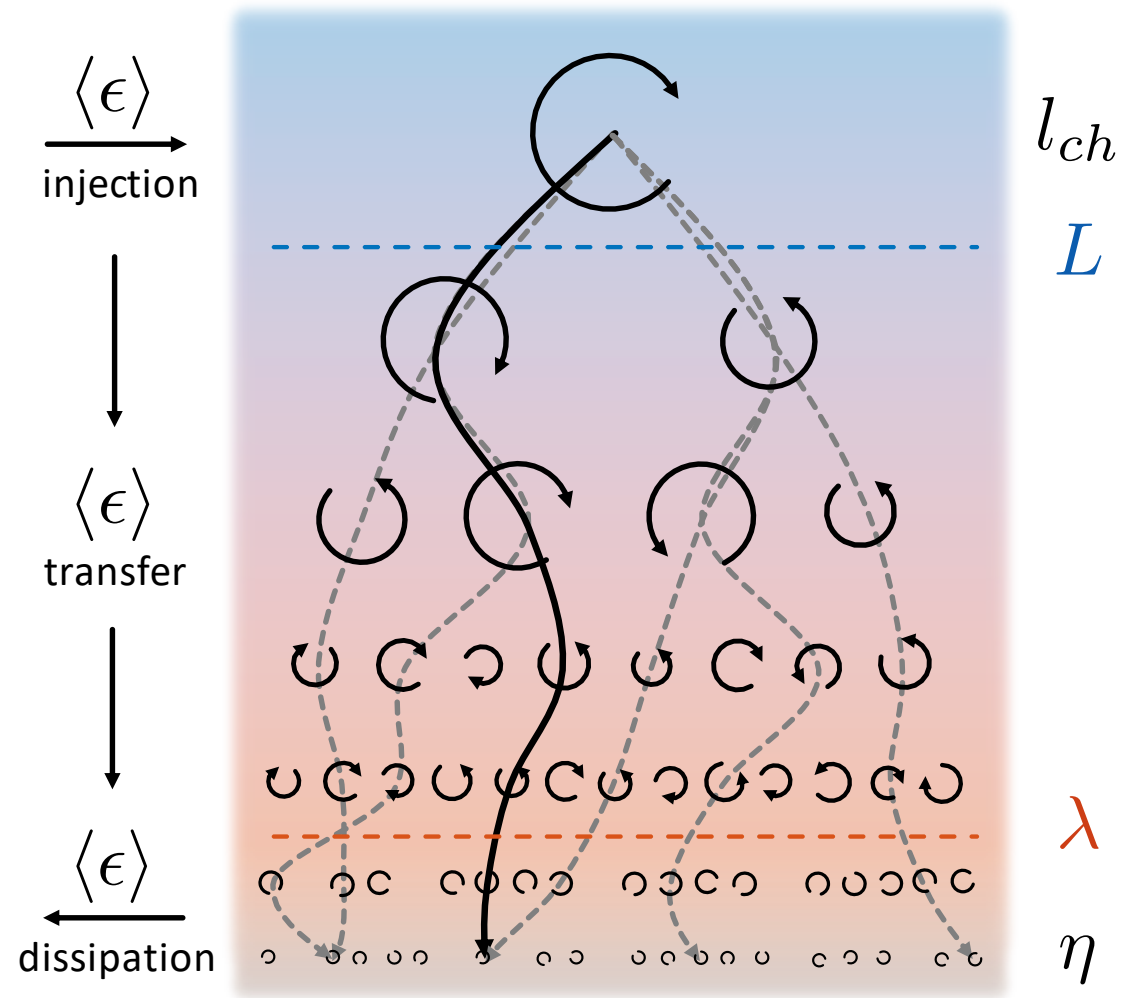
How to describe the stochastic evolution of u_r from scale to scale?



Scaling of structure functions:

$$S^k(r) = \langle u_r^k \rangle \propto r^{\zeta_k}$$

Estimation of fundamental length scales



Input

Enter the kinematic viscosity in m²/s

15.32e-6

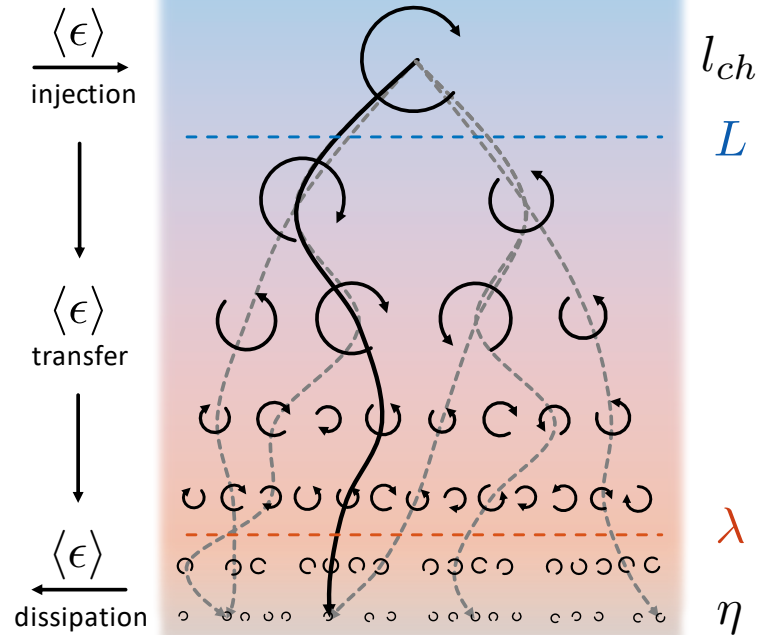
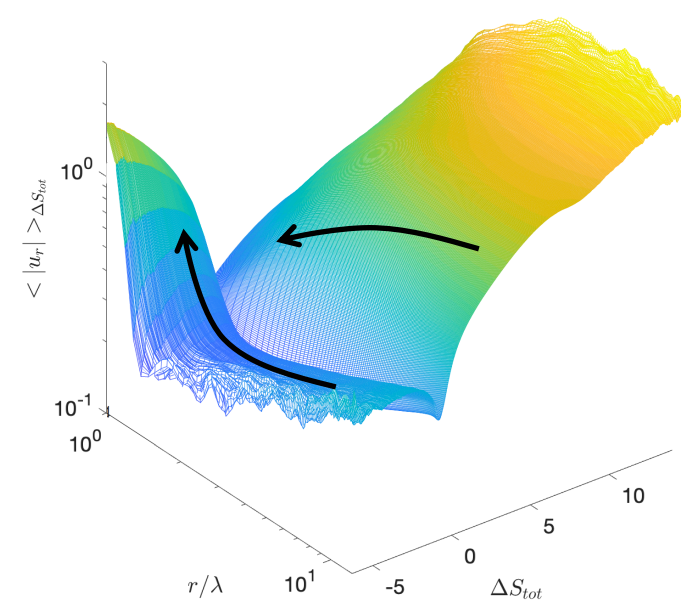
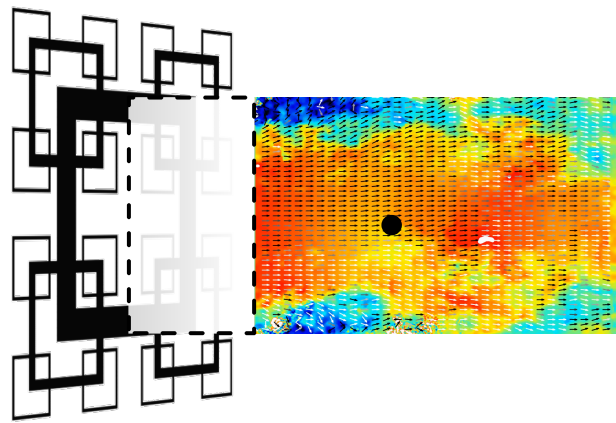
OK Cancel

Input

Enter the value of Kolmogorov constant, C₂ associated with second order structure function(in between 2.0 to 2.2)

2.1

OK Cancel



PHYSICAL REVIEW FLUIDS

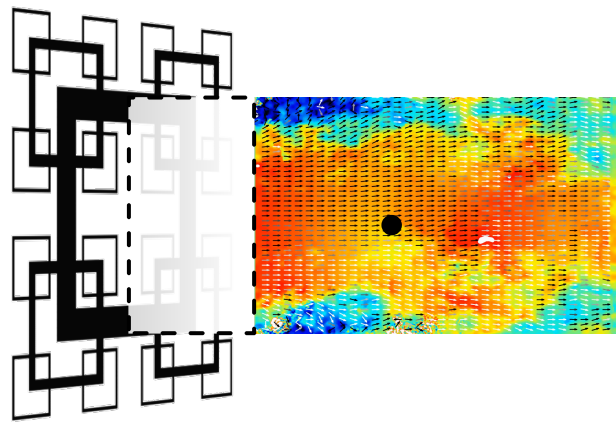
Small scale structures of turbulence in terms of entropy and fluctuation theorems

André Fuchs, Sílvia M. Duarte Queirós, Pedro G. Lind, Alain Girard, Freddy Bouchet, Matthias Wächter, and Joachim Peinke
Phys. Rev. Fluids **5**, 034602 – Published 11 March 2020

ANNUAL REVIEWS

The Fokker–Planck Approach to Complex Spatiotemporal Disordered Systems

J. Peinke,^{1,2} M.R.R. Tabar,³ and M. Wächter¹

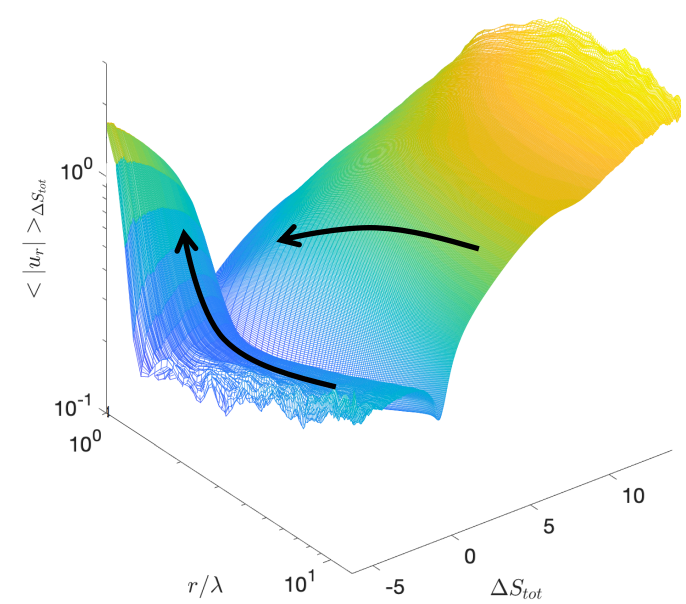


structures
statistics

Friedrich
Peinke

Fokker-Planck description

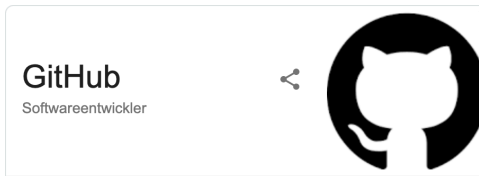
- Markov picture of turbulence
- Multi-point statistics



