Finding better numerical solutions for circulation along piecewise-constant coastlines in ocean models

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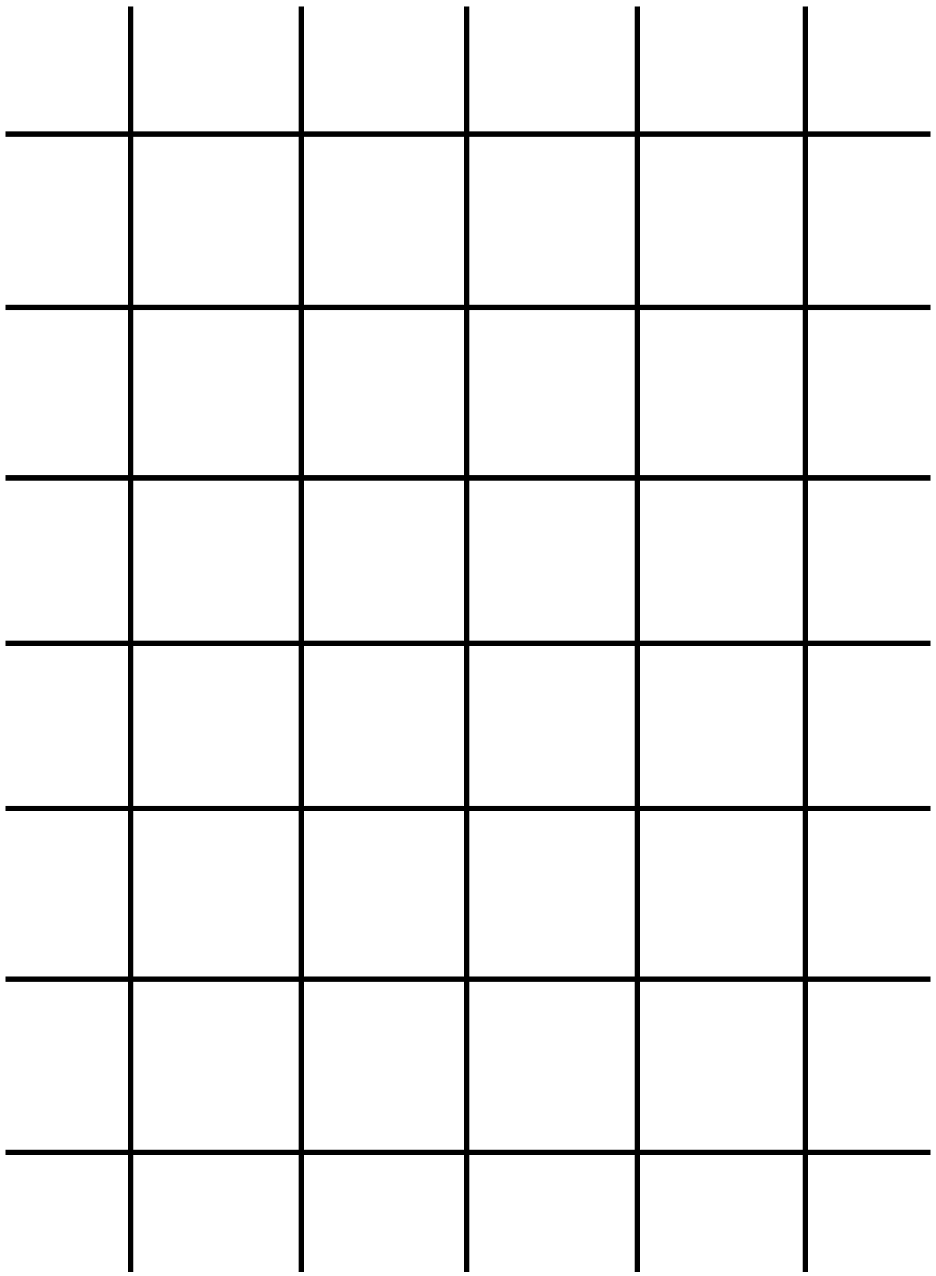
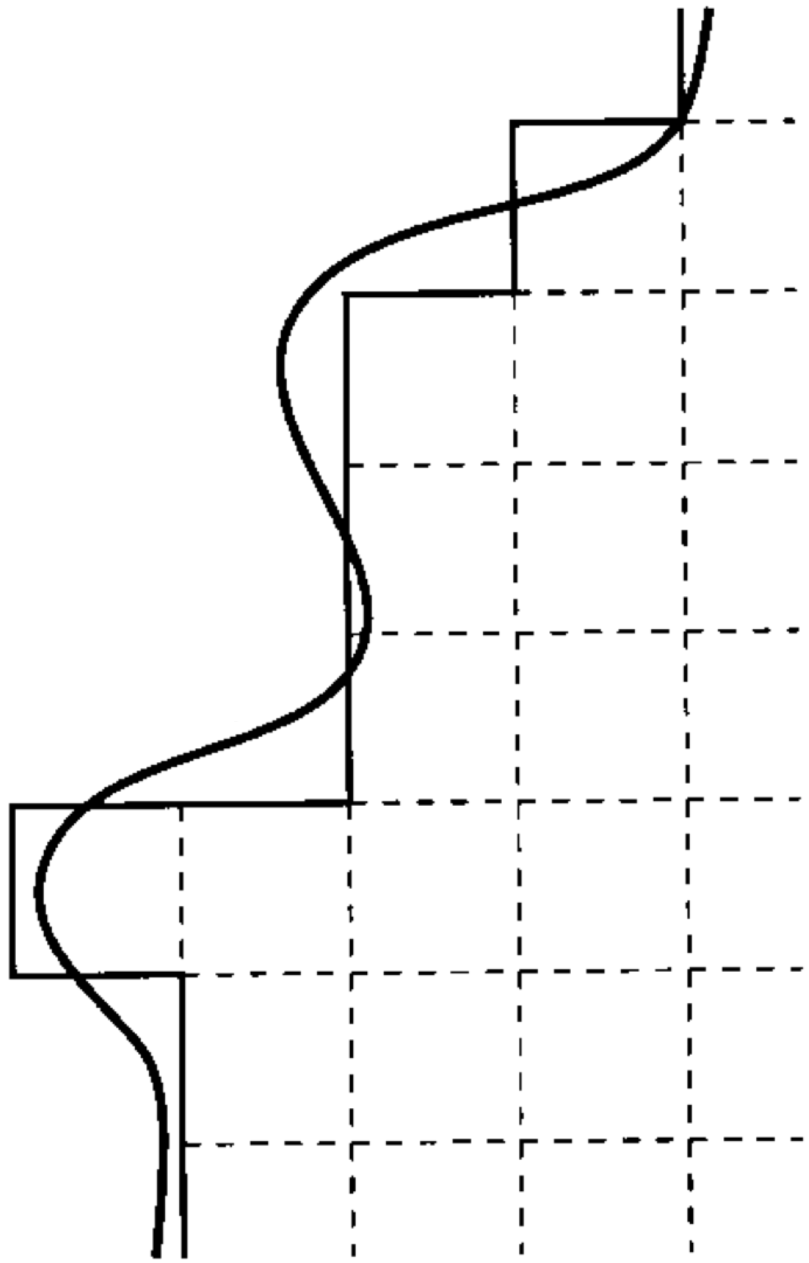
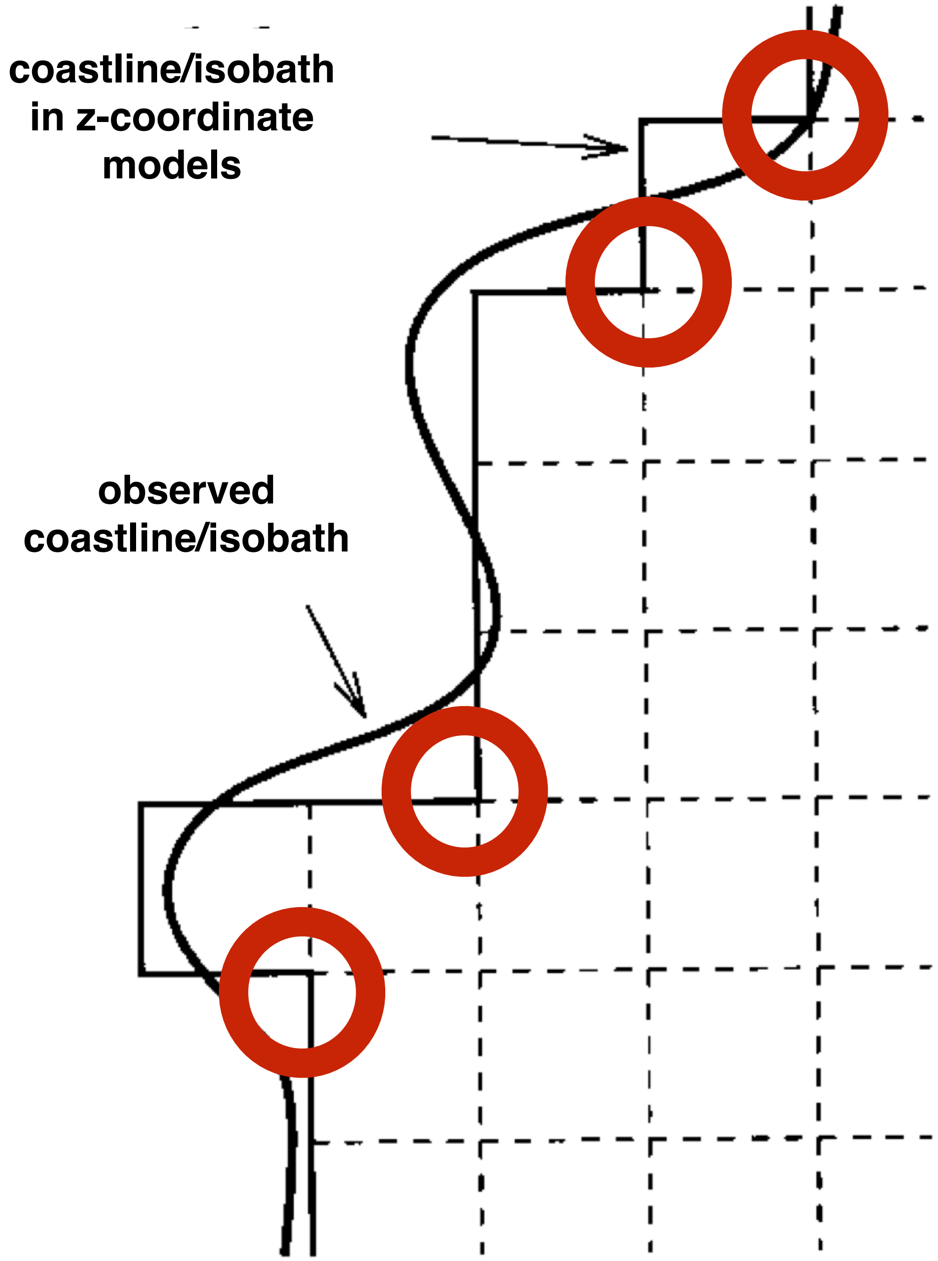




