Spectral evolution of Alfvénic turbulence

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Alfvénic turbulence denotes a regime of MHD turbulence in which Alfvén waves propagating in a given direction along the mean field are dominant, as commonly found in polar regions/coronal holes/fast solar wind.

We show here using direct MHD simulations that the wind expansion changes strongly the turbulent evolution, not only the turbulence dissipation, but also probably the nature of the turbulent cascade and the role of the mean magnetic field.

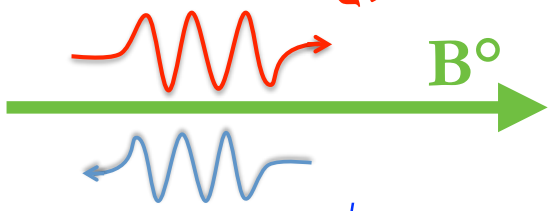
PLAN

Introduction
Homogeneous Alfvénic turbulence
Expanding Alfvénic turbulence
Conclusion

Definitions and aim of this work

Elsasser variables $z_\pm$

$z_+ = u - \delta B/\sqrt{\varrho}$, $E_+ = z_+^2/2$



$z_- = u + \delta B/\sqrt{\varrho}$, $E_- = z_-^2/2$

cross helicity measures $E\pm$ imbalance:

$\sigma_c = (E_+ - E_-)/(E_+ + E_-)$

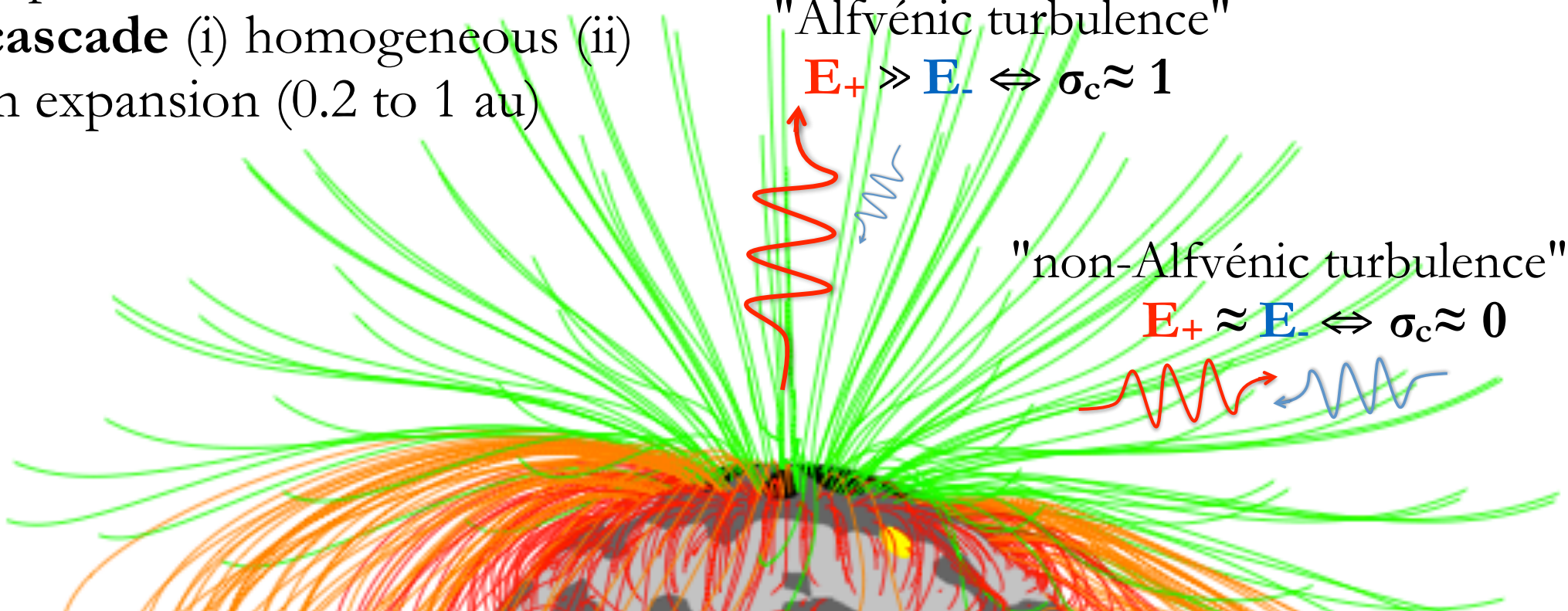
Aim: use MHD simulations to explore the **Alfvénic turbulent cascade** (i) homogeneous (ii) in expansion (0.2 to 1 au)

"Alfvénic turbulence"

$E_+ \gg E_- \Leftrightarrow \sigma_c \approx 1$

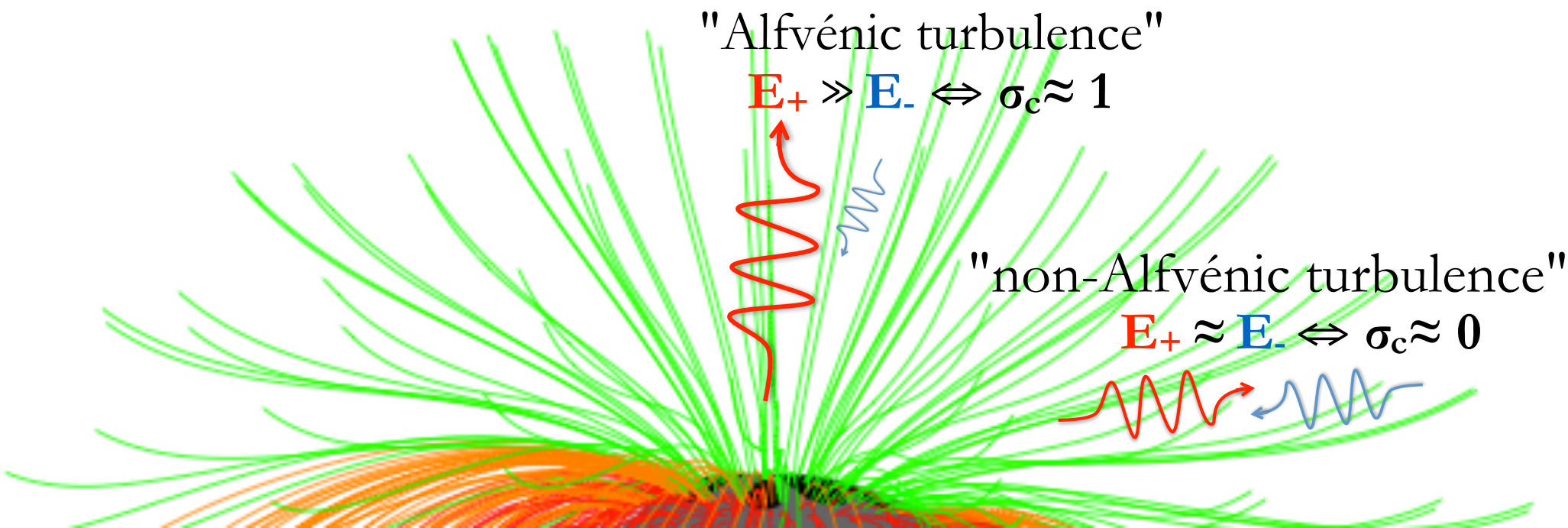
"non-Alfvénic turbulence"

$E_+ \approx E_- \Leftrightarrow \sigma_c \approx 0$



Why compare homogeneous with expanding turbulence?

…because expanding MHD is specific of the accelerated solar wind, while homogeneous MHD is a priori simpler, easier to study, and perhaps able to bring light on turbulence closer to the Sun.
Moreover, some believe there is no big difference, in spite of evidence (*Dong et al 2014, Verdini Grappin 2015, 2016*). Our results also points to large differences for Alfvénic turbulence.



Homogeneous simulations

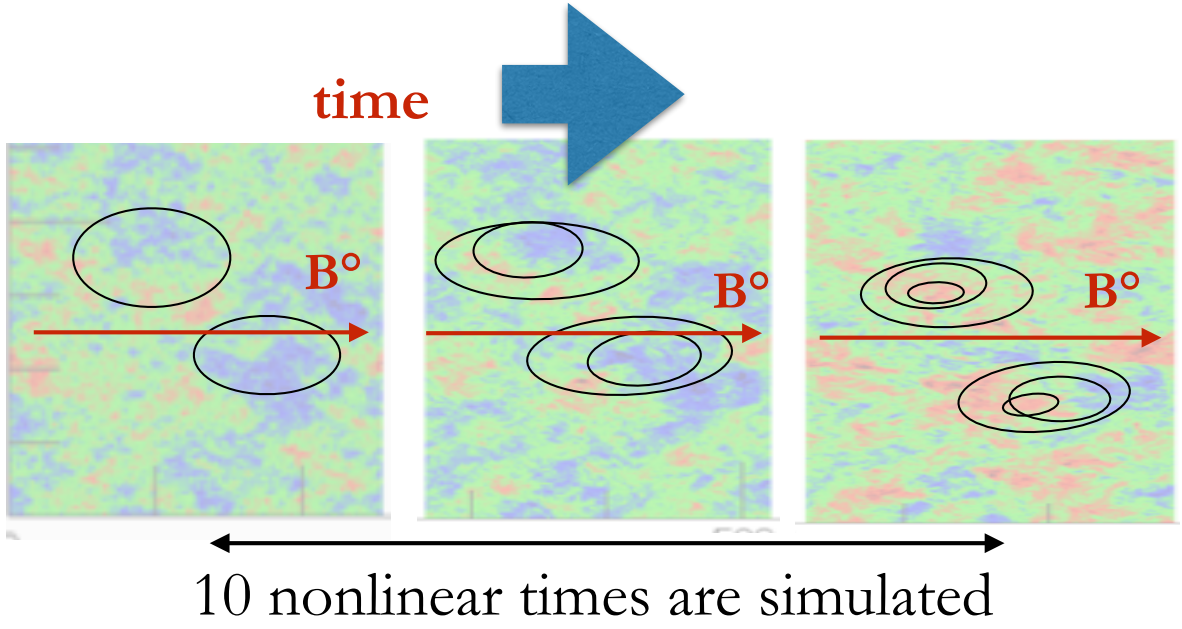
3D simulations: compressible MHD equations with low Mach number ($M°=0.12$)

Nonlinear parameter (Alfvén/nonlinear time):

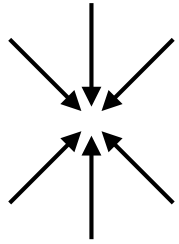
$\chi = t_A/t_{NL} = (k° u_{rms}°)/(k°_{//}(B°/\sqrt{\rho°}) \leq 1$

eddies ***collapse in directions $\perp$ to mean field $\vec{B}°$***

eddies ***expand along $\vec{B}°$***



Nonlinear coupling + wave propagation (Alfvén waves mainly) → direct cascade



Expanding simulations

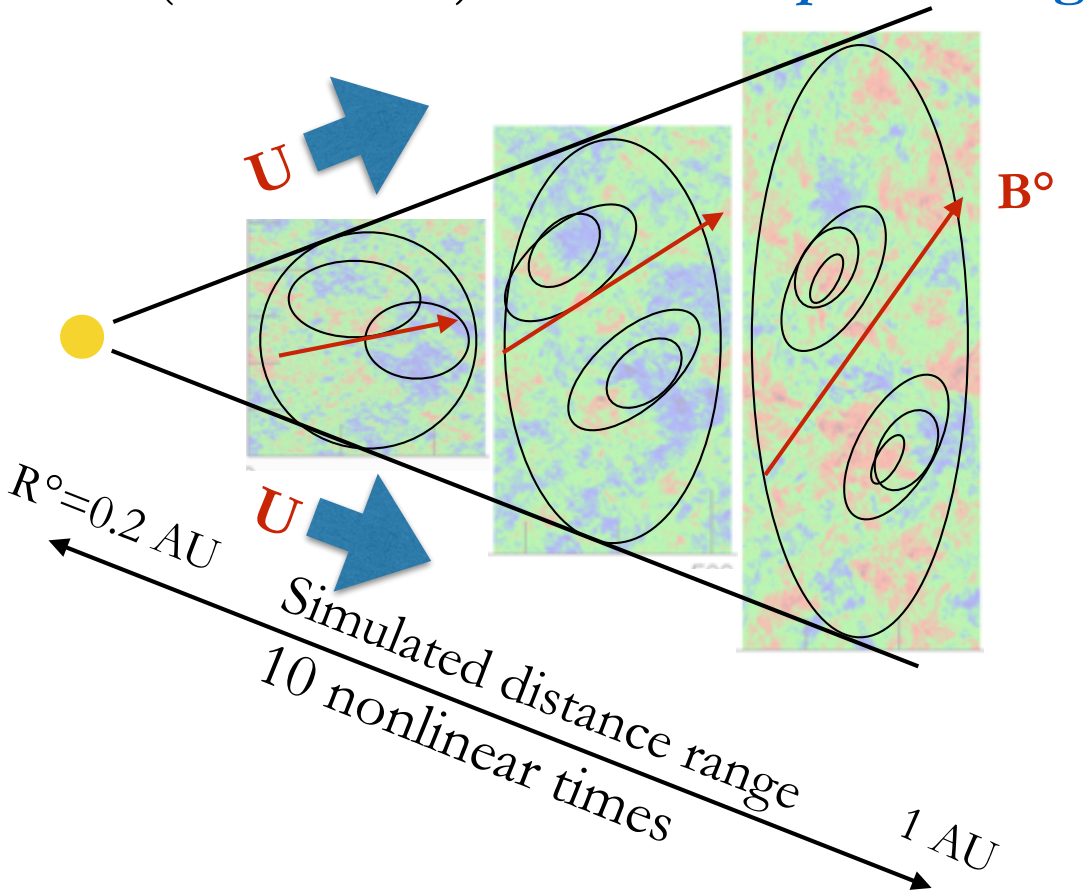
3D MHD +coordinates comobile with expansion (EBM, *Grappin Mangeney Velli 1993*)

