

Hamiltonian Monte Carlo inversion of seismic reflection data in the acoustic approximation

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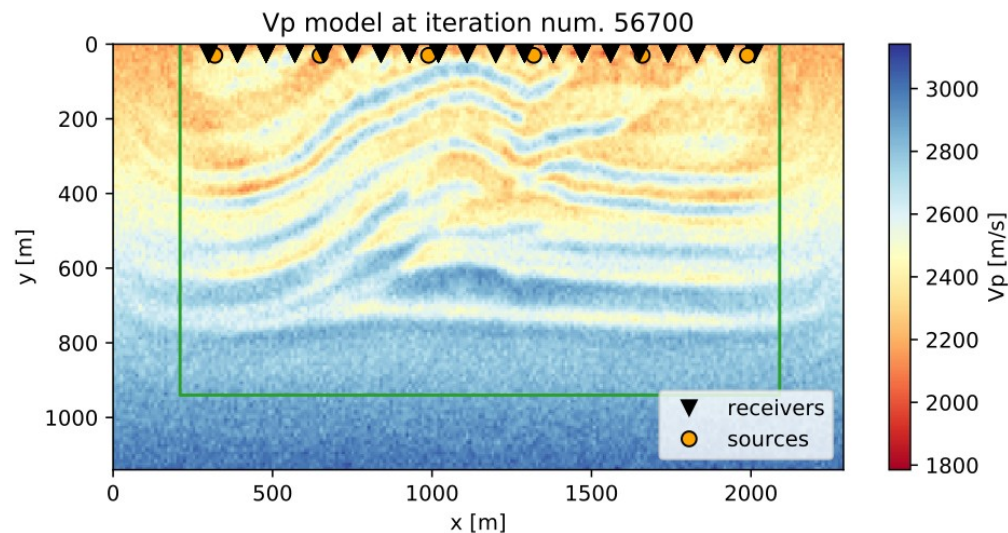
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Summary

We address the problem of inversion of seismic reflection data in the acoustic approximation in 2D using a probabilistic approach. Specifically, we employ a Hamiltonian Monte Carlo method (HMC) to sample solutions to the inverse problem, i.e., P-wave velocity models. HMC requires the computation of the gradient of the misfit functional, which is carried out by the efficient adjoint method.

In this preliminary work we developed the necessary code and initiated synthetic tests to find good tuning parameters. Because of its simplicity the described methodology can become a numerical laboratory to investigate probabilistic FWI and also be used in certain real cases in exploration geophysics.