Adcroft&Marshall (1998) How slippery are piecewise-constant coastlines in numerical ocean models?



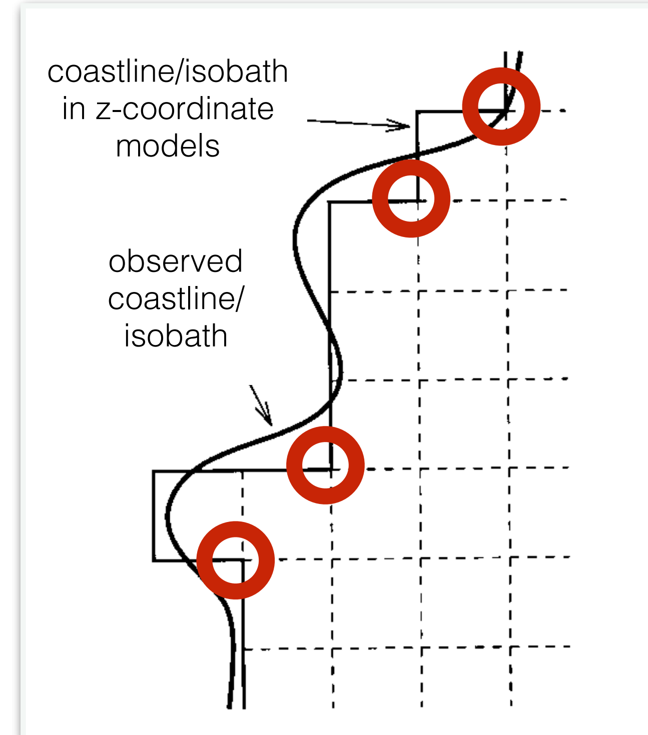
Adcroft&Marshall (1998) How slippery are piecewise-constant coastlines in numerical ocean models?