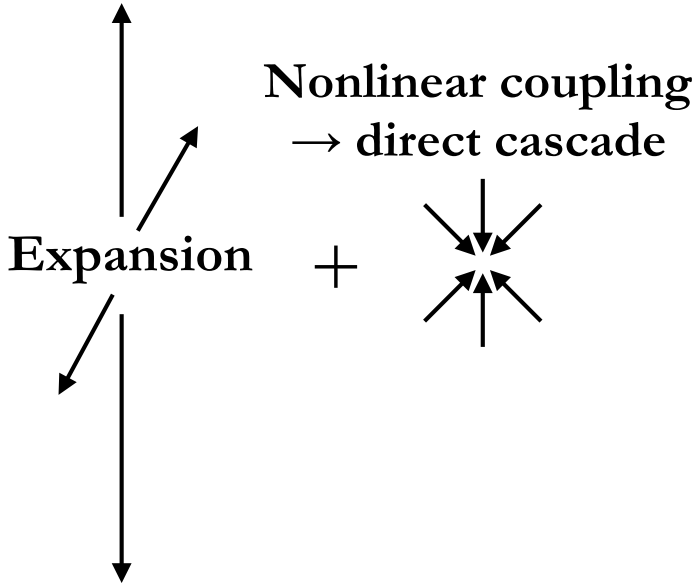
Expansion parameter:

$\varepsilon^\circ = (U/R^\circ)/(k^\circ u^\circ_{rms}) = 0.4$

$\varepsilon \approx 1$ (large scales) → eddies ***expand in directions ⊥ to the radial***

$\varepsilon \ll 1$ (small scales) → eddies ***expand along B° and collapse ⊥ to B°***





A first difference: imbalance evolution

Two inviscid invariants: $E_\pm = z_\pm^2$ → Two cascades of E+ and E-

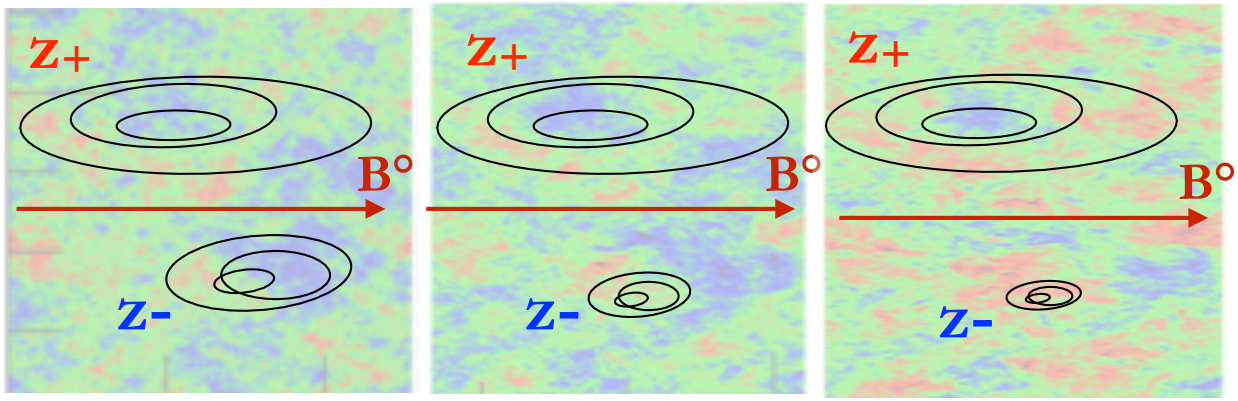
$\mathbf{z_+ \gg z-}$ → z_+ decay time ≈ $1/(kz-) \gg$ z- decay time ≈ $1/(kz_+)$

→ $z_+/z-$ imbalance ***should*** **increase with time** (*Dobrowolny Mangeney Veltri 1980*)

1) no expansion

YES, imbalance **increases**

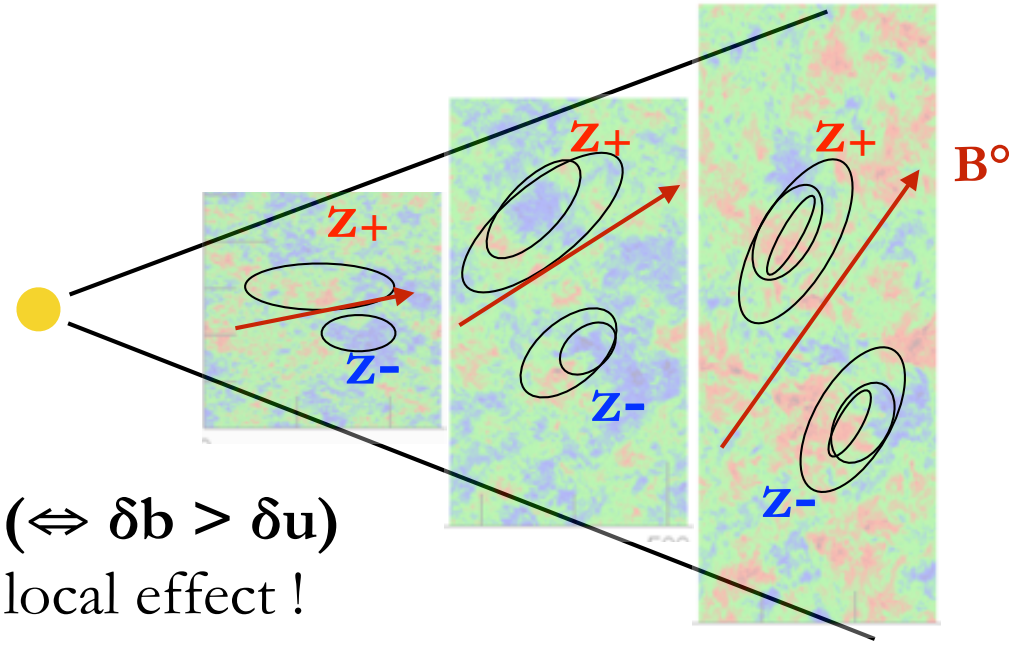
- edqnm closure (*Grappin Pouquet Léorat Frisch 1982*)
- 2D MHD simulations *Ting Matthaeus Montgomery 1986*)



2) with expansion

NO, imbalance **decreases**

-Helios observations (*Roberts et al 1987, Bavassano et al 2000*)



Probable explanation:

expansion → **large scale injection of z- (⇔ δb > δu)**

NB also shear flow (*Roberts et al 1992*) but local effect !

Some possible cascades

Energy flux $Q_\pm$ and scaling

1) fast cascades: $t_A > t_{NL\pm}$

$t^*_\pm = t_{NL\pm} = 1/kz_\mp$

$Q_\pm = E_\pm/t^*_\pm = kz_\mp E_\pm = cstt$

$\rightarrow$ m+=m-=-5/3 **(1)**

(*Lithwhick and Goldreich, 2003*)

2) slow cascades: $t_A < t_{NL\pm}$

$t^*_\pm = t_{NL\pm}\,(t_{NL\pm}/t_A) = (k_{//}B°)/(kz_\mp)^2$

$Q_\pm = k^2E_+E-/(k_{//}B°) = cstt$

+Isotropic cascade ($k_{//} \propto k_\perp$) :

$\rightarrow$Pinning relation **$m_+ + m- = -3$** **(2)**

(*Grappin Pouquet Léorat, 1983*)

3) mixed cascades: $t_{NL-} < t_A < t_{NL+}$

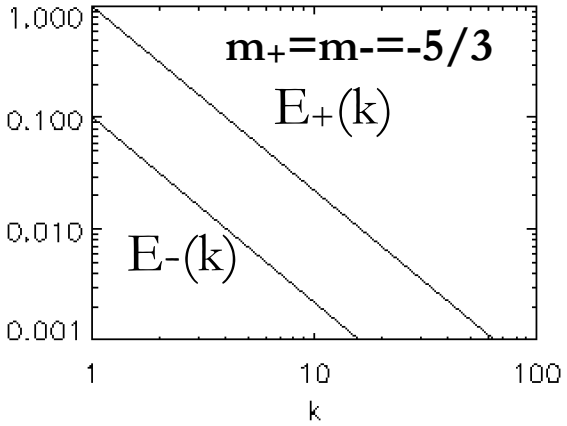
Verdini Grappin Pinto Velli (2012)

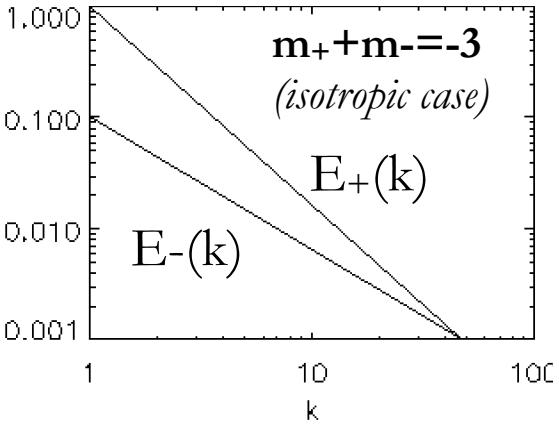
$Q_+ = k^2E_+E-/(k_{//}B°) = cst \rightarrow m_+ + m- = -3$

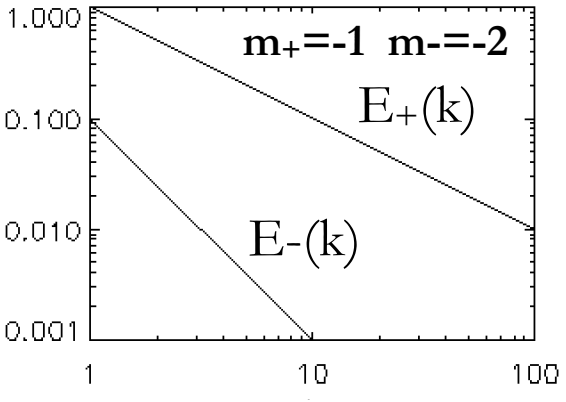
$Q- = kz_+ E- = cst \rightarrow m_+ + 2m- = -5$

$\rightarrow$ $m_+ = -1$, $m- = -2$ **(3)**

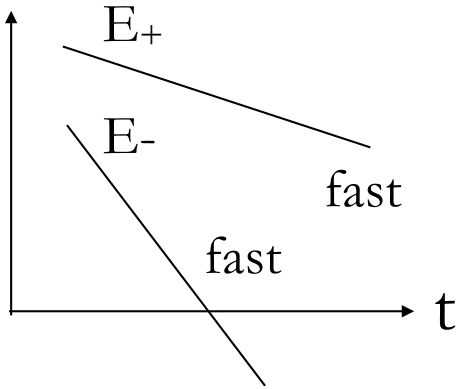
scaling

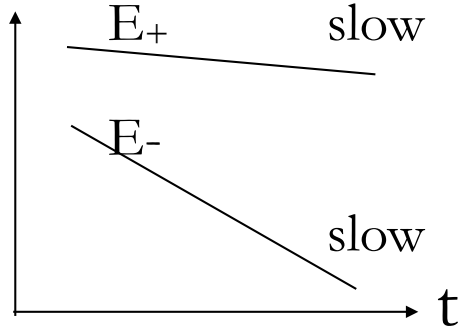


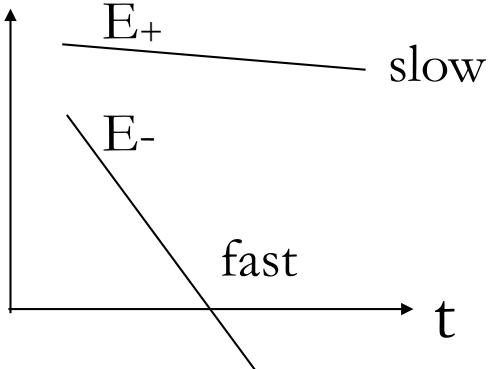




fast/slow decay





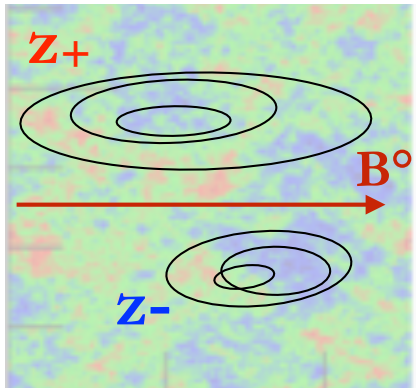


Parameters

Determine how spectral properties (and possibly cascade regimes) change when:

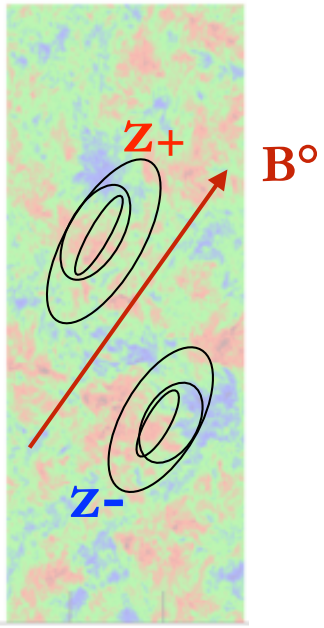
(i) **no expansion**
spectral index **m°=-1,-5/3**
cross helicity **σ_c°=0.5, 0.8**
mean field **B_x°=1,2,3,5**
($\vec{B}^\circ$ aligned with Ox)

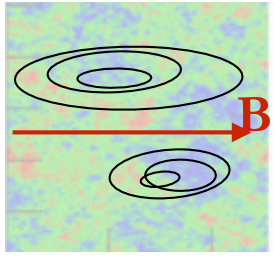
…varying
B°, σ_c°, m°



(ii) **with expansion**
at 0.2 au (t=0):
expansion
$\varepsilon=(U/R^\circ)/(k^\circ u^\circ_{rms})=0.4$
spectral index **m°=-1,-5/3**
cross helicity **σ_c°=0, 0.83,1**
mean field **$\vec{B}^\circ$=[2,0.4,0]**
(rotates as distance increases)

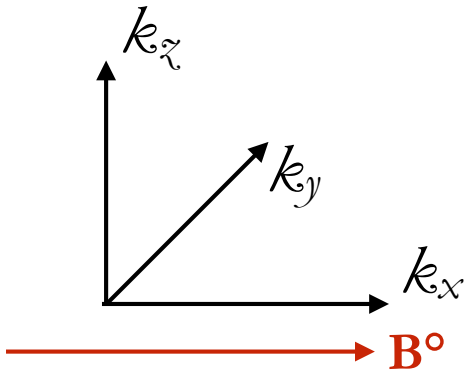
…varying σ_c°, m°





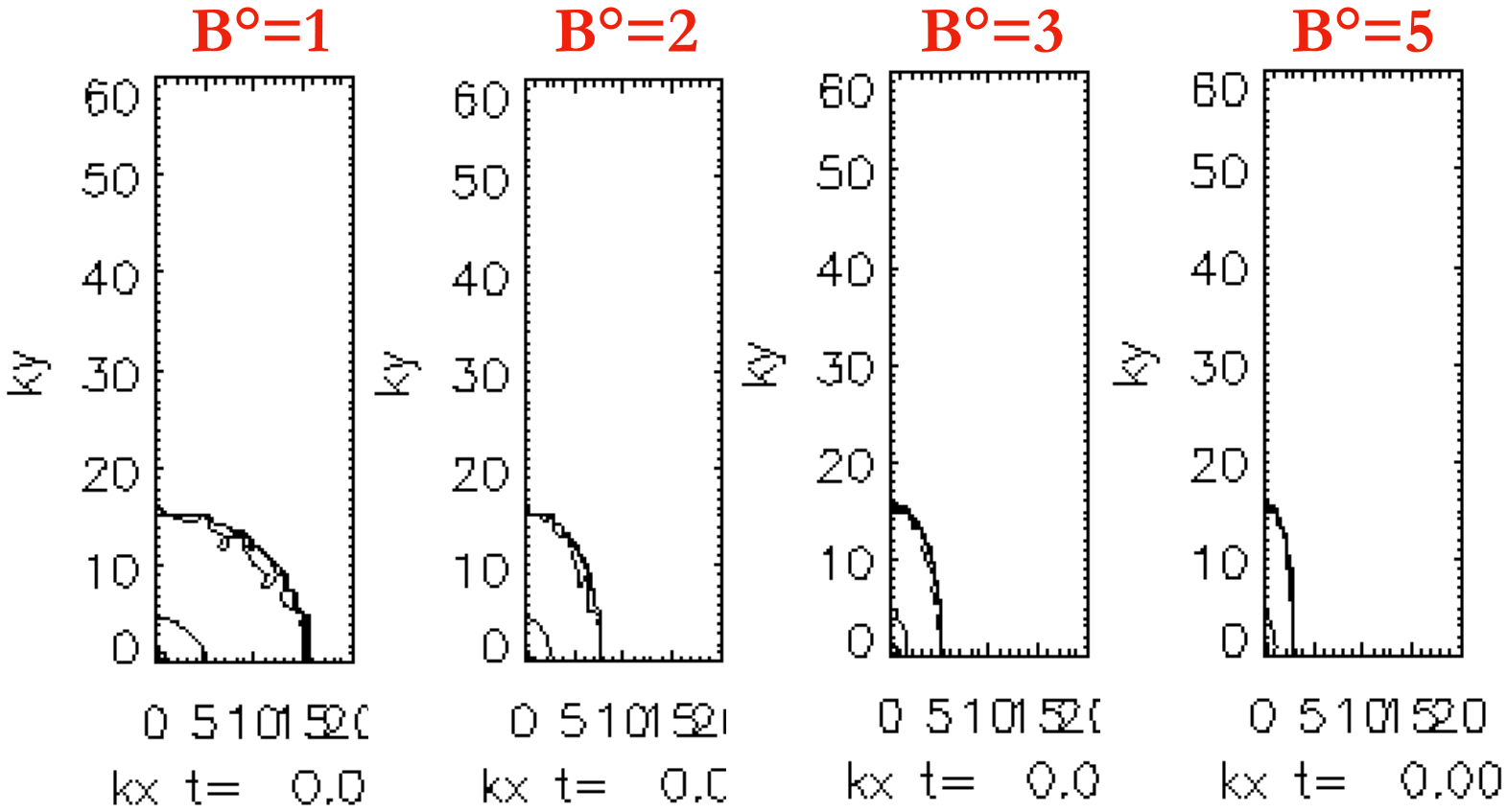
Part 1: homogeneous turbulence

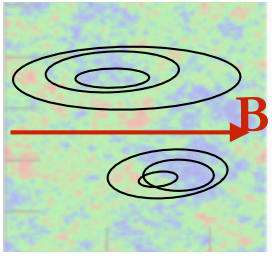
Initial spectral anisotropy



t=0

Total energy spectra $E_T^{3D}(k_x,k_y,k_z=0)$

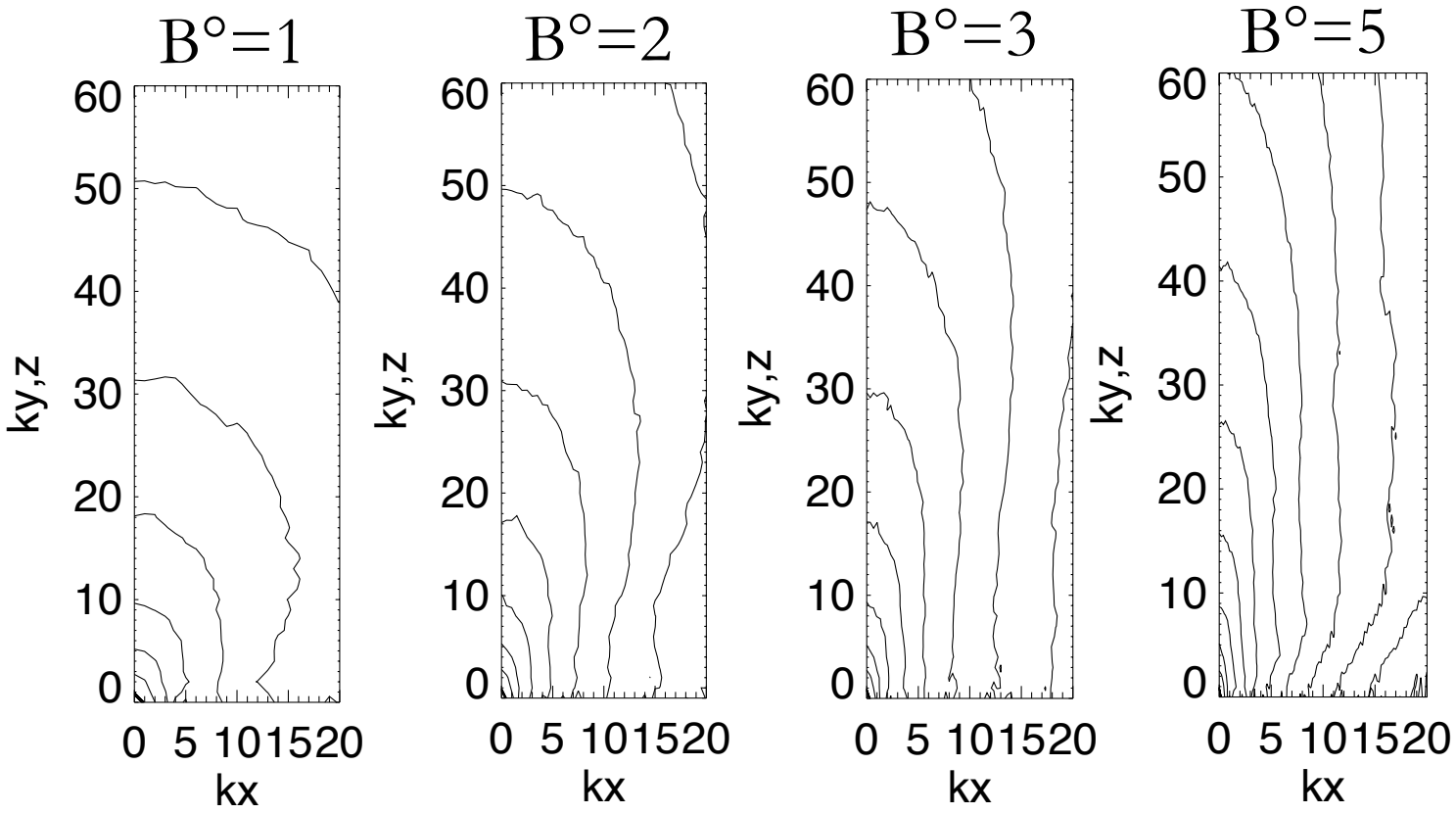




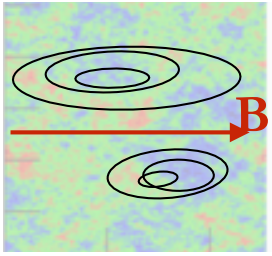
B° and spectral anisotropy $m°=-5/3$ $\sigma_c°=0.8$