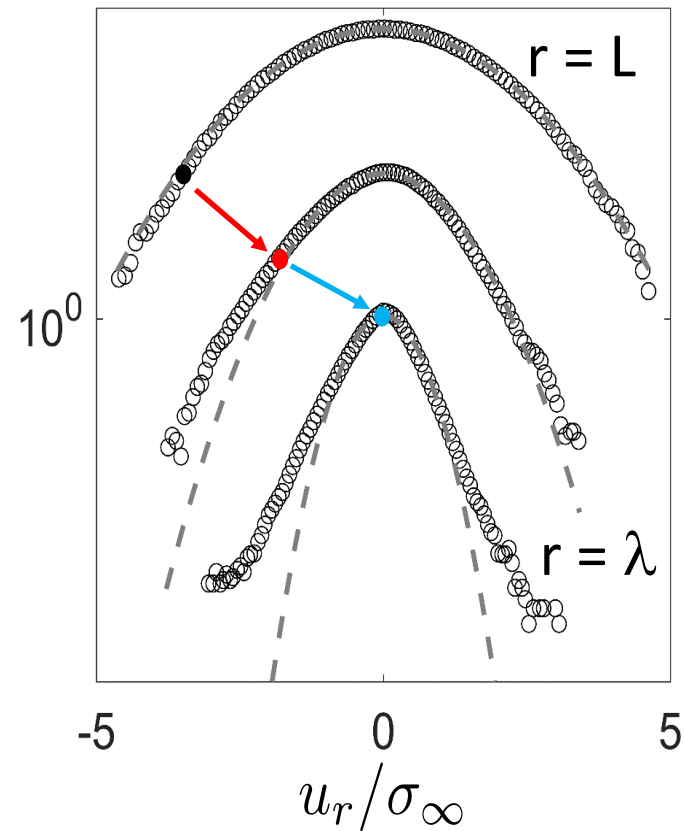
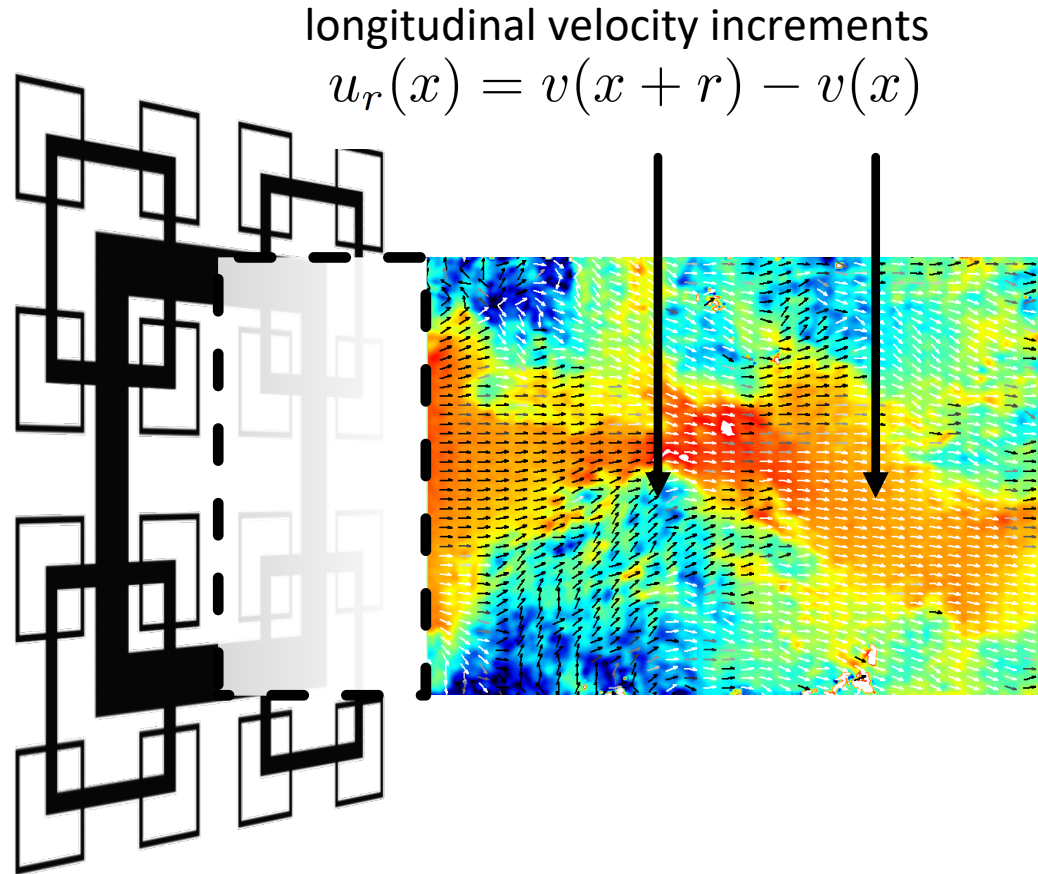
Seifert
Engel,
Nickelsen