“Staircase problem”

How slippery are piecewise-constant coastlines in numerical ocean models?

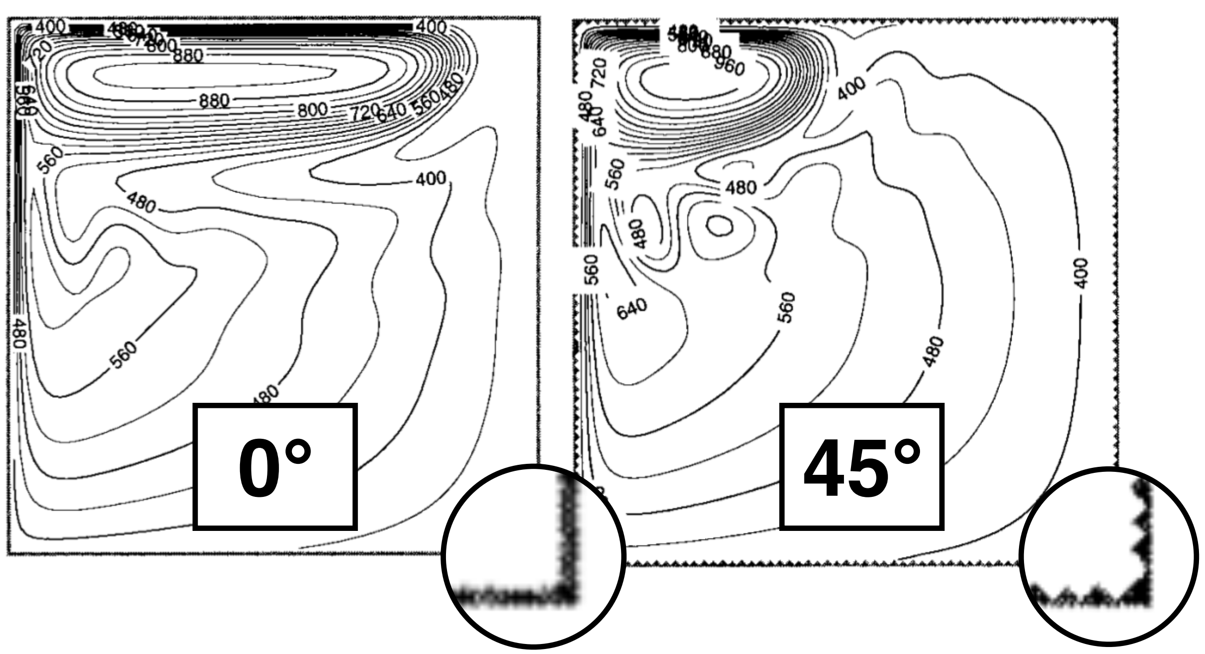
By ALISTAIR ADCROFT, *Department of Earth, Atmospheric and Planetary Sciences, Massachusetts Institute of Technology 54-1523, Cambridge, Massachusetts 02139, USA* and DAVID MARSHALL*, *Department of Meteorology, University of Reading, P.O. Box 243, Reading, RG6 6BB, UK*

(Manuscript received 31 July 1996; in final form 15 August 1997)