t=10

3D spectra E_+^{3D} after azimutal averaging in planes k_y,k_z



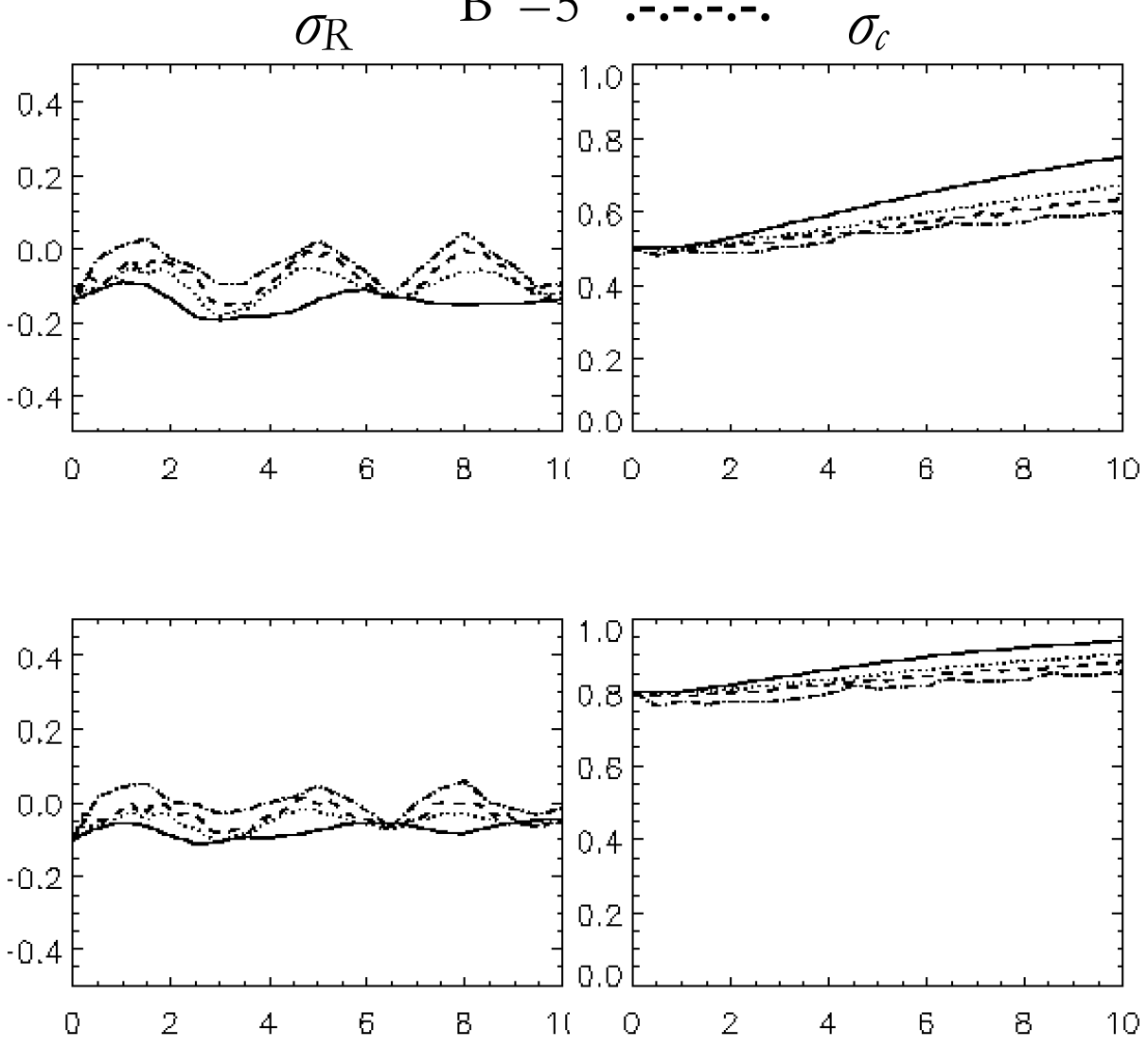
Spectral anisotropy of E_+ grows with B° (similar for E-)
This suggests a strong perpendicular cascade with $k^{-5/3}$ scaling …



Imbalance (σ_c) and residual energy (σ_R) vs B°

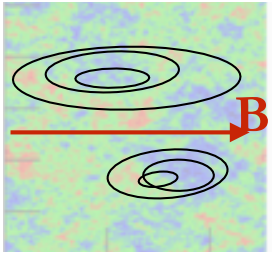
B°=1 ——
B°=2 ·····
B°=3 - - - - -
B°=5 -·-·-·-·-

m°=-5/3



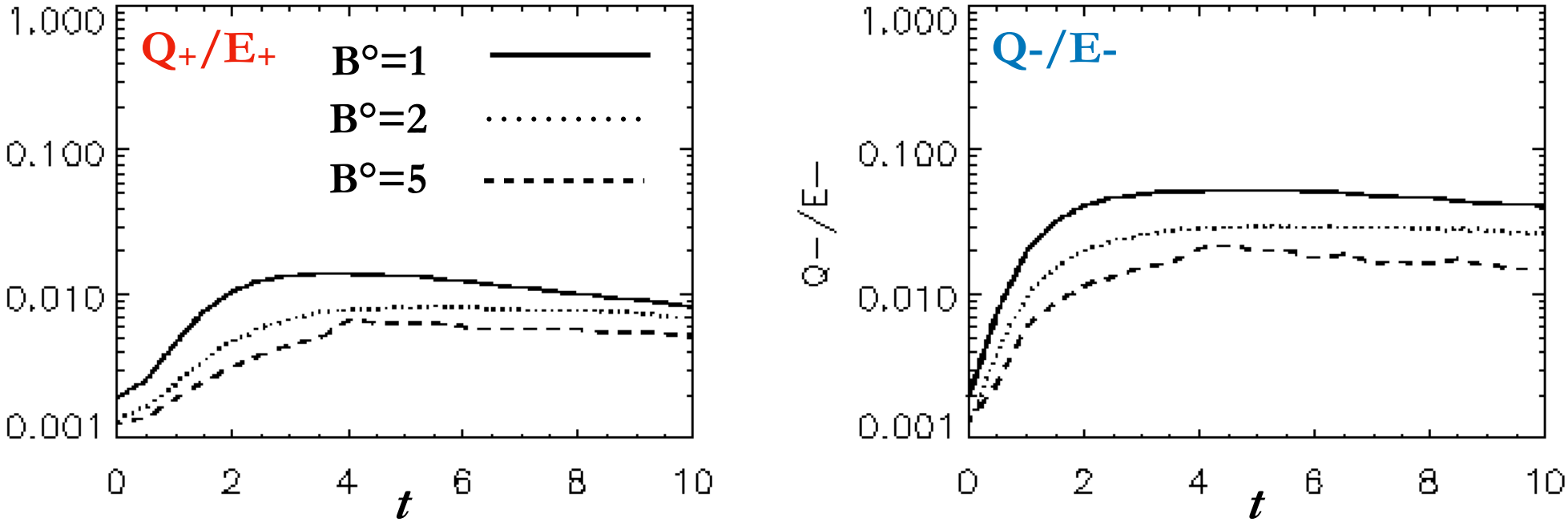
A permanent magnetic excess is visible when B°=1, while large oscillations of residual energy $\sigma_R=(u^2-b^2)/(u^2+b^2)$ around zero are visible when B°=5, revealing the growing importance of large scale Alfvén waves with large B°.

The fastest growth of cross helicity $\sigma_c=(E_+-E-)/(E_++E-)$, mainly due to rapid turbulent decay of E-, is observed when B° is small: increasing B° indeed slows down turbulent dissipation as confirmed in next slide.



$E_{\pm}$ log decay rates vs B° (**m°=-5/3 σ_c°=0.8**)

Log decay rates $Q_{\pm}=-dE_{\pm}/dt$

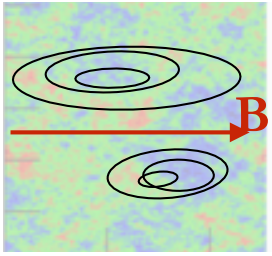


(1) **Q-/E- ≫ Q+/E+ ⇒** $E_+/E-$ (and **σ_c**) **grows with time** (*Dobrowolny Mangeney Veltri 1980)*

(2) **Decay time $\tau^*_{\pm}$** ($=Q_{\pm}/E$) **grows with 1/B°** (see *Bandyopadhyay et al* (2018) for **σ_c°=0**)

→ signature of **Alfvén waves**

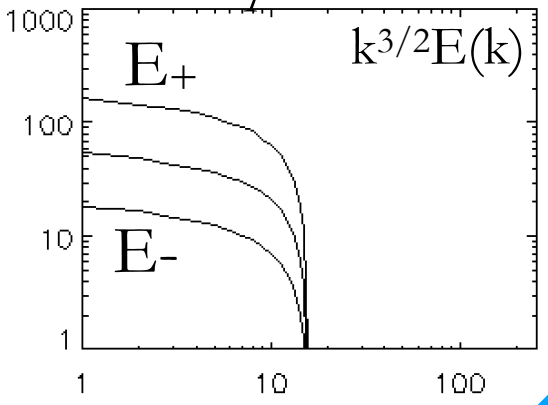
→ **weak cascade ?** (*see discussion*)



Scaling vs B° (m°=-5/3, σ_c°=0.8)

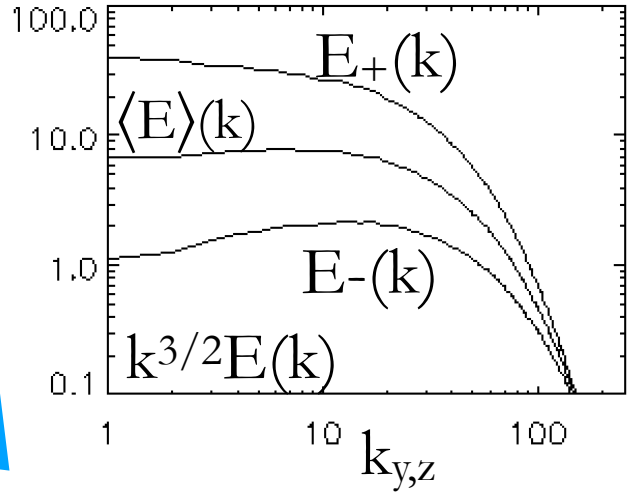
t=0

Initial spectra ∝ $k^{-5/3}$

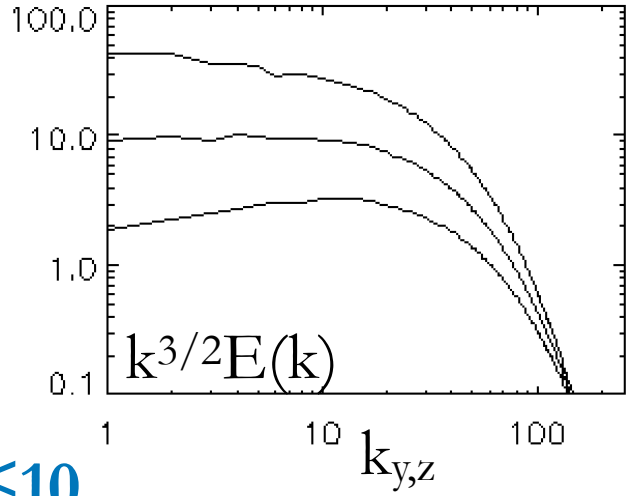


1D ⊥ spectra E+, E- and $\langle E\rangle=(E_+E_-)^{1/2}$

B°=1



B°=2



8≤t≤10

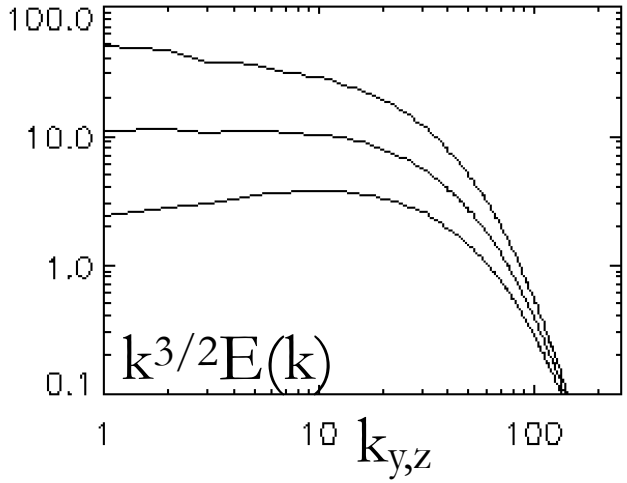
1≤B_x°≤3 → mean spectrum $\langle E\rangle\approx k^{-3/2}$ ⇔ **m_++m-=-3**

(see pinning relation slide 8)

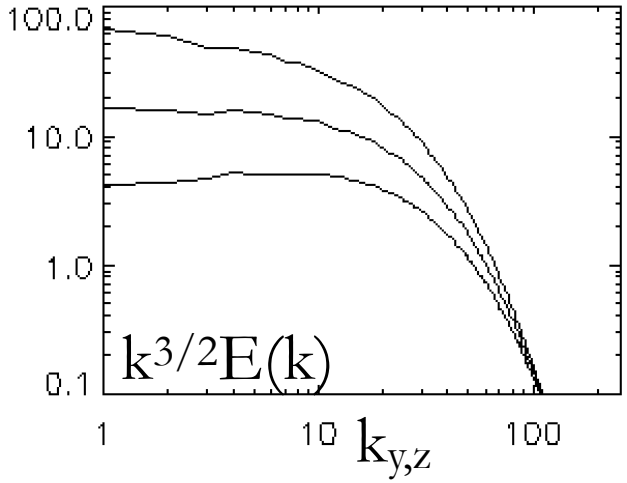
B_x°=5 → The pinning relation becomes **m_++m-≈-10/3** for

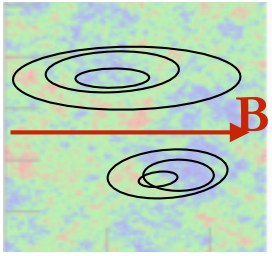
→ both relations suggest weak cascade, *see slide 8*