stochastic thermodynamics
of cascade process

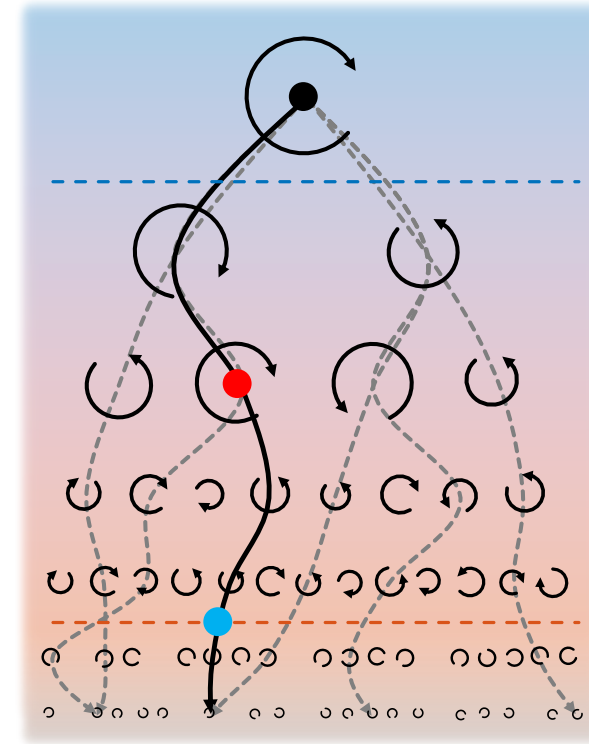
- fluctuation theorem

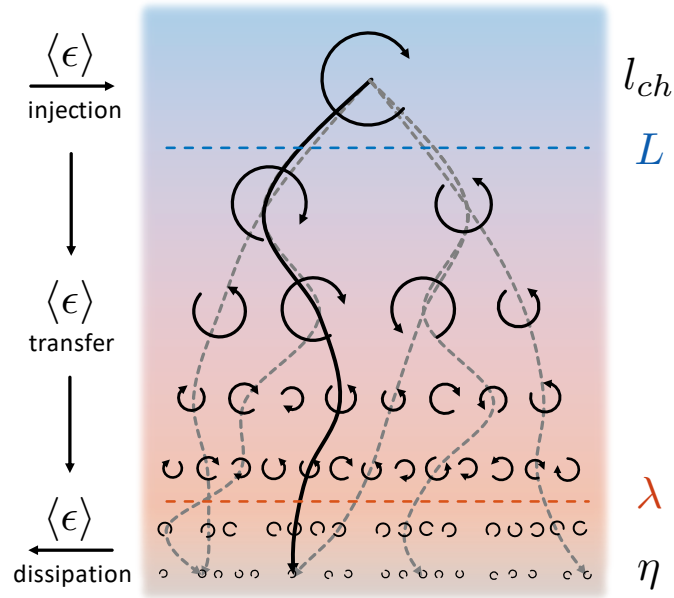


Trajectories of velocity increments $[u(\cdot)]$



one realization of the cascade process





Friedrich
Peinke

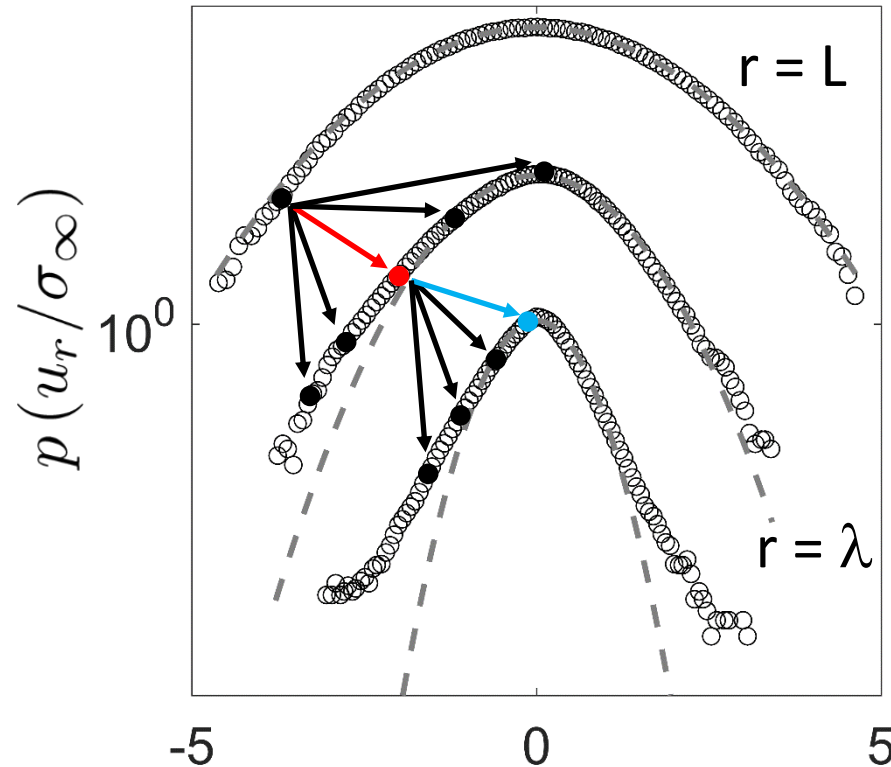
Fokker-Planck description

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Seifert
Engel,
Nickelsen

- stochastic thermodynamics
of cascade process
- fluctuation theorem

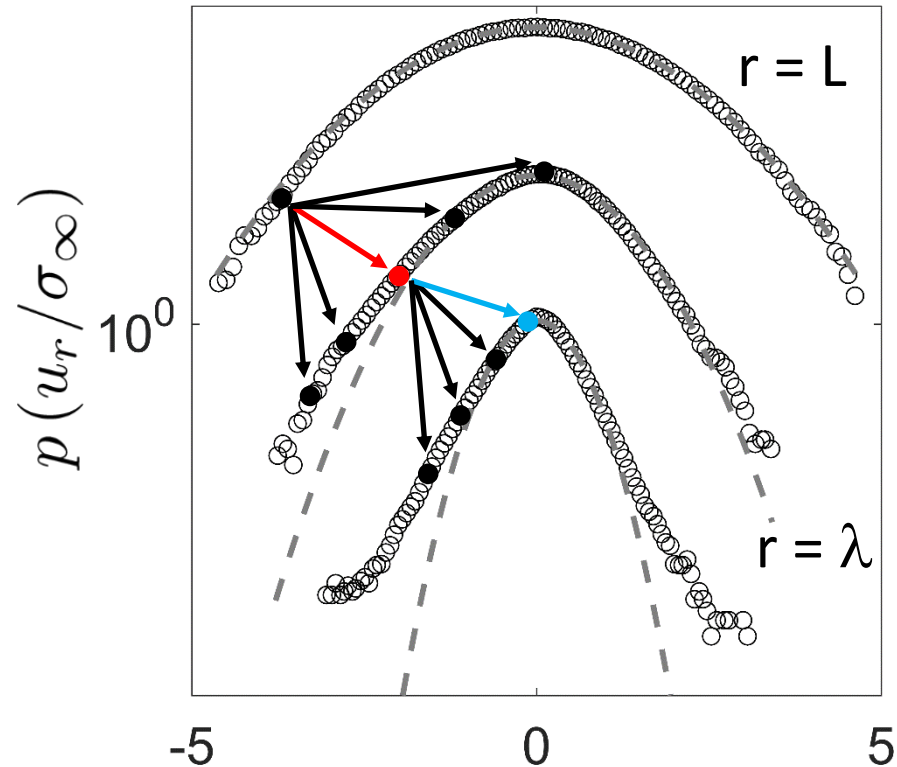
How to describe the stochastic evolution of u_r from scale to scale?



Markov picture of turbulence
Fokker-Planck description:

$$-\partial_r p(u_r | u_{r_0}) = \left(-\partial_u D^{(1)}(\underline{u_r}, r) + \partial_u^2 D^{(2)}(\underline{u_r}, r) \right) p(u_r | u_{r_0})$$

How to describe the stochastic evolution of u_r from scale to scale?

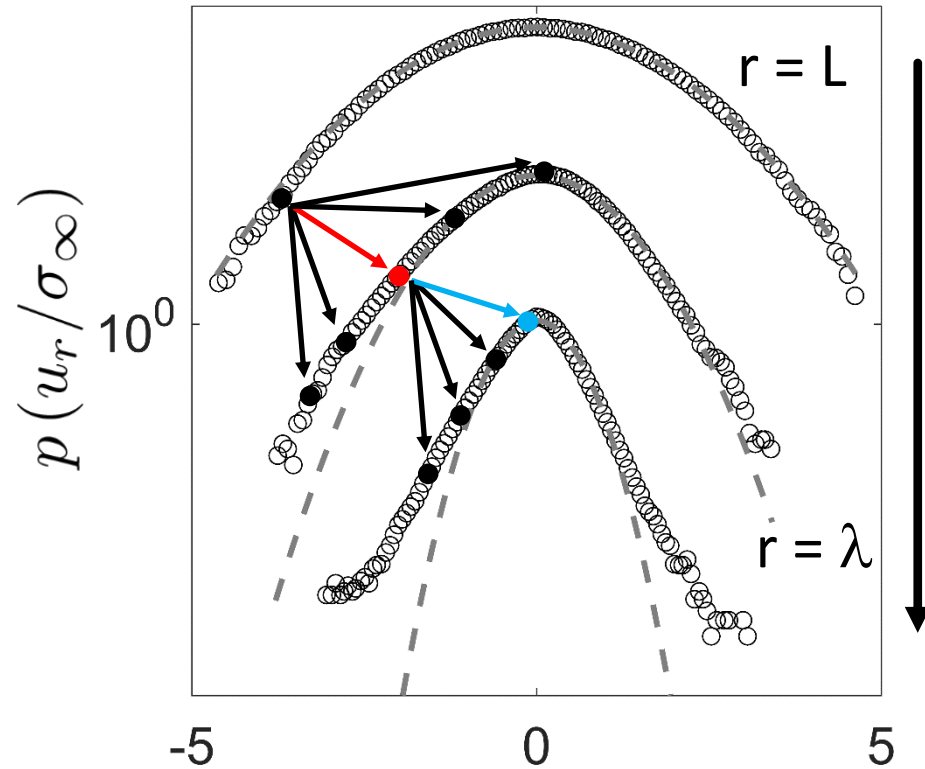


$$\underline{p(u_r)} | \underline{u_{r'}} = \overset{?}{p(u_r | u_{r'}, u_{r''})}$$

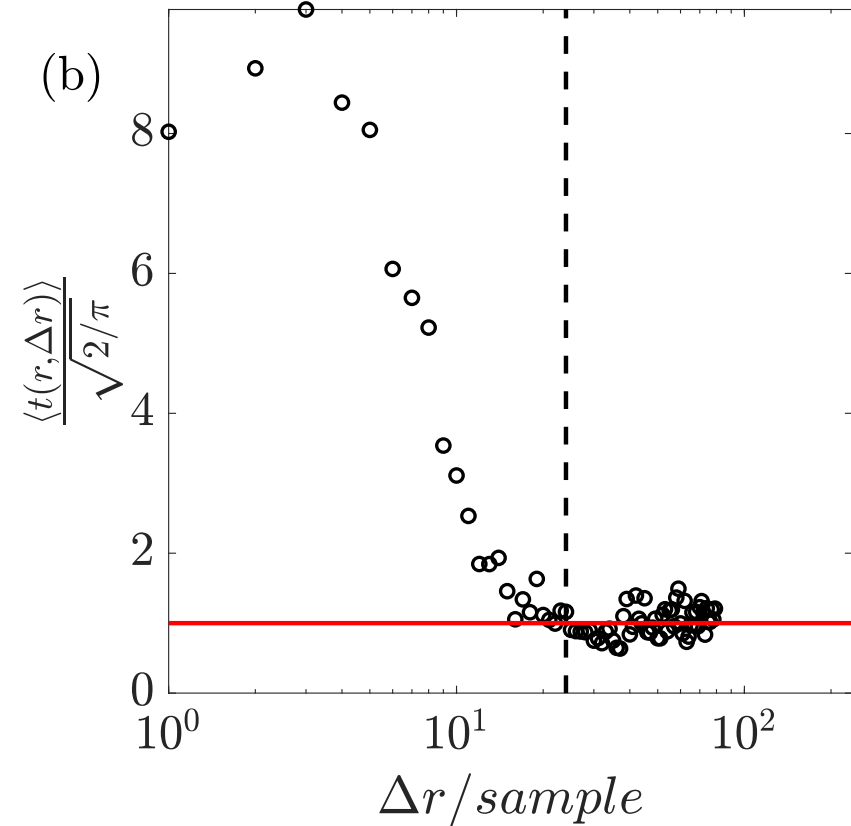
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Einstein-Markov length: wilcoxon test

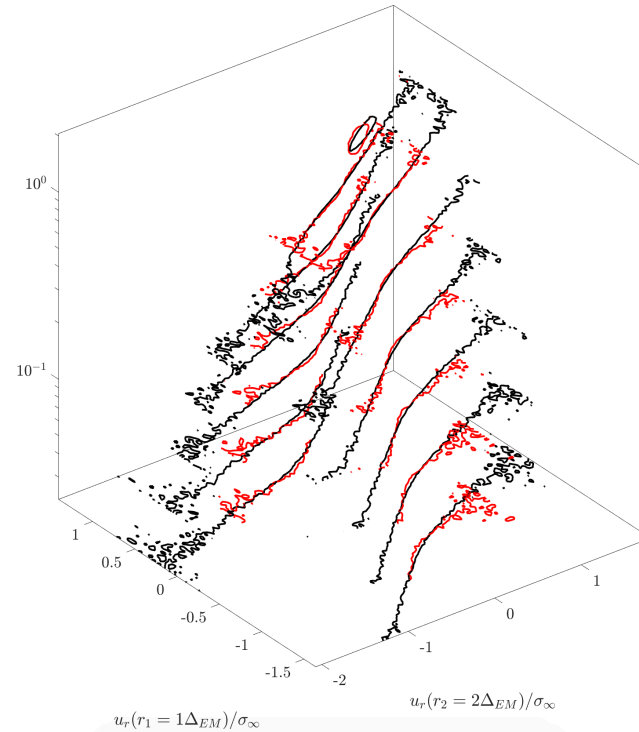
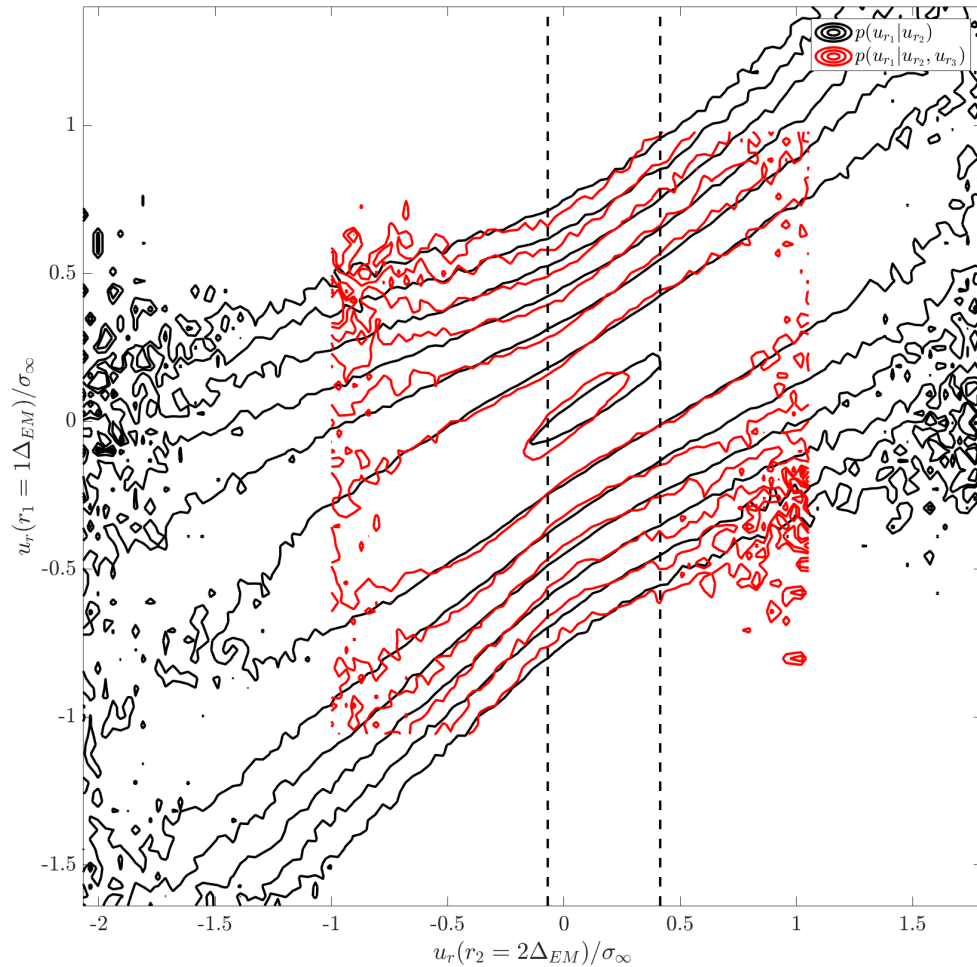


$$p(\underline{u_r} | \underline{u_{r'}}) \stackrel{?}{=} p(\underline{u_r} | \underline{u_{r'}}, \underline{u_{r''}})$$