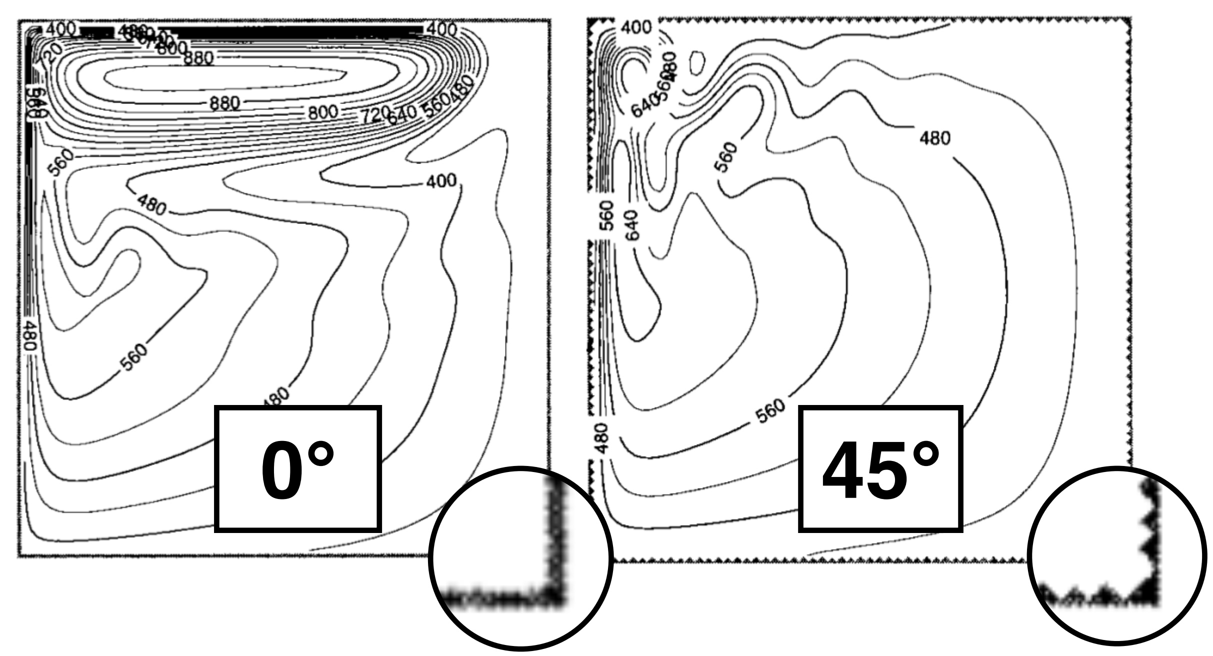


1/4° Resolution

Div-rot diffusion’s formulation



Symmetric diffusion’s formulation



Shallow water model (with reduced gravity)

$$\partial_t \boldsymbol{u} + \left(\frac{f+\zeta}{h}\right)\boldsymbol{k}\times h\boldsymbol{u} + \nabla\frac{1}{2}(\boldsymbol{u}.\boldsymbol{u}) = -g'\nabla h - r\boldsymbol{u} + \frac{1}{h}\mathrm{div}(\nu h\sigma) + \frac{\boldsymbol{\tau}}{\rho h}$$

$$\partial_t h + \mathrm{div}(h\boldsymbol{u}) = 0$$

Here, free-slip or no-slip boundary condition apply both on advection (*dynamical)* and dissipation *(viscous)* terms.

Leap Frog
ENS
C grid
Beta plan

$$\sigma_{D,\zeta} = \begin{pmatrix} \chi & -\zeta \\ \zeta & \chi \end{pmatrix} = \begin{pmatrix} (\partial_x u + \partial_y v) & -(\partial_x v - \partial_y u) \\ (\partial_x v - \partial_y u) & (\partial_x u + \partial_y v) \end{pmatrix}$$