B°=3



B°=5

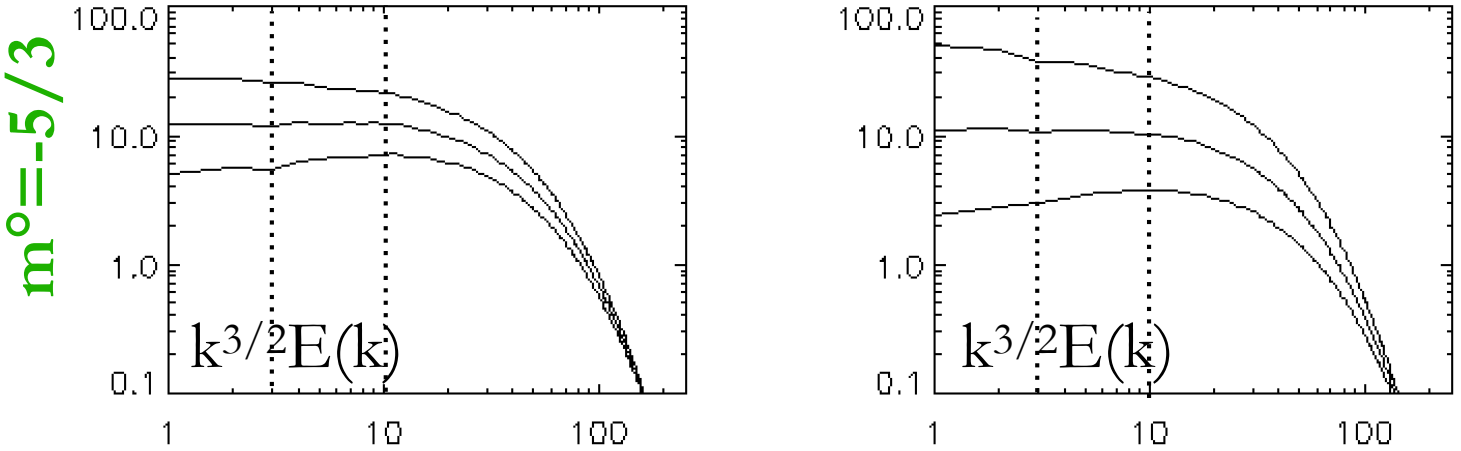




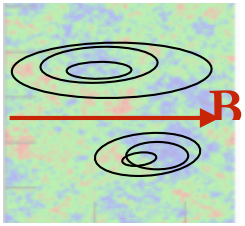
Scaling vs cross helicity σ_c ($B^\circ=3$)

1D ⊥ spectra E+, E- and $\langle E\rangle=(E_+E_-)^{1/2}$

$\sigma_c^\circ=0.5$ $8\le t\le 10$ $\sigma_c^\circ=0.8$



When σ_c° grows, E+ steepens and E- flattens: this indicates that the two energy flux Q+ and Q- adopt the quasi-isotropic weak form



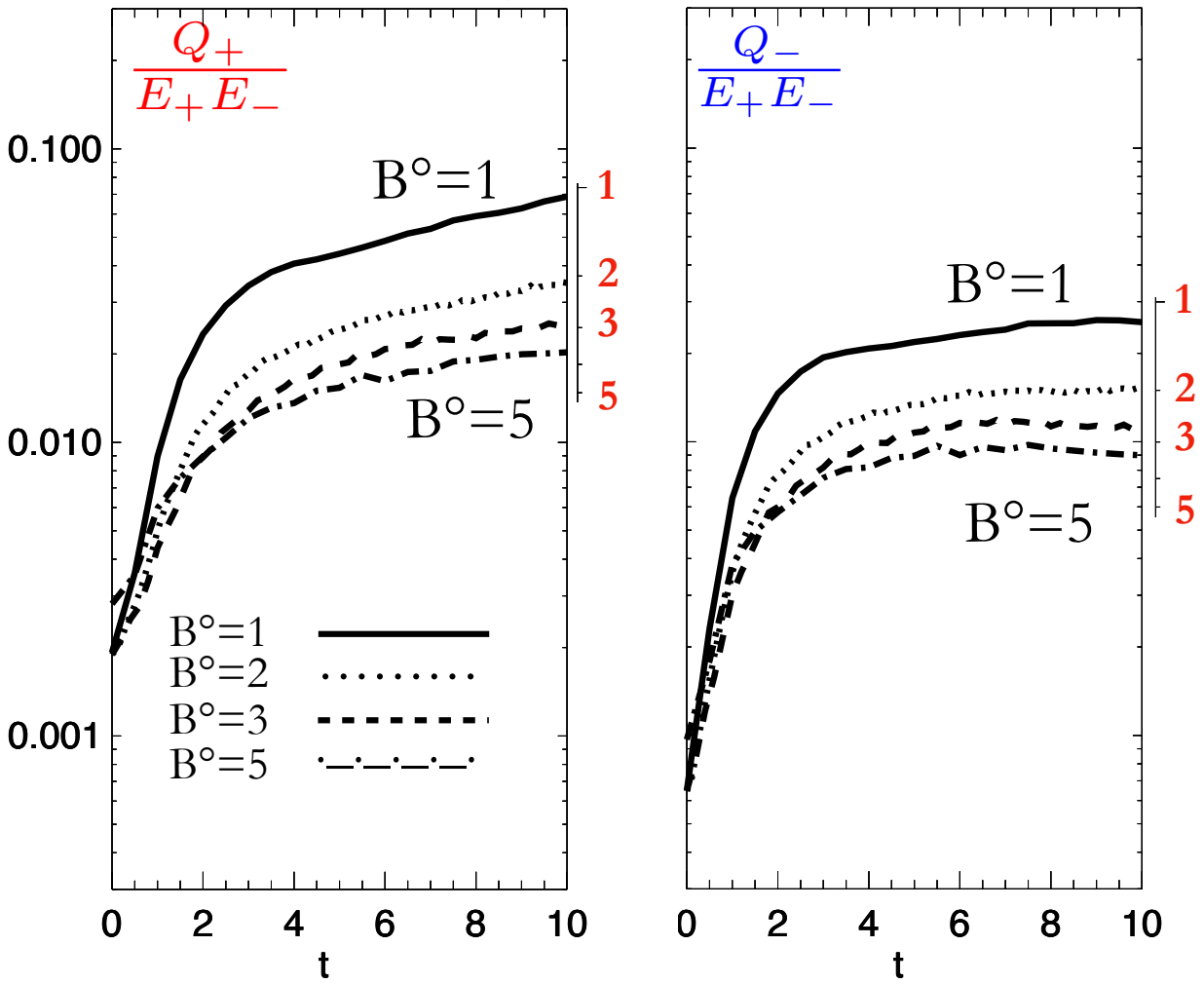
$Q_{\pm}$ vs B°

Assume the isotropic weak form of the energy flux (thus with $k_{//} \propto k_{\perp}$):

$Q_{\pm}=kE_{+}E_{-}/B°$ (1)

$y_{\pm}=Q_{\pm}/(E_{+}E_{-}) \propto 1/B°$ (2)

$\sigma_c°=0.5$



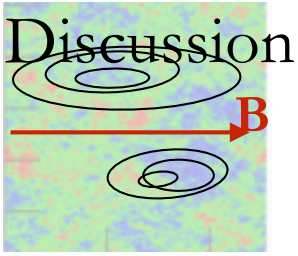
For $\sigma_c°=0.5$ (same for 0.8), and $1 \le B° \le 3$:

$y_{+} \propto 1/B°$

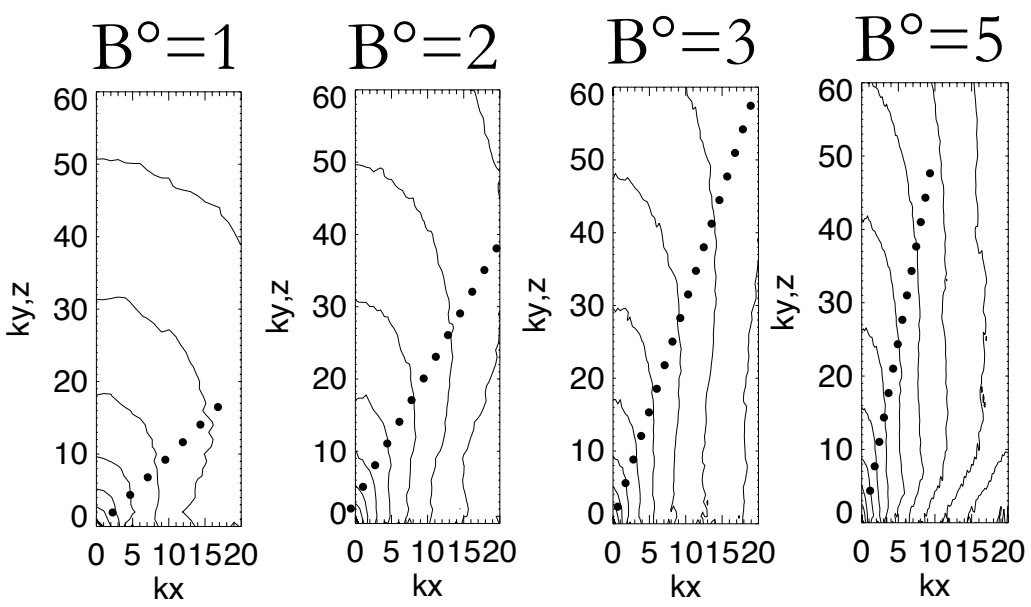
$y_{-} \propto 1/B°^{\alpha}$ with $\alpha \approx 0.8$

NB

- Relation (1) should hold in the inertial range, while here *total energies* $E_{\pm}$ are considered
- if considering inertial range, wavenumber k could be B°-dependent



B° and spectral anisotropy $m°=-5/3$ $\sigma_c°=0.8$

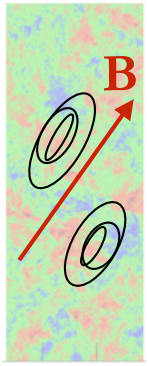


Our previous findings are contradicted by the evidence given by the anisotropic 3D spectra which are in support of a constant Alfvén frequency Alfvén frequency $k_{//}°B°$, in favor of a anisotropic version of weak turbulence, which lead to $\mathbf{m_+ + m- = -4}$, not -3 (*Galtier et al 2000*).

A possible issue would be to interprete the above figure as showing the cascade to be confined not completely within the perpendicular plane, but instead filling a 3D volume defined by $\mathbf{k_{//}B° \leq k_\perp b_{rms}}$ (see dotted lines in figure above).

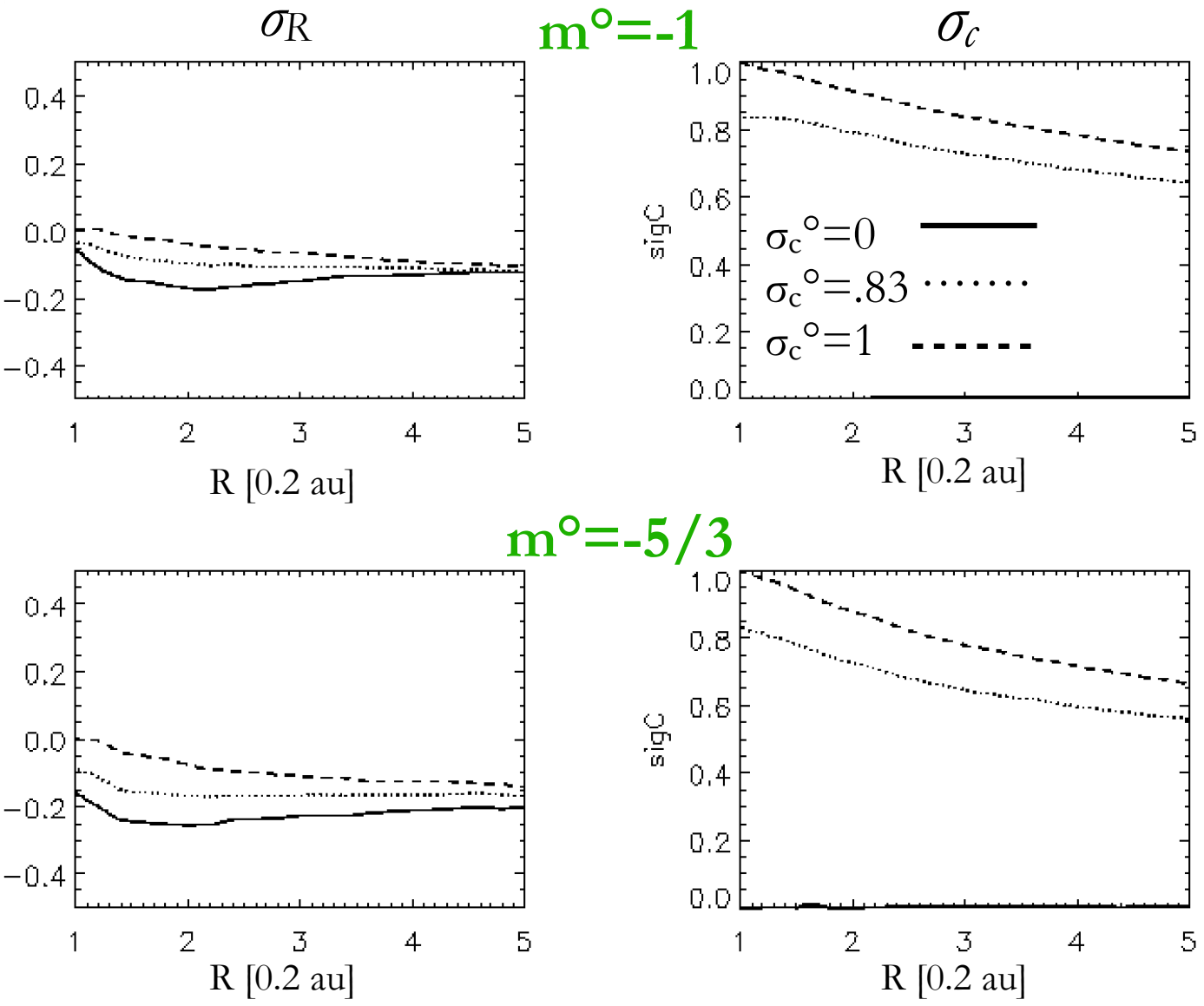
This would allow to reconcile the ensemble of properties observed here in the range $1 \leq B° \leq 3$:

- $Q_\pm \propto 1/B°$
- $m_+ + m- = -3$
- quasi-⊥ cascade



Part 2: expanding turbulence

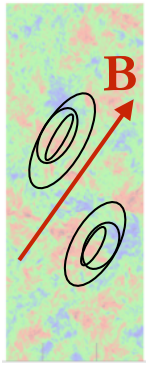
Cross helicity σ_c and residual energy σ_R

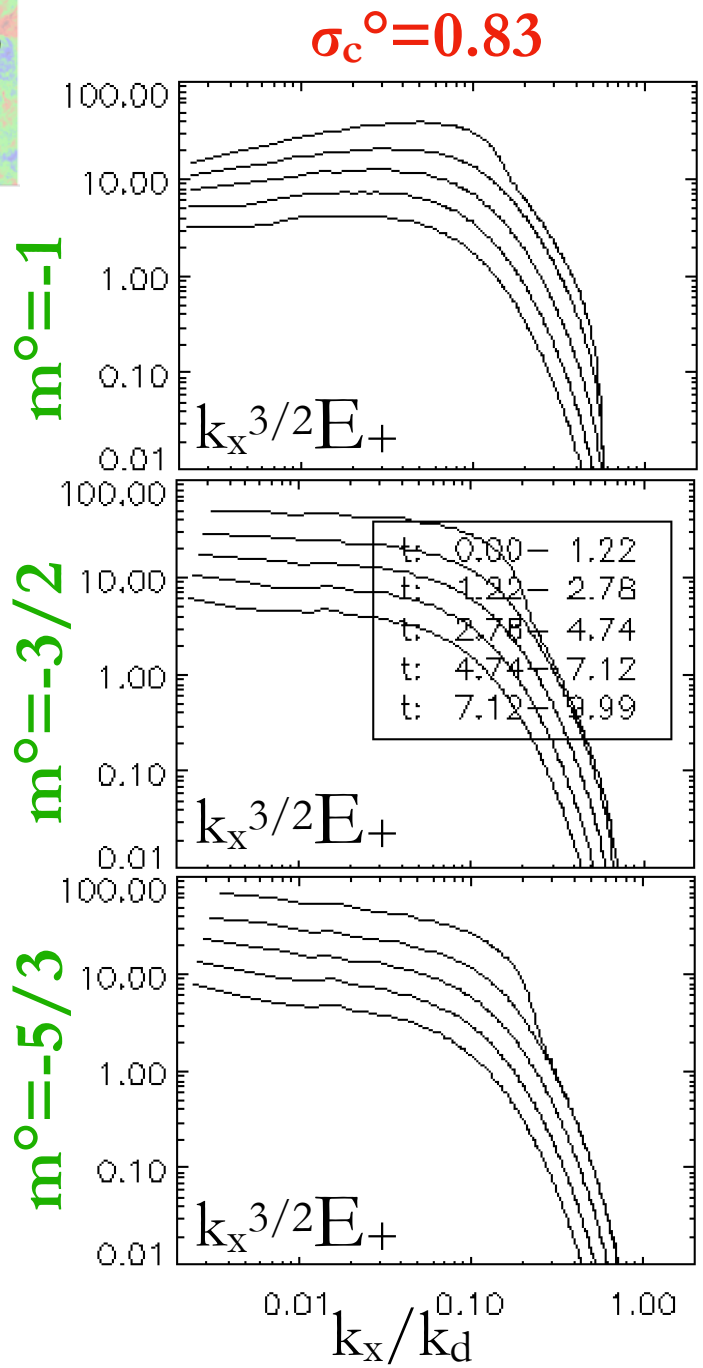


One sees here no longer the **Alfvén-like oscillations** of residual energy observed for the same ratio $b_{rms}/B°$ in homogeneous runs.

Indeed, **expansion favors magnetic fluctuations** compared to velocity fluctuations, thus working against the linear Alfvén equipartition effect.
This also reduces σ_c as seen in the right figures by introducing continuously new E- energy.

E_+ relaxation from t=0 to 10 (0.2 to 1 au)



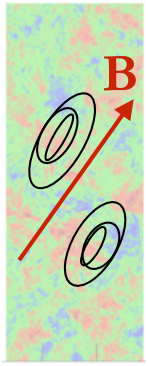


Successive spectra are averaged within five time intervals from t=0 to t=10.
All spectra are compensated by $k_x^{-3/2}$

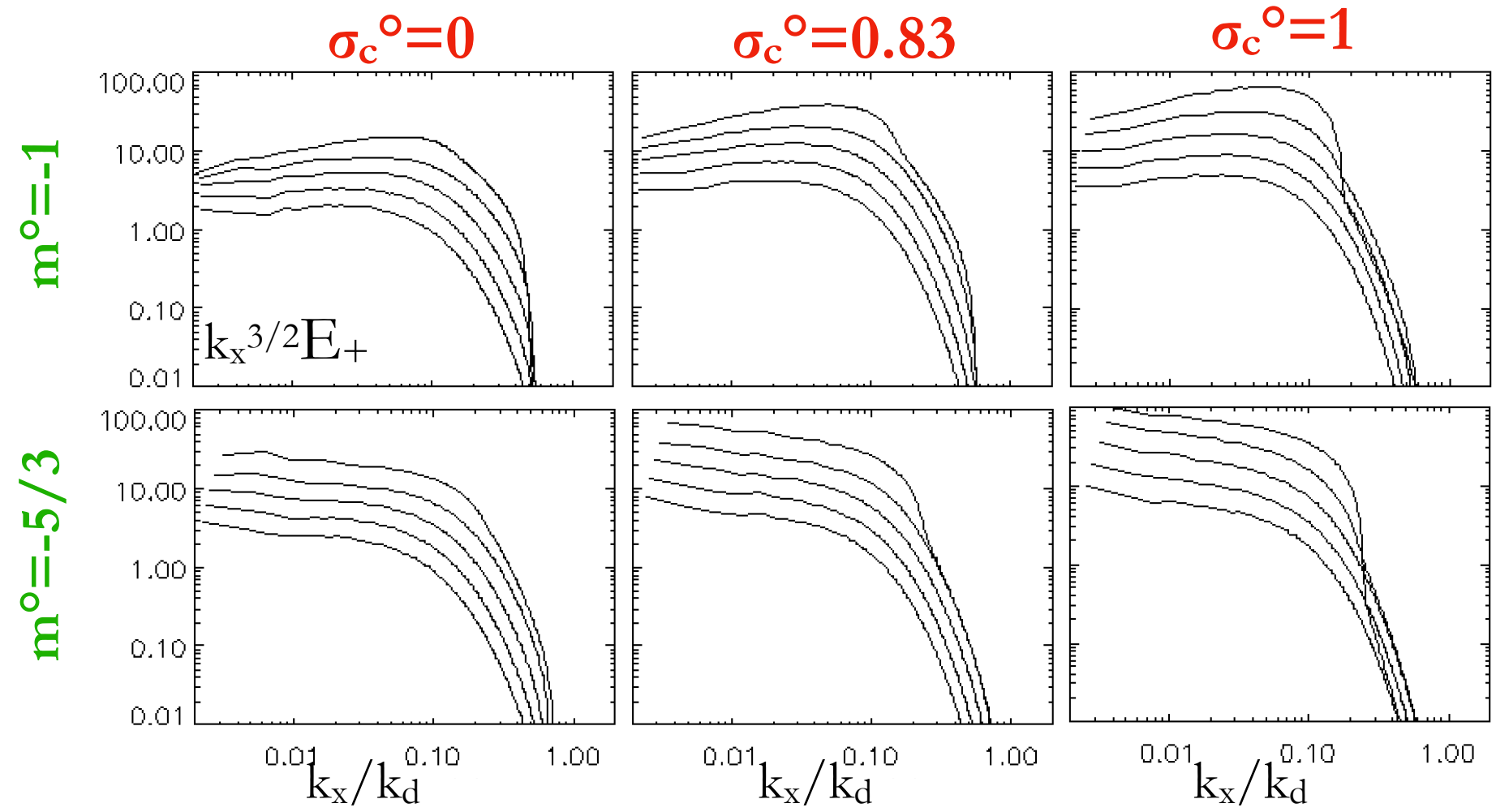
The final indices of E_+ are as follows
m°=-1 → m_+=-1.5
m°=-1.5 → m_+=1.7
m°=-5/3 → m_+=1.75

Thus:
- the 1 au index depends strongly on the initial (0.2 au) index
- Convergence to the final index is fast for the two cases m°=-3/2 and -5/3

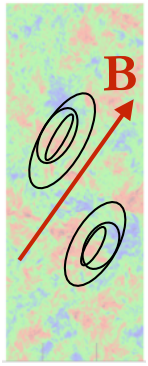
Note that $1 \le |m°| \le 3/2$ at 0.2 au gives $1.5 \le |m| \le 1.7$ which corresponds to observations



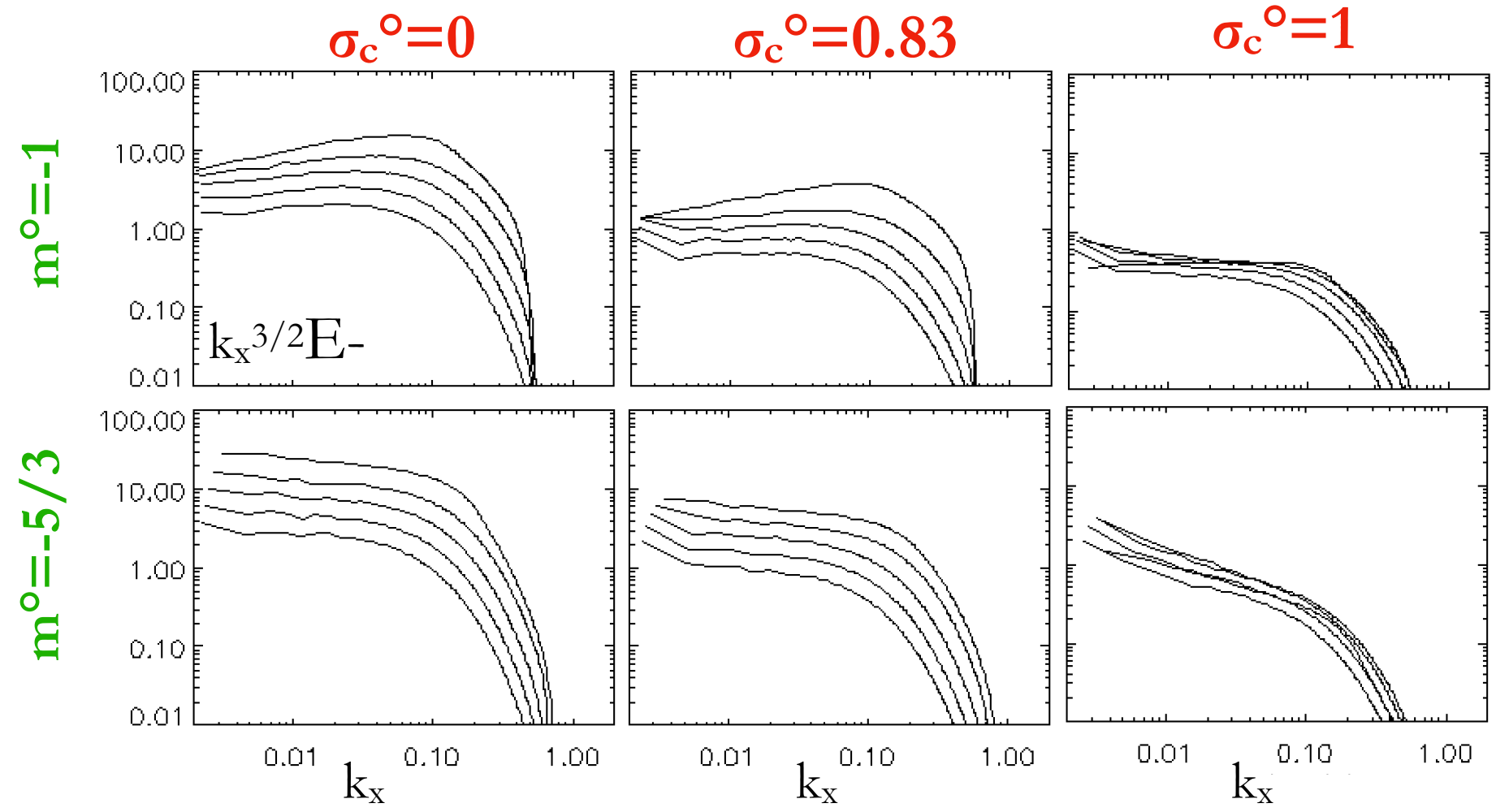
E_+ relaxation from t=0 to 10 (0.2 to 1 au)