Einstein-Markov length: conditional PDF

$$p(u_r | u_{r'}) \stackrel{?}{=} p(u_r | u_{r'}, u_{r''})$$



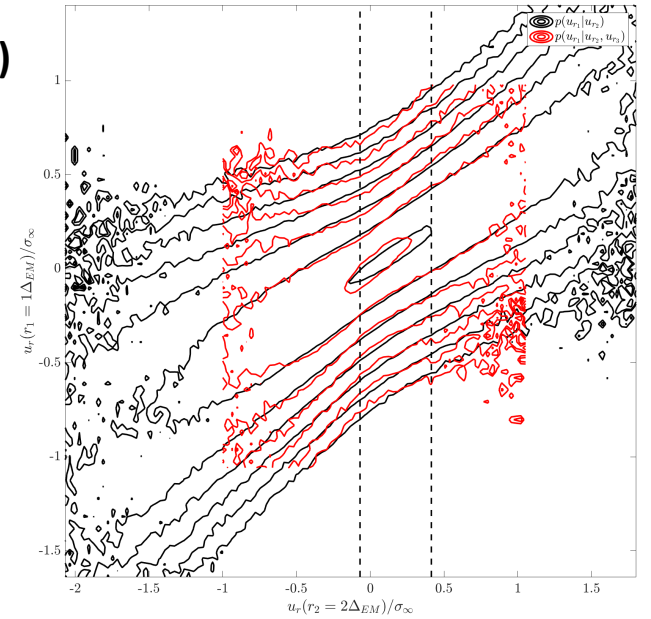
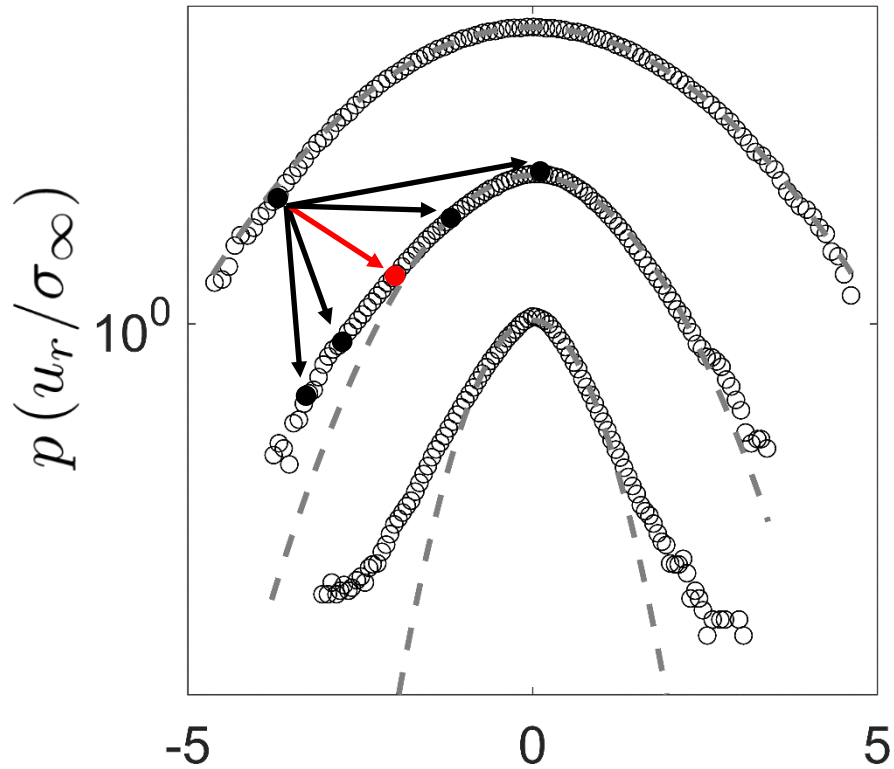
Increment of the second condition:

OK Cancel

Number of Bins:

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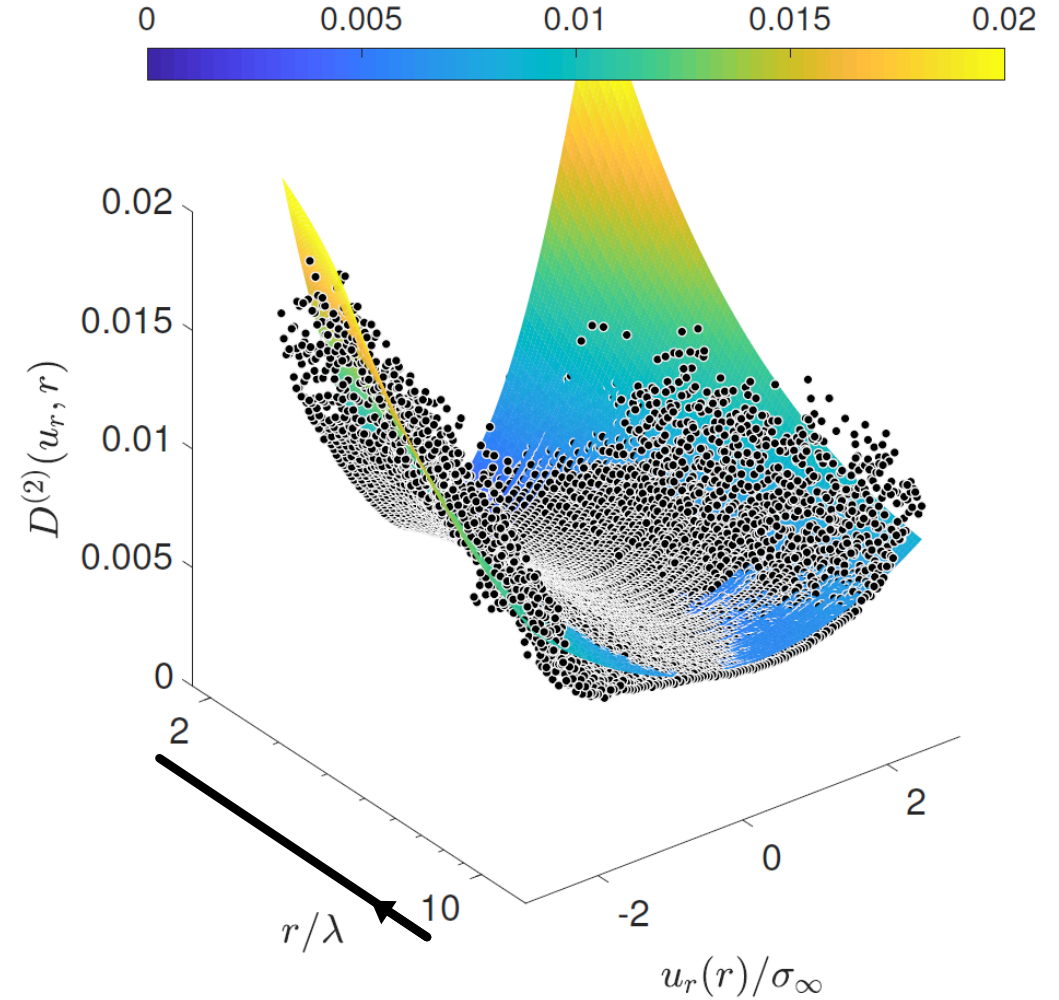
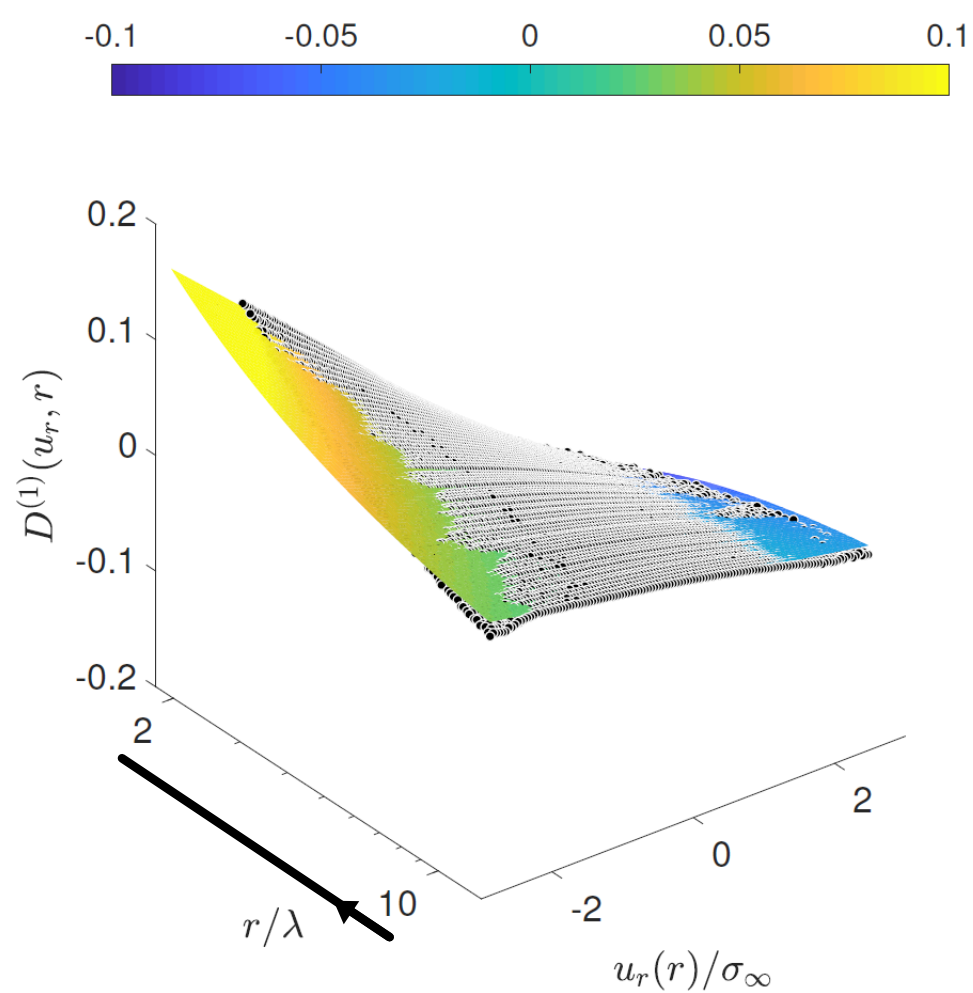
Markov picture of turbulence: Kramers-Moyal coefficients $D^{(1;2)}$