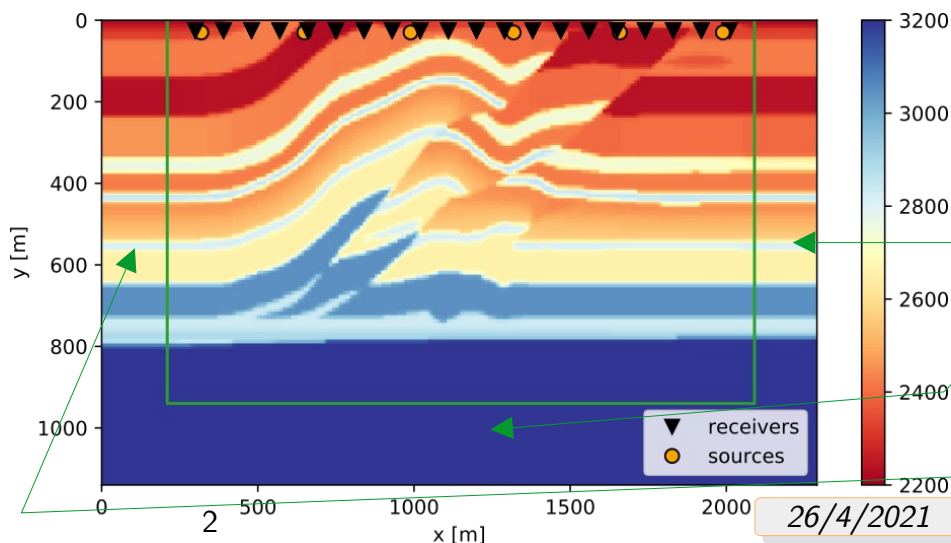
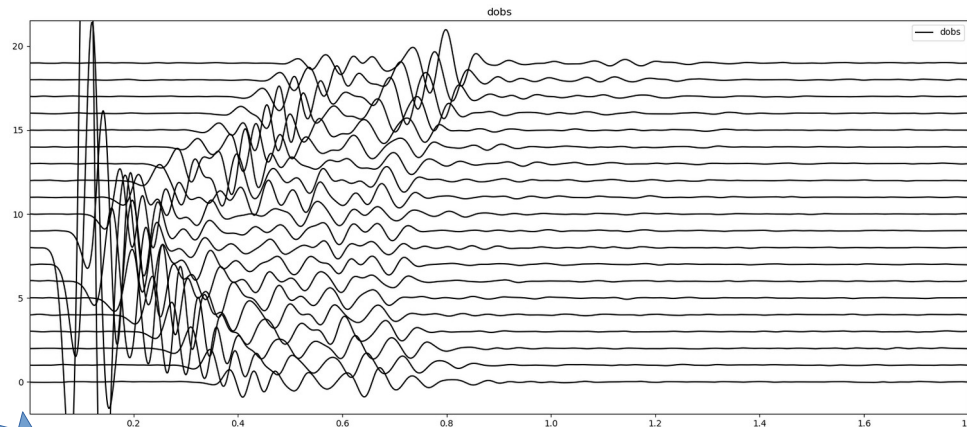
The forward problem

Seismic waves in the acoustic approximation

We consider the acoustic wave equation in 2D (constant density formulation):

$$\frac{1}{v^2} \frac{\partial^2 p}{\partial t^2} = \frac{\partial^2 p}{\partial x^2} + \frac{\partial^2 p}{\partial z^2} + s$$

where p is pressure, v the P-wave velocity and s the source term.



Forward code

Finite-difference method in 2D

Absorbing boundary conditions on left, right and bottom sides: perfectly matched layer (PML)

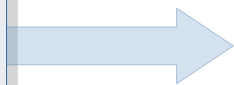
Written entirely in the **Julia** programming language, parallelised “by source”

The inverse problem

Probabilistic inverse approach

All the information is expressed in terms of probabilities.

Sampling the *posterior distribution*, to fully address nonlinear problems.



Collection of solutions (V_P)

Statistical analysis of V_P models
Uncertainty quantification



Hamiltonian Monte Carlo method

Hamiltonian dynamics as a proposal mechanism using auxiliary momentum variables

Samples the Hamiltonian $H = U(m) + K(m)$, where $U = -\log(\text{posterior})$ and K are the potential and kinetic energies, respectively.

Uses information from the **gradient** of U (misfit) with respect to P-wave velocity (the unknown)