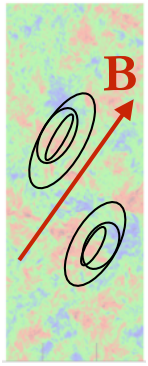
Note that relaxed spectral index slightly increase with σc°



E- relaxation from t=0 to 10



- E- converge (as E_+) towards very different scaling depending on m°
- relaxed spectral index slightly increase with σc°



Scaling of E_+, E_- and $\langle E \rangle$: (1) m°=-1 and -5/3

$8 \le t \le 10$

σ_c°=0 | σ_c°=0.83 | σ_c°=1

m°=-1 | m°=-5/3

$k_x^{3/2} E_\pm$

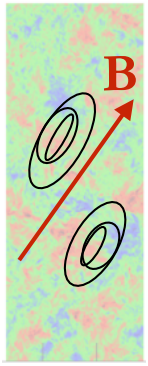
$E_+(k)$, $\langle E \rangle(k)$, $E_-(k)$

1000, 100, 10, 1

1, 10, 100

k_x

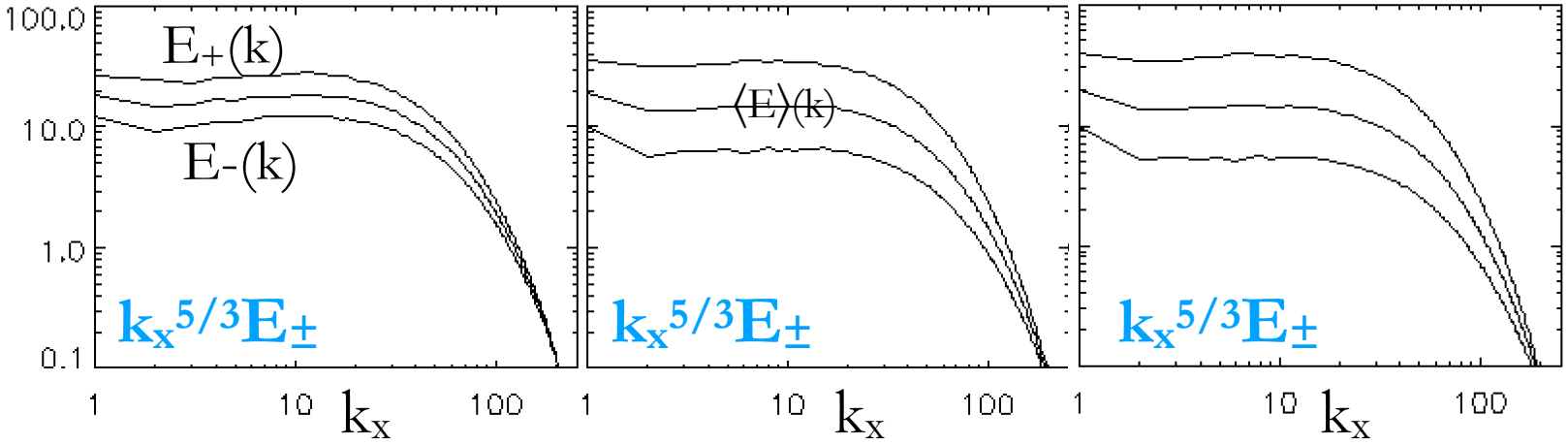
- Spectra become steeper with larger σ_c°
- Final scaling at 1 au depends on initial scaling



Scaling of E_+, E- and $\langle E \rangle$ (2) $m°=-1.5$, $0.5 \le \sigma c° \le 0.9$

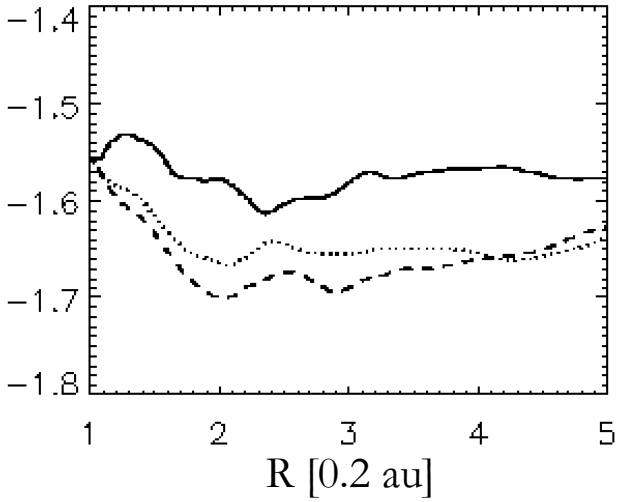
Final spectra E+ E- (7≤t≤10)

$\sigma_c°=0.5$ $\sigma_c°=0.83$ $\sigma_c°=0.9$

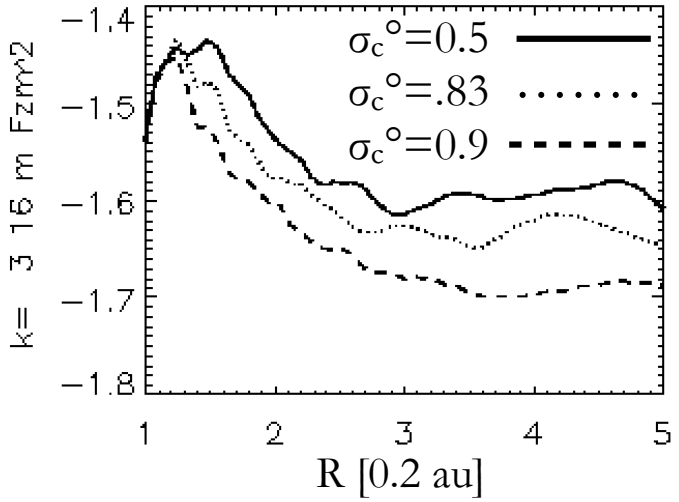


With large enough $\sigma_c°$ ($0.8 \le \sigma_c° \le 0.9$), final spectral indices are ≈**-5/3** for both E+ and E-

m_+ inside $3 \le k_x \le 16$

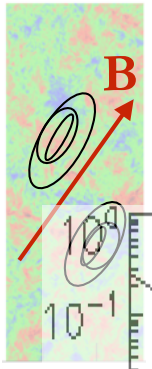


m_- inside $3 \le k_x \le 16$

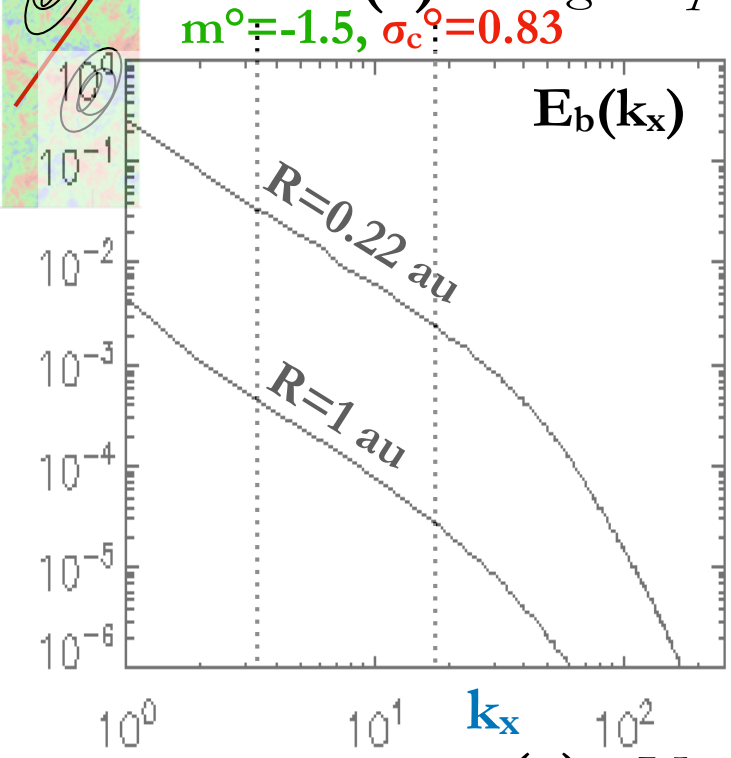


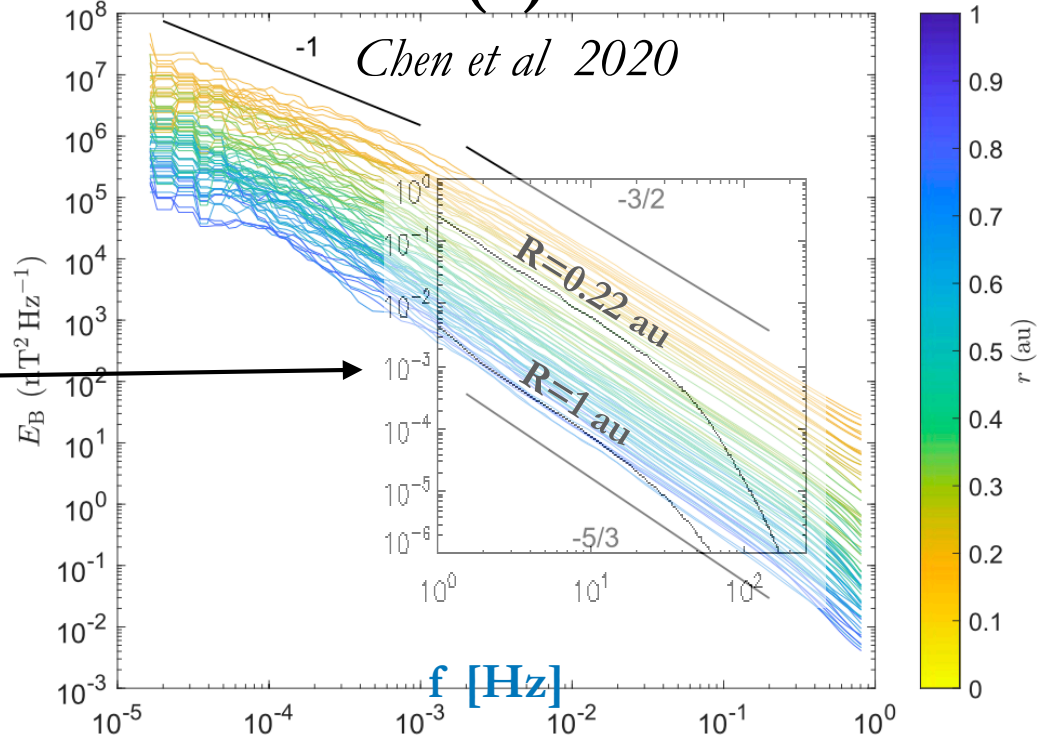
- Short transient: spectra adopt the no-expansion configuration with $|m_+| > 1.5 > |m-|$
- Relaxation: all three runs adopt indices with $|m\pm| \ge 1.6$