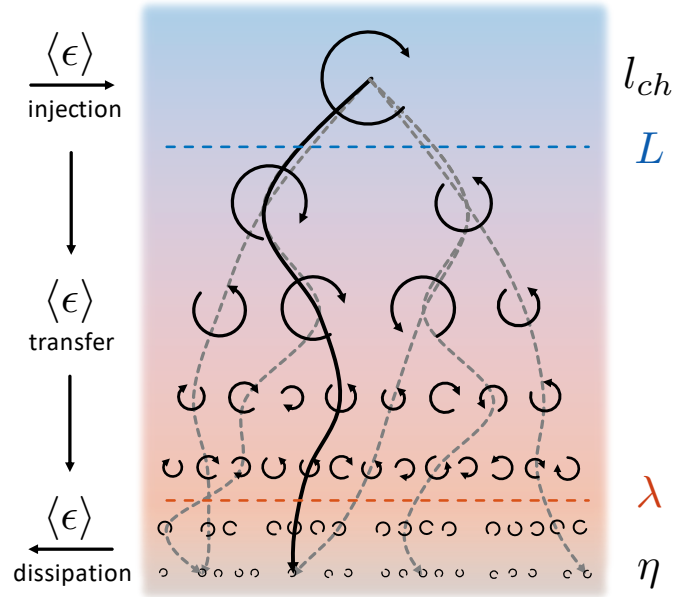


conditional moments

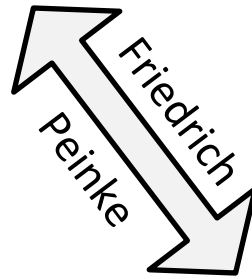
$$D^{(k)}(u_r, r) = \lim_{r' \rightarrow r} \frac{\int_{-\infty}^{\infty} (u_{r'} - u_r)^k p(u_{r'} | u_r) du_{r'}}{k! (r' - r)}$$

Markov picture of turbulence: Kramers-Moyal coefficients $D^{(1;2)}$





Vielleicht erst hier die
Animation mit den
Trajektorien
einfügen



Fokker-Planck description

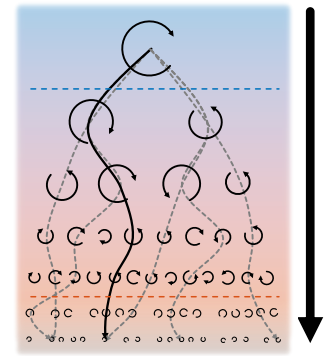
- Markov picture of turbulence
- Multi-point statistics



stochastic thermodynamics
of cascade process

- fluctuation theorem

Fluctuation theorem for **non-thermal** noise in **macroscopic** system: developed turbulence



total entropy production for single trajectories $[u(\cdot)]$

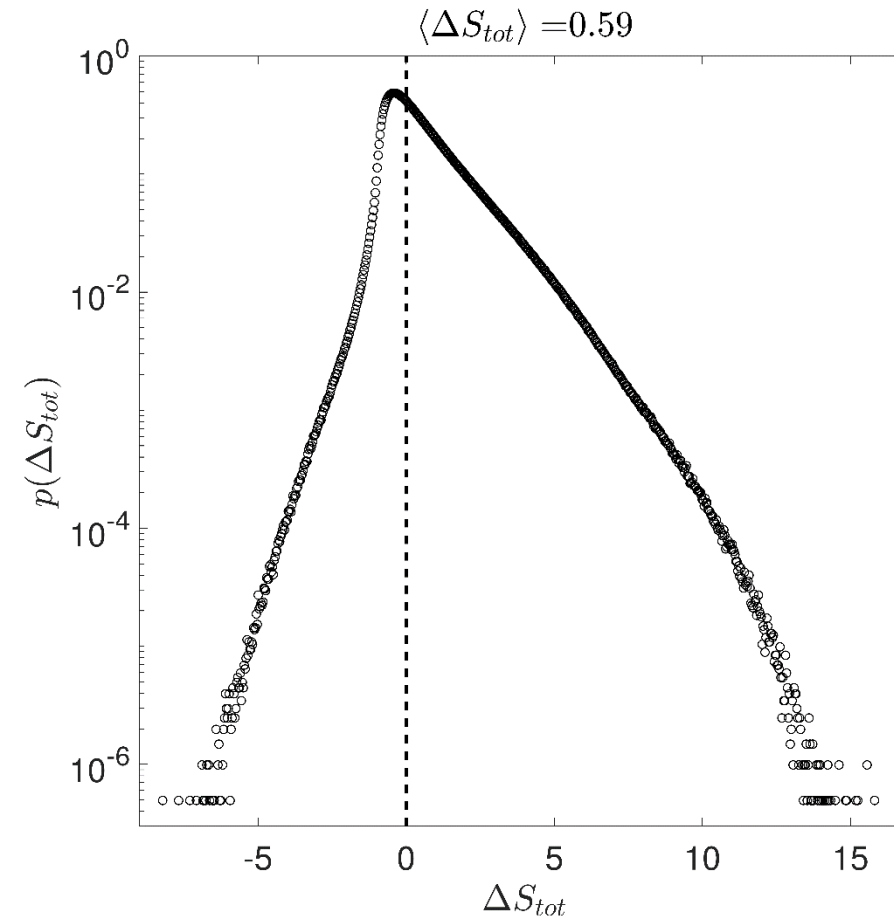
$$\Delta S_{tot} [u(\cdot)] = \int_L^\lambda \partial_r u_r \frac{D^{(1)}(u_r, r) - \partial_u D^{(2)}(u_r, r)}{D^{(2)}(u_r, r)} dr - \ln \left(\frac{p(u_\lambda, \lambda)}{p(u_L, L)} \right)$$

integral fluctuation theorem (IFT)