Inverse code

Constrained Hamiltonian Monte Carlo sampler

Adjoint method for gradient computations

Written entirely in the **Julia** programming language, runs on a laptop



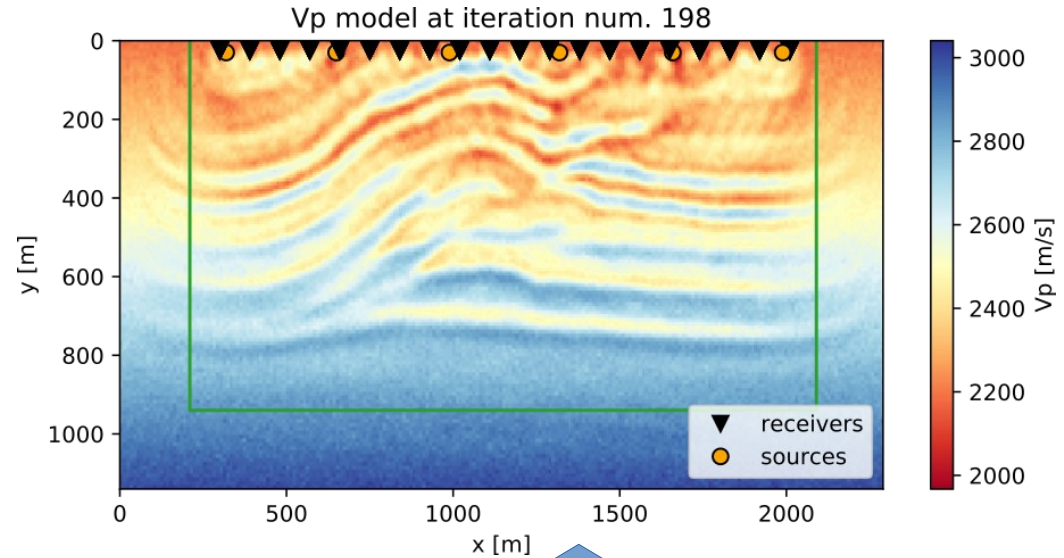
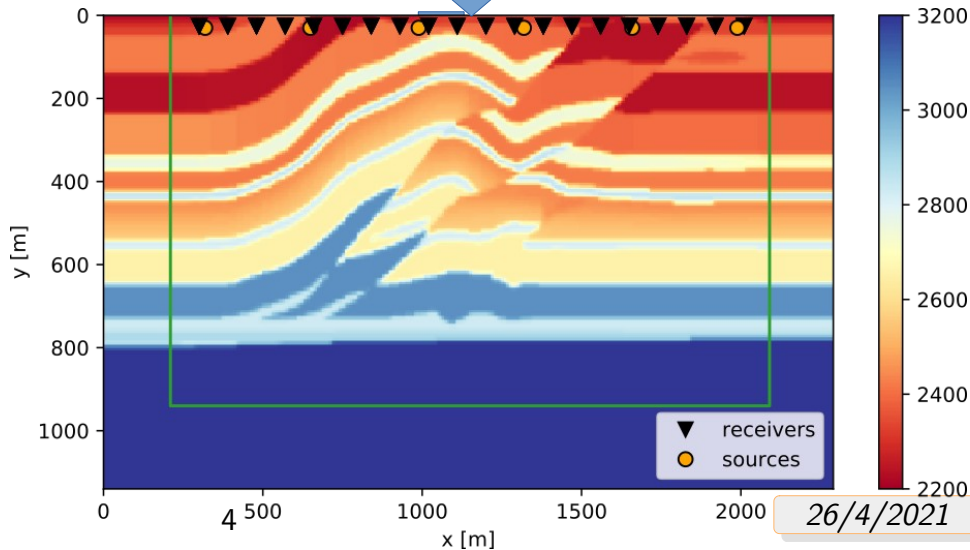
Current work

Synthetic tests

A modified version of the overthrust model

About 26000 model parameters

Starting model: a laterally homogenous model, V_P changing only with depth



Tuning of the algorithm

Fairly fast to find a good model

Still slow to explore once a good model is found: optimal tuning under study...

Conclusions

Hamiltonian Monte Carlo inversion in the acoustic approximation:

- A numerical lab to explore the behaviour of HMC for FWI
- Simple to use, runs on a laptop, Julia language
- Currently under testing on synthetic data sets
- Potential applications in exploration geophysics

HMCTomo package

The code utilised in this study is part of **HMCTomo**, a framework for HMC inversion of various geophysical data written in Julia and Python programming languages.

See: Gebraad, L., Zunino, A., Fichtner, A., and Mosegaard, K.: HMCTomo: A framework for Hamiltonian Monte Carlo sampling of Bayesian geophysical inverse Problems, EGU General Assembly 2020, EGU2020-9394, 2020

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