Comparing (a) run $m°=-1.5$, $\sigma_c°=0.83$ with (b) first two PSP orbits

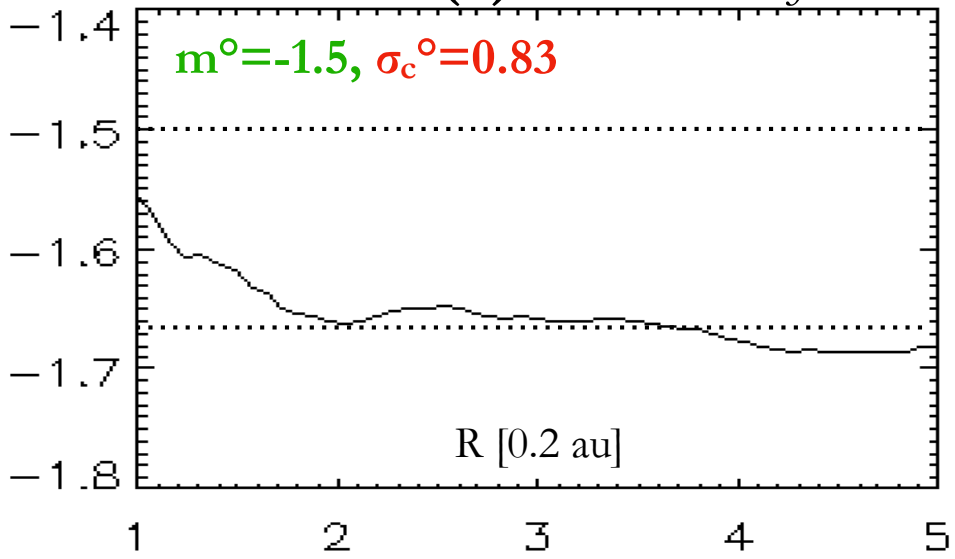


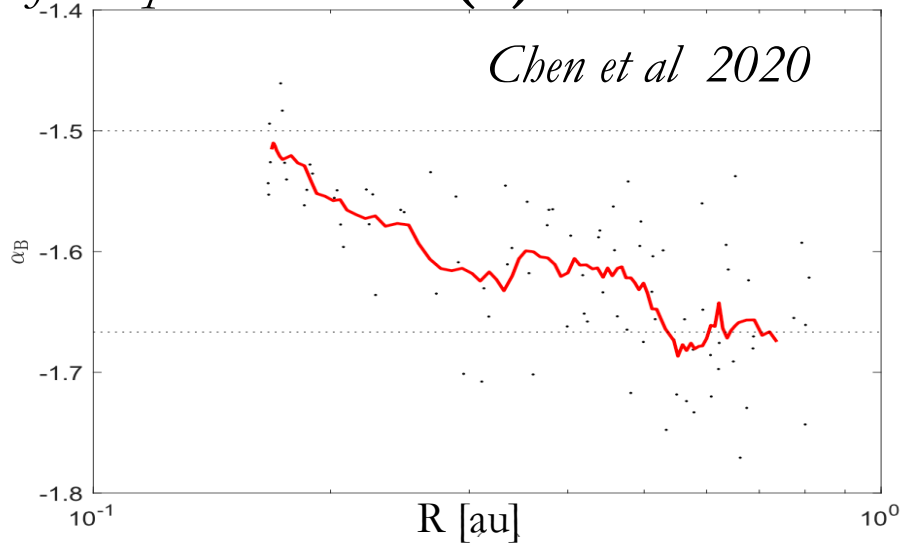
(a) *Magnetic power spectrum vs heliocentric distance* **(b)**

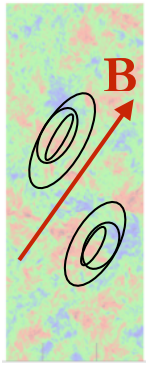




(a) *Variation of the magnetic field spectral index* **(b)**



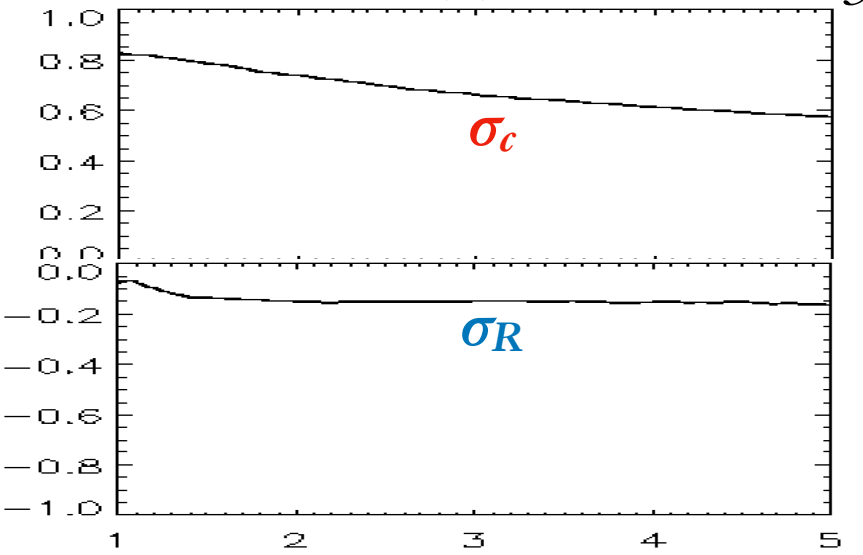




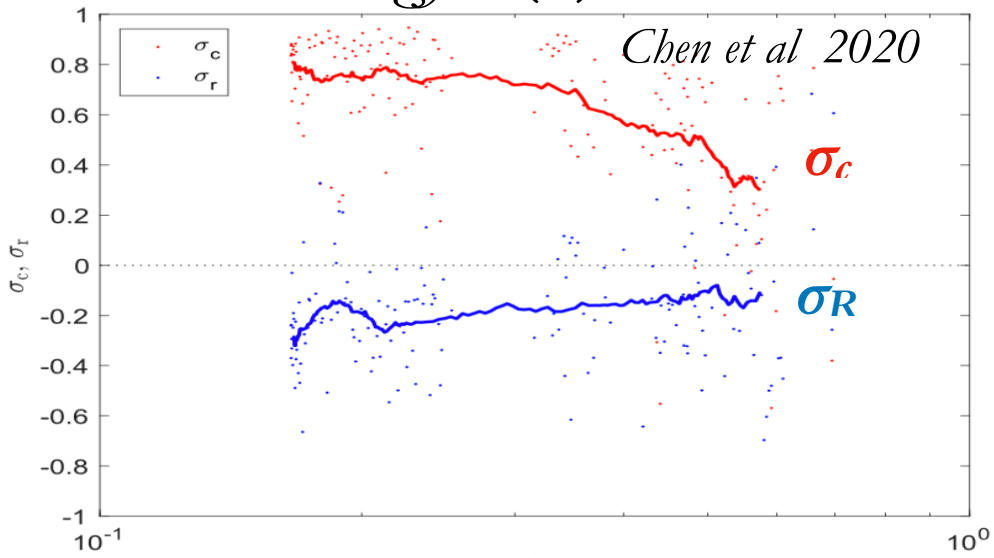
Comparing (2)

(a) $m^{\circ}=-1.5$, $\sigma_c^{\circ}=0.83$ with (b) first two PSP orbits

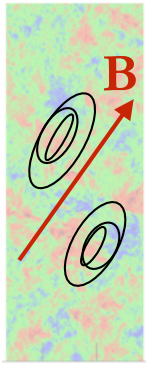
(a) *Cross helicity and residual energy* **(b)**



R [0.2 au]



R [au]



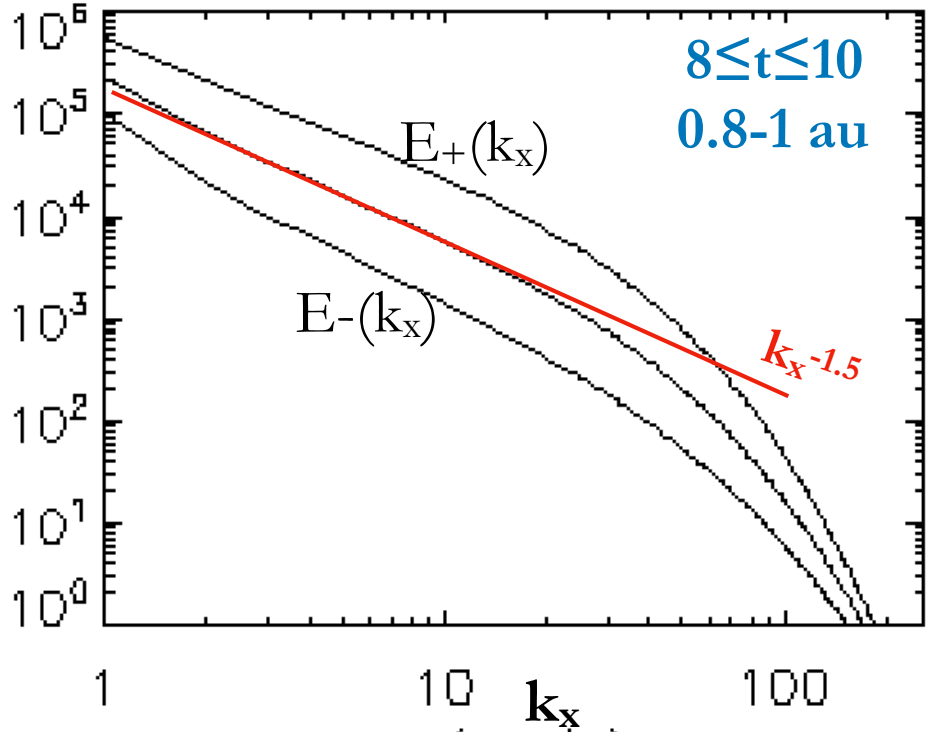
Comparing (3):

(a) run $m^\circ=-1$, $\sigma_c^\circ=0.83$ at 1 au

(b) first two PSP orbits at 0.17 au

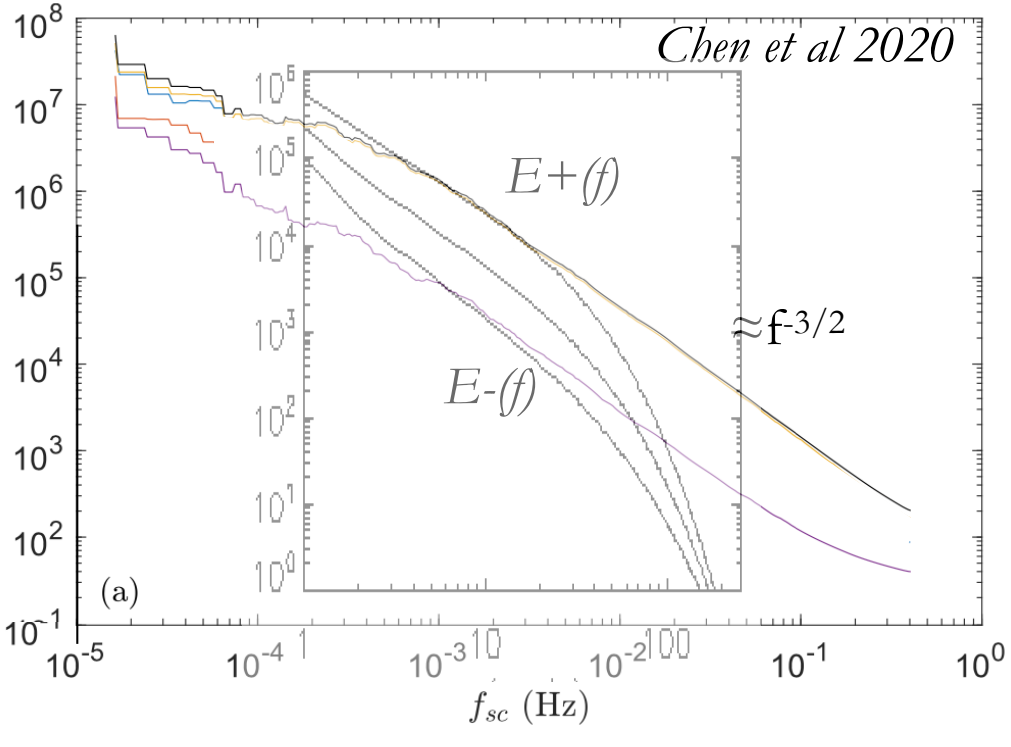
(a)

E+, ⟨E⟩, E- around 1 au



(b)

E± spectra at 0.17 au



Conclusion

We considered Alfvénic MHD turbulence with moderate mean field B°. Our preliminary study shows that with similar initial conditions, such a turbulence develops differently, whether it is imbedded in an expanding wind or not.

Without expansion, turbulence shows a strong dependance (as turbulent dissipation) on the mean magnetic field, sharing several properties with the weak isotropic turbulence of Iroshnikov and Kraichnan, in spite of a significant anisotropy along the mean magnetic field. In particular, for moderate mean field values, the two Elsasser spectra show pinning around the -3/2 scaling.

With expansion, turbulence rapidly develops properties which differ strongly with the previous ones: the two spectra are quasi-parallel. The index -3/2 is not central. Instead, the spectral scaling at 1 au depends on the initial (0.2 au) scaling: as a rule, the spectra steepen with distance.

The steepening of spectra with distance, already observed in Helios mission, has been confirmed recently in PSP mission, with scaling close to the Sun being close to a -3/2 scaling, which could thus be a fossil of turbulence born in a plasma devoid of expansion.