$$\langle e^{-\Delta S_{tot}} \rangle [u(\cdot)] = 1$$

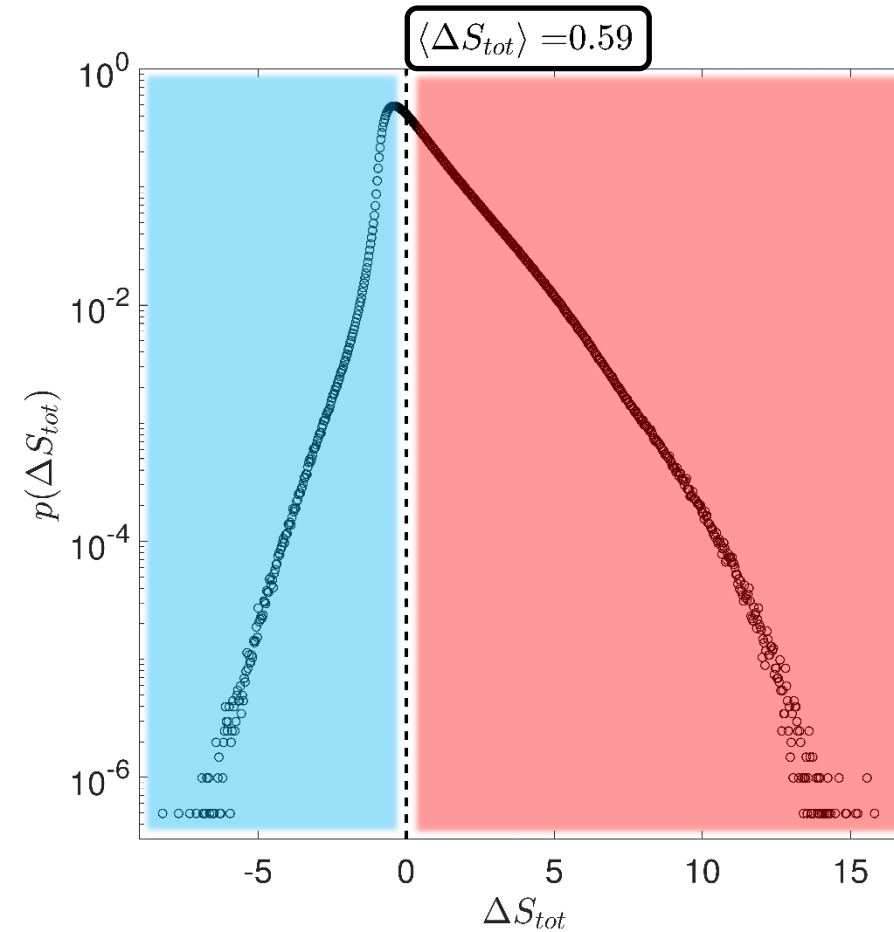
Entropy balance of cascade process

Verification of the measured FPE: **Validity of the IFT**



Entropy balance of cascade process

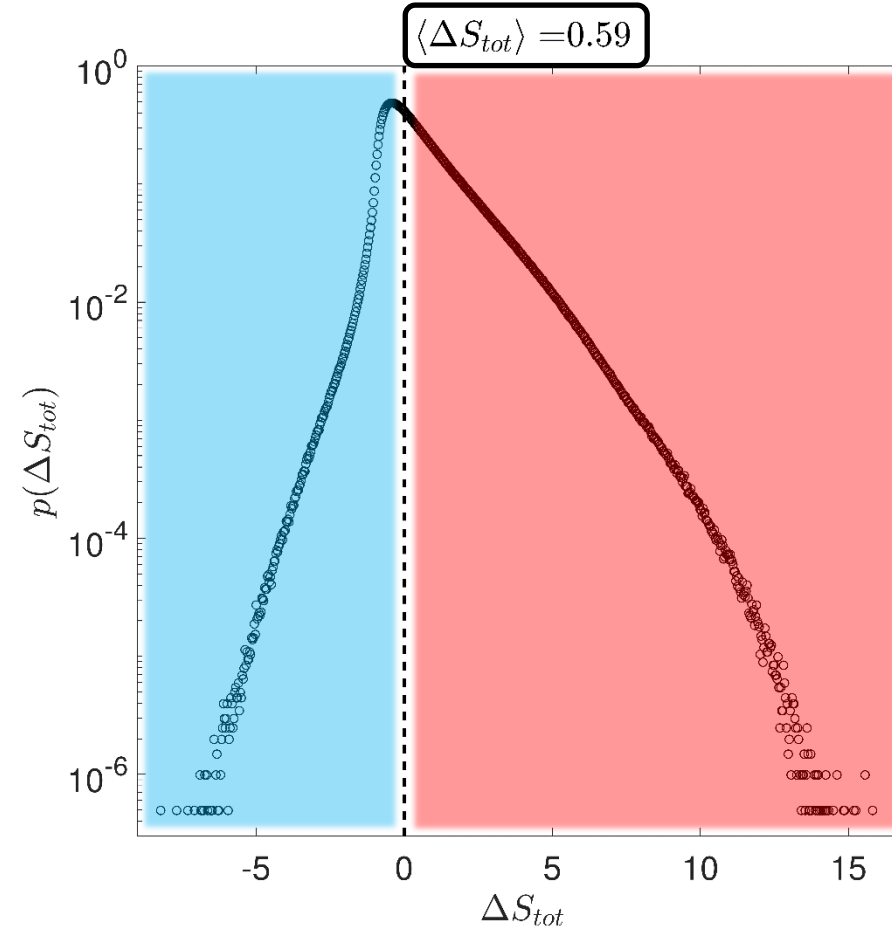
Verification of the measured FPE: **Validity of the IFT**



Entropy balance of cascade process

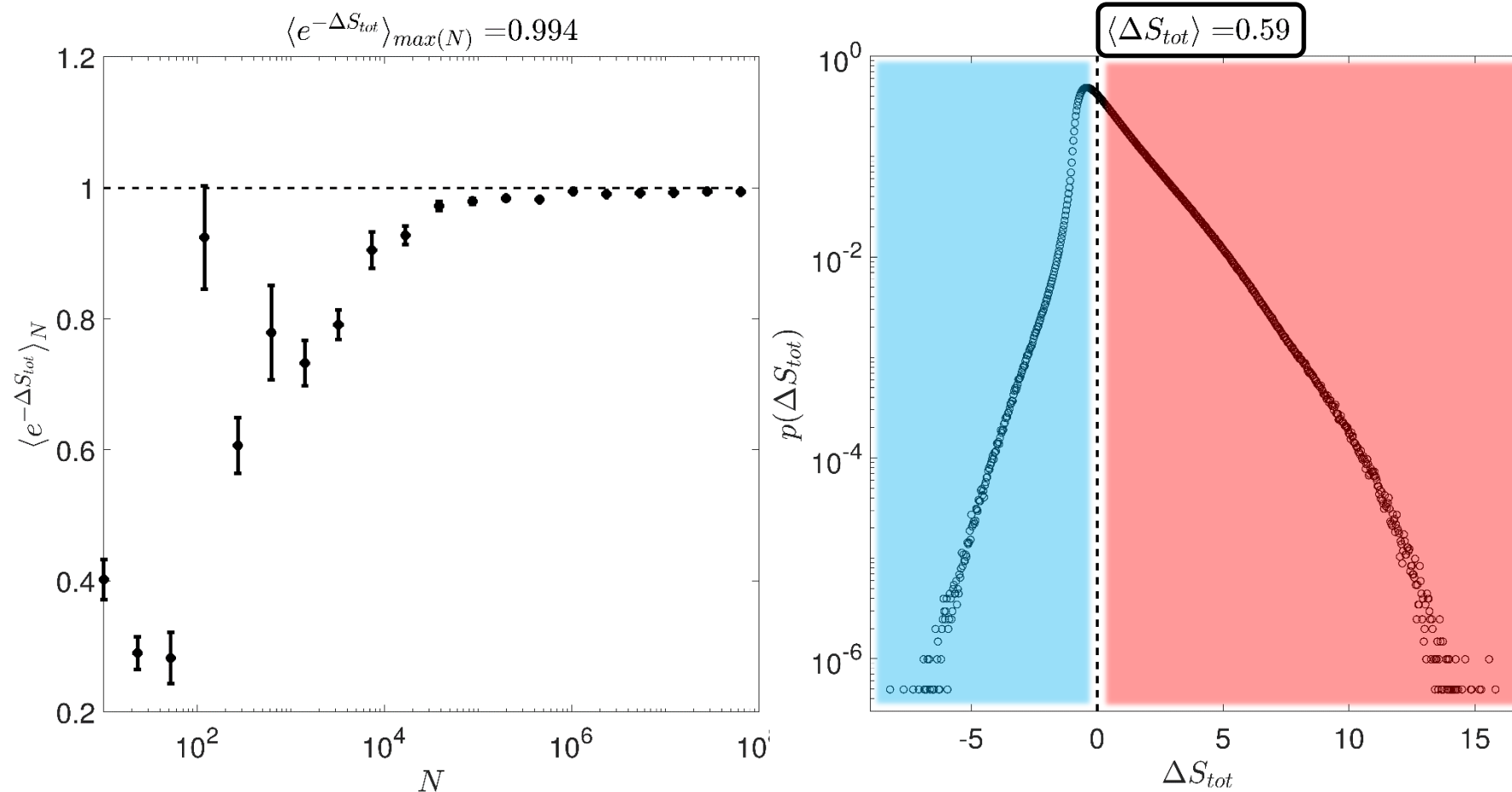
Verification of the measured FPE: **Validity of the IFT**

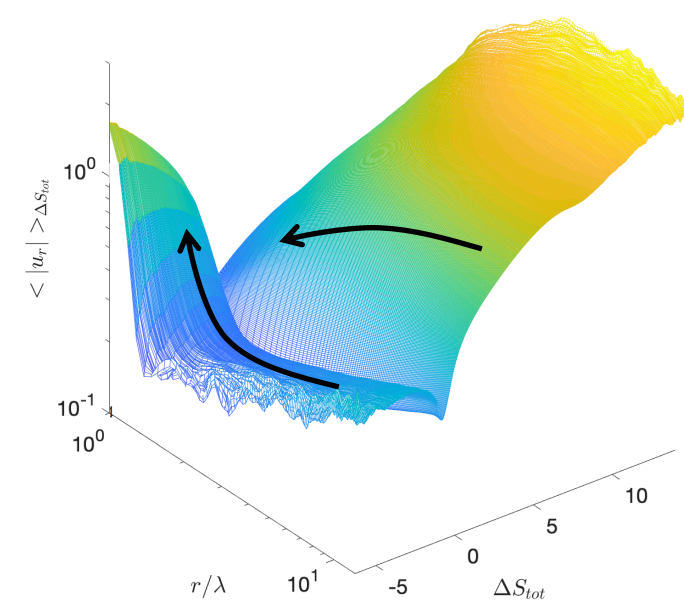
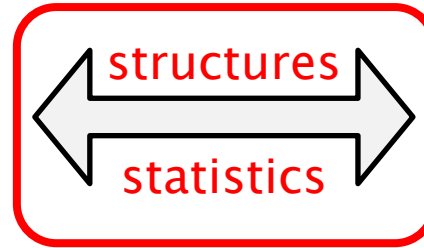
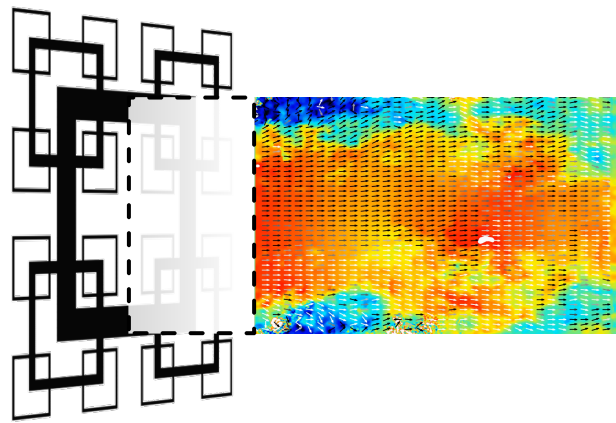
$$\langle e^{-\Delta S_{tot}} \rangle_{[u(\cdot)]} = 1$$



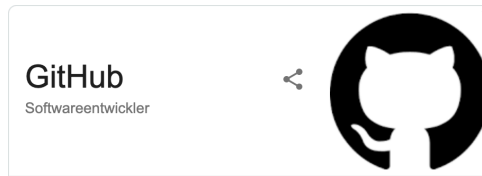
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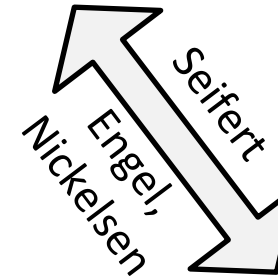


Open source MATLAB package



Fokker-Planck description

- Markov picture of turbulence
- Multi-point statistics

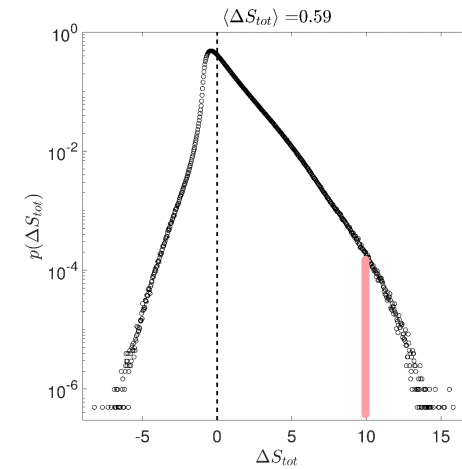
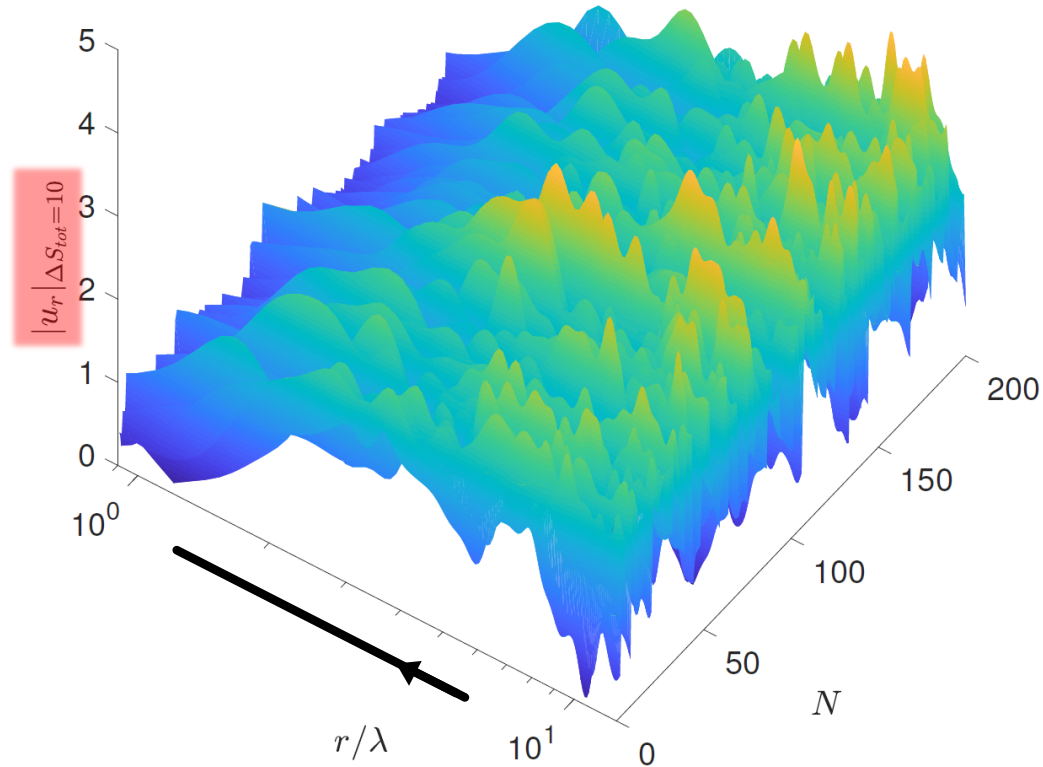


stochastic thermodynamics
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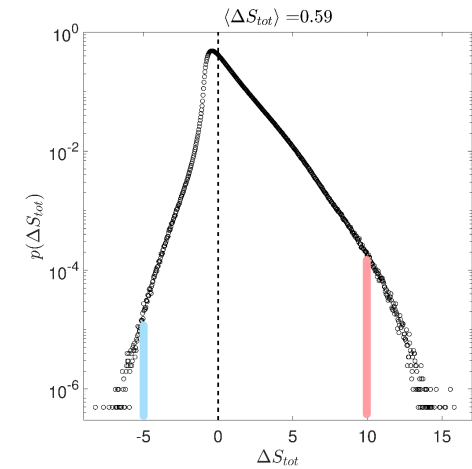
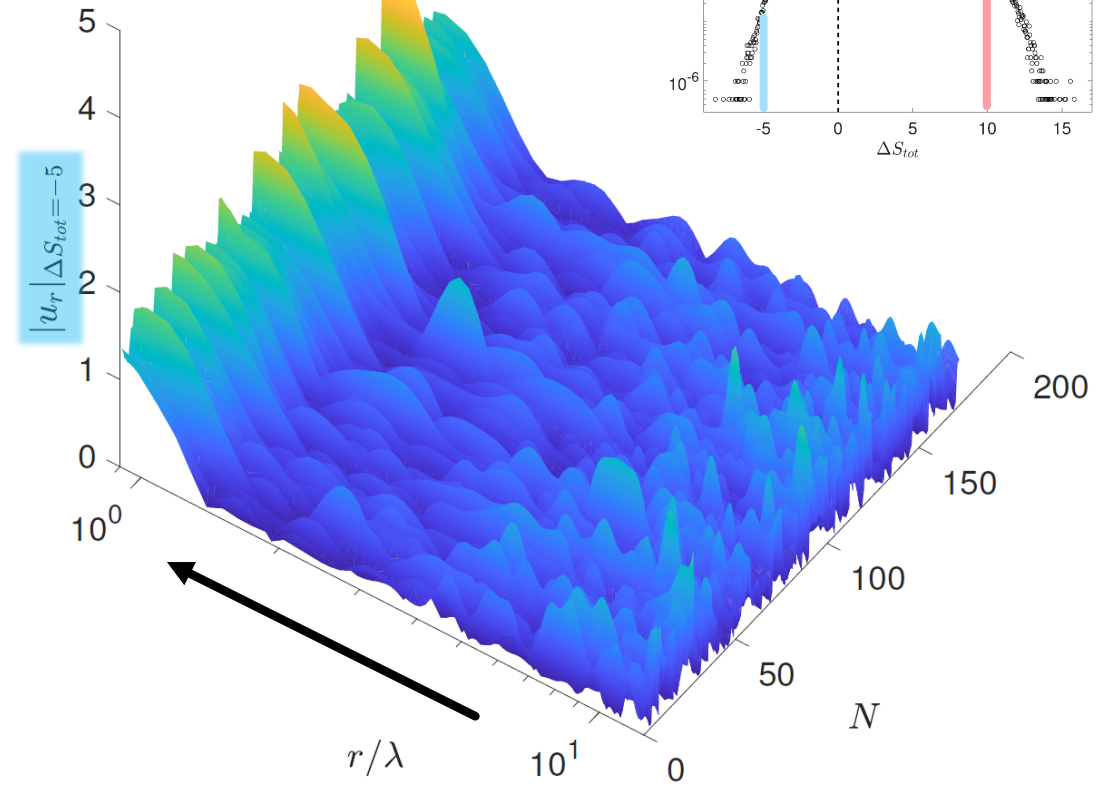
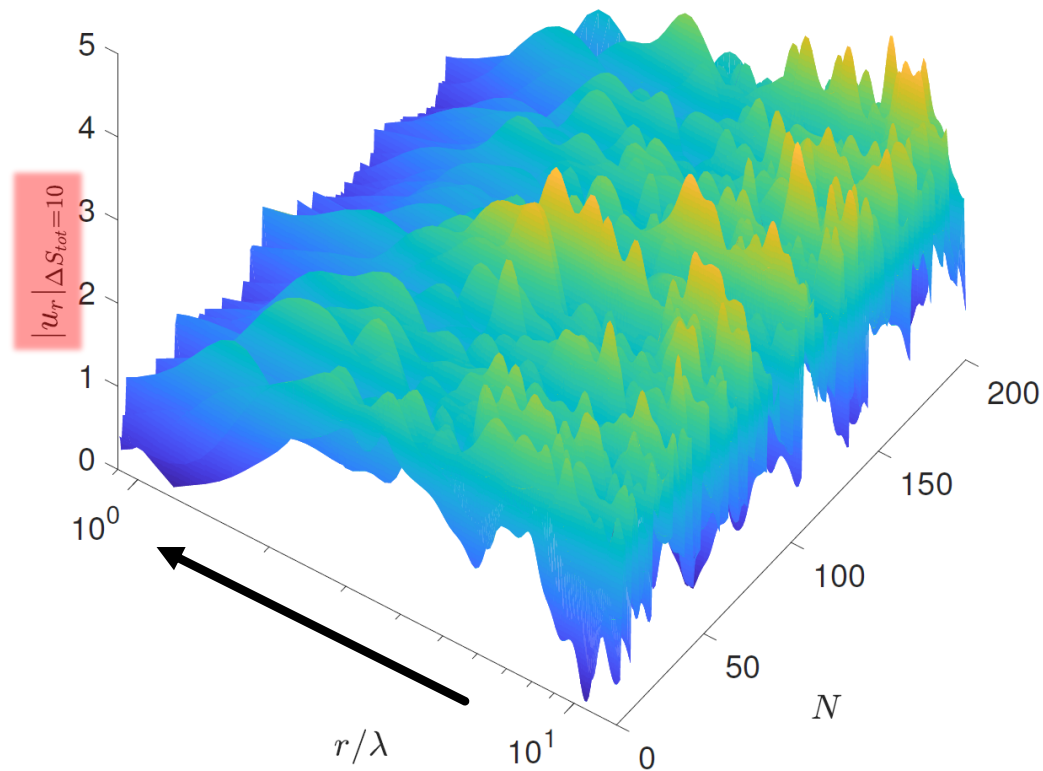
200 realization of turbulent cascade process

Entropy **producing** trajectories



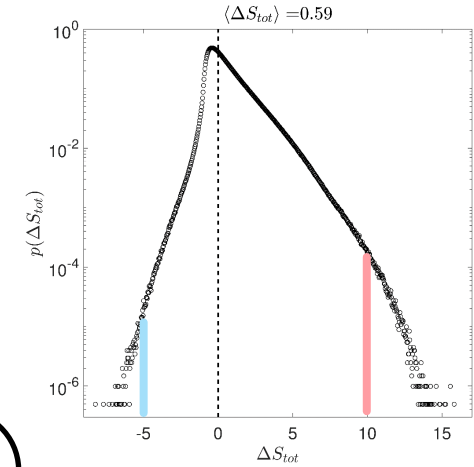
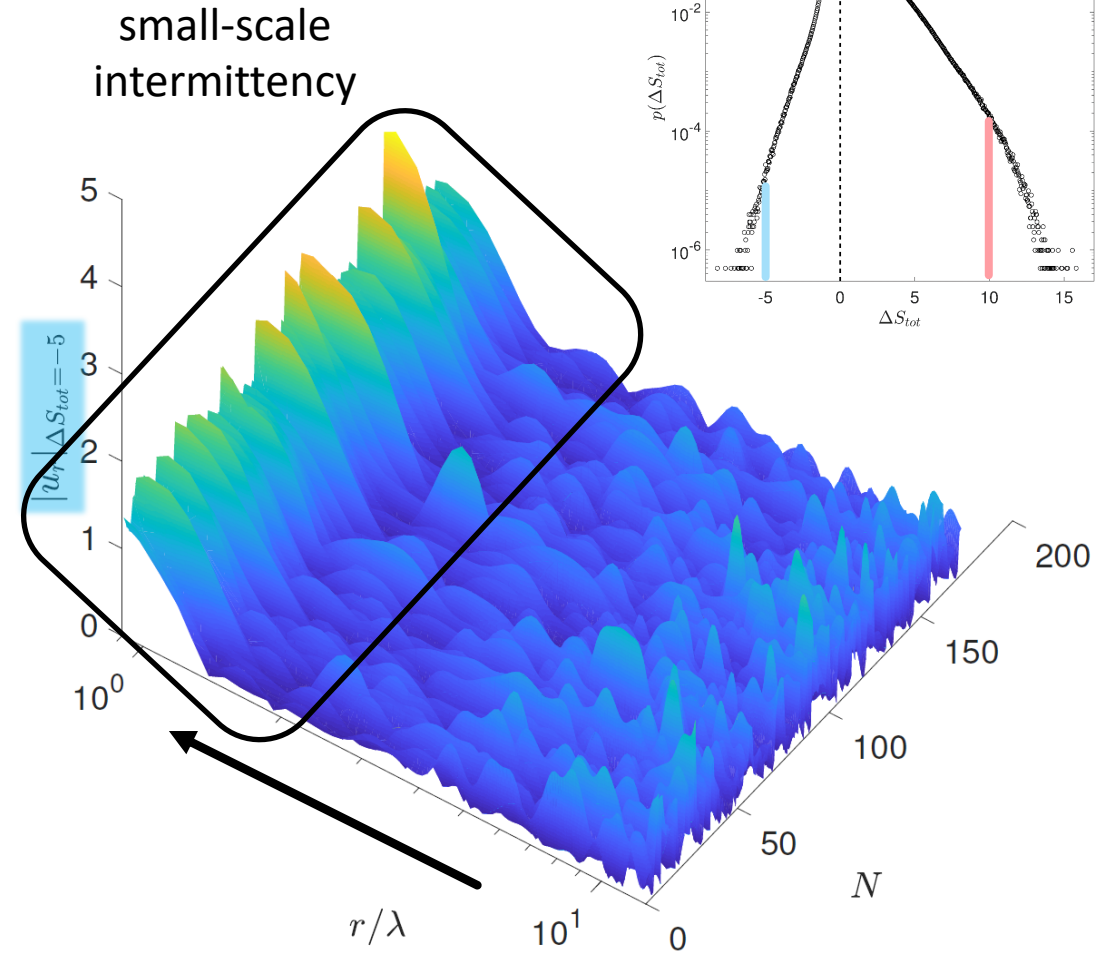
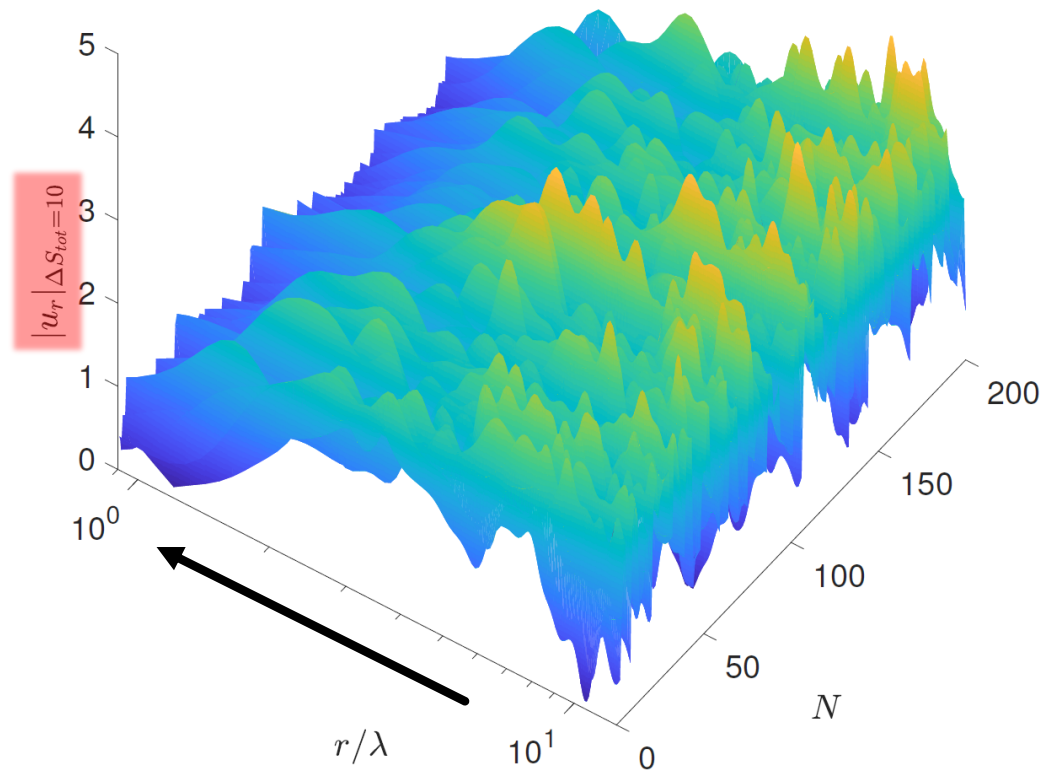
200 realization of turbulent cascade process

Entropy producing / consuming trajectories



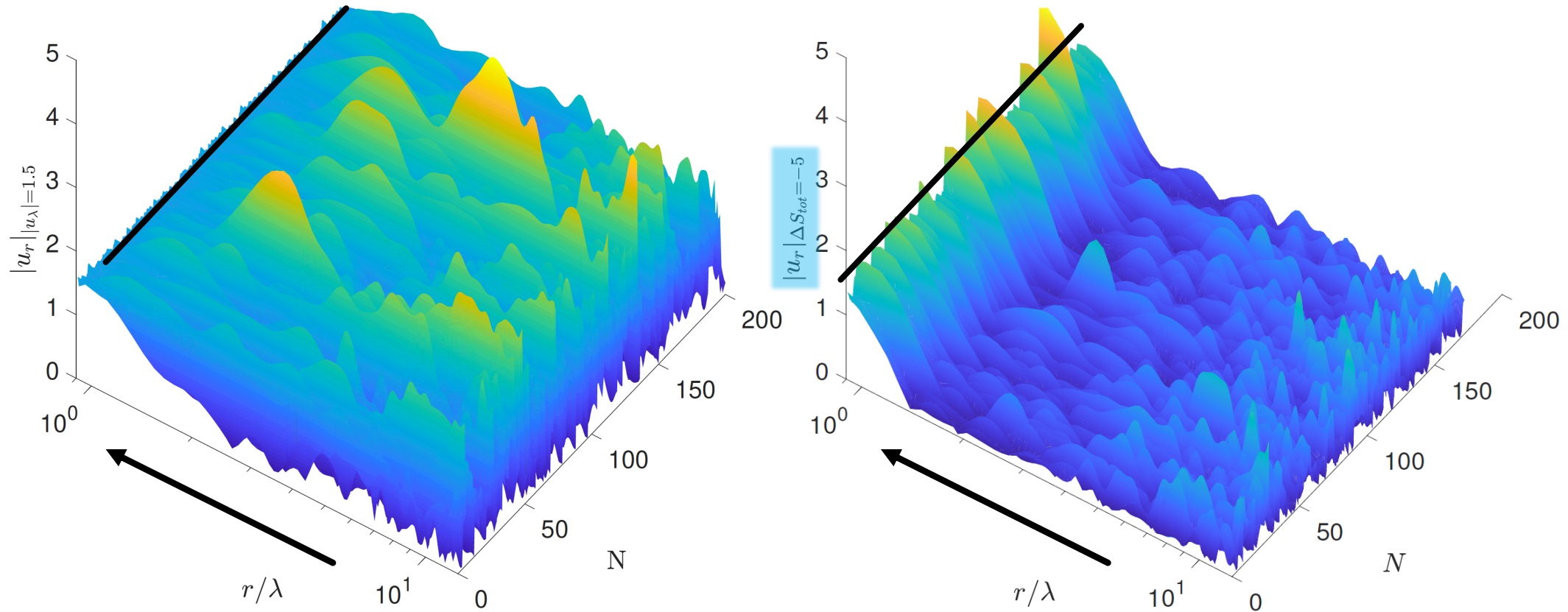
200 realization of turbulent cascade process

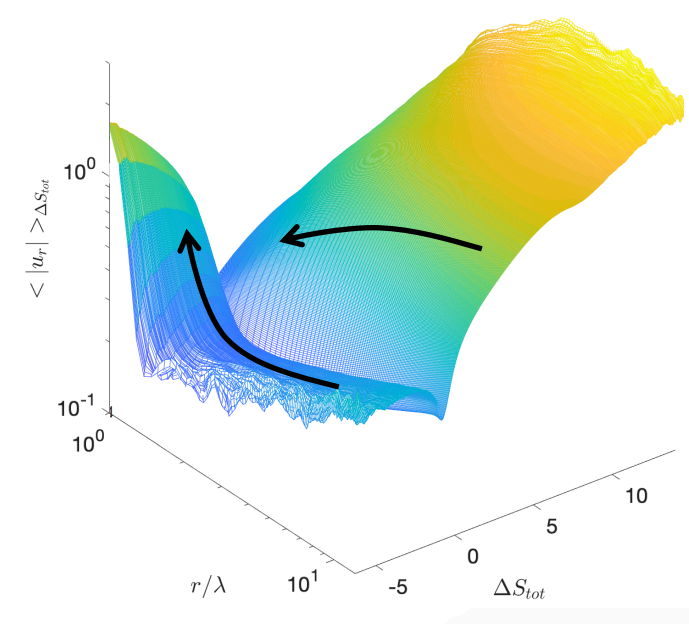
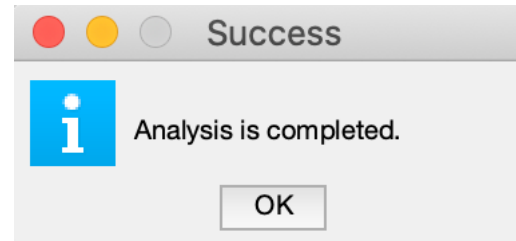
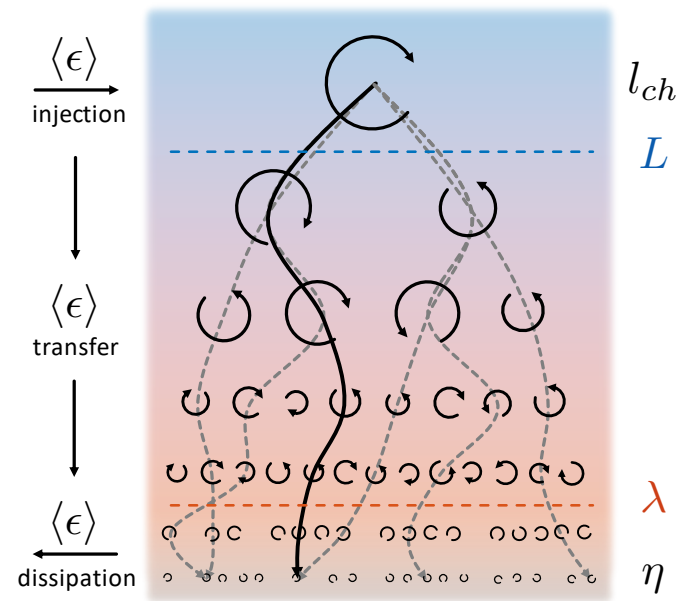
Entropy producing / consuming trajectories



200 realization of turbulent cascade process

“trivial” extreme trajectories at $|u_\lambda| = 1.5$





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 (Dated: August 7, 2020)