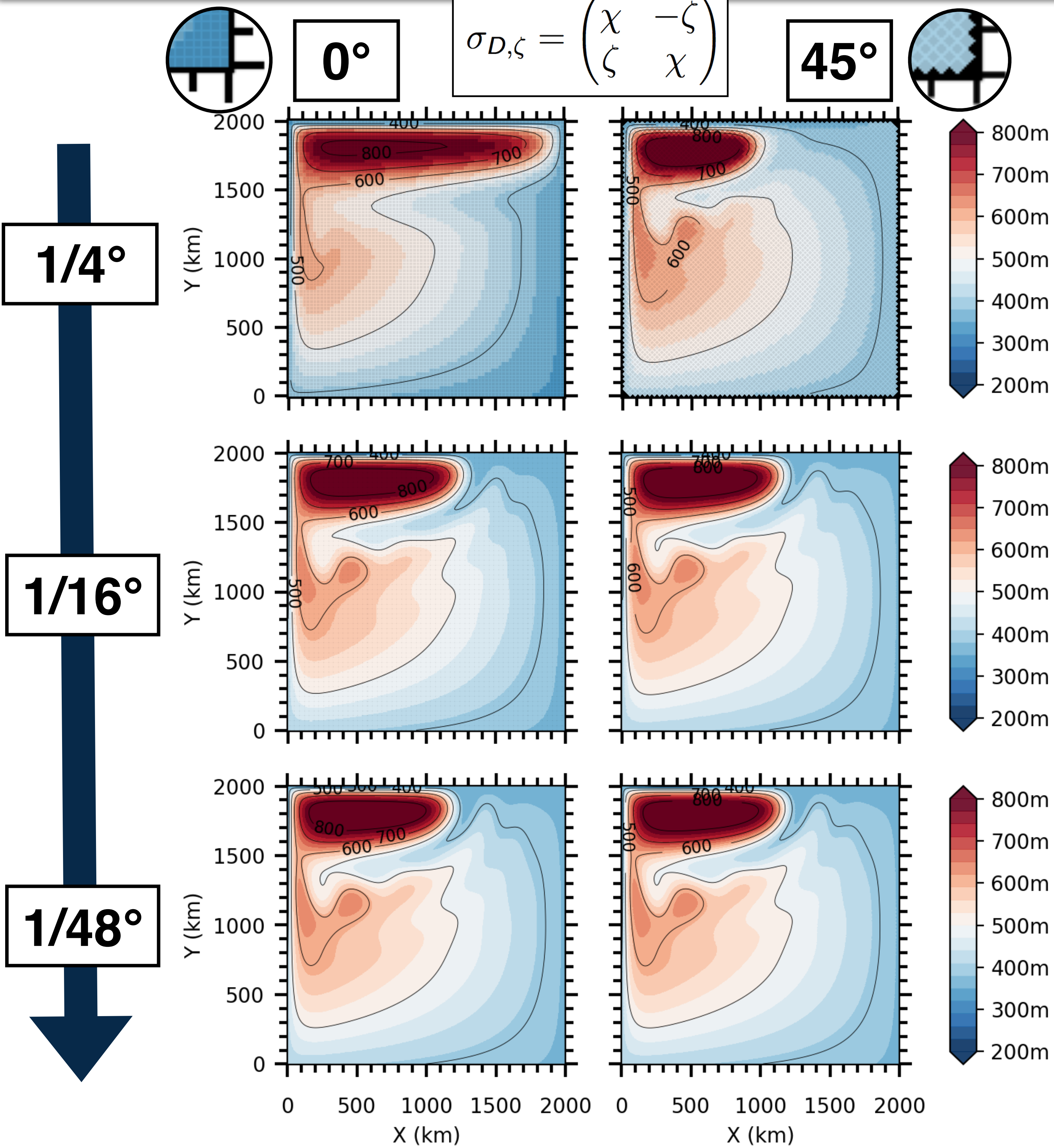
—> rotational-divergence form or "rot-div"

$$\sigma_{sym} = \begin{pmatrix} D_T & D_S \\ D_S & -D_T \end{pmatrix} = \begin{pmatrix} (\partial_x u - \partial_y v) & (\partial_y u + \partial_x v) \\ (\partial_y u + \partial_x v) & -(\partial_x u - \partial_y v) \end{pmatrix}$$

—> symmetric form or "sym"
In continuous, preserves angular momentum

Numerical Convergence

$$\sigma_{D,\zeta} = \begin{pmatrix} \chi & -\zeta \\ \zeta & \chi \end{pmatrix}$$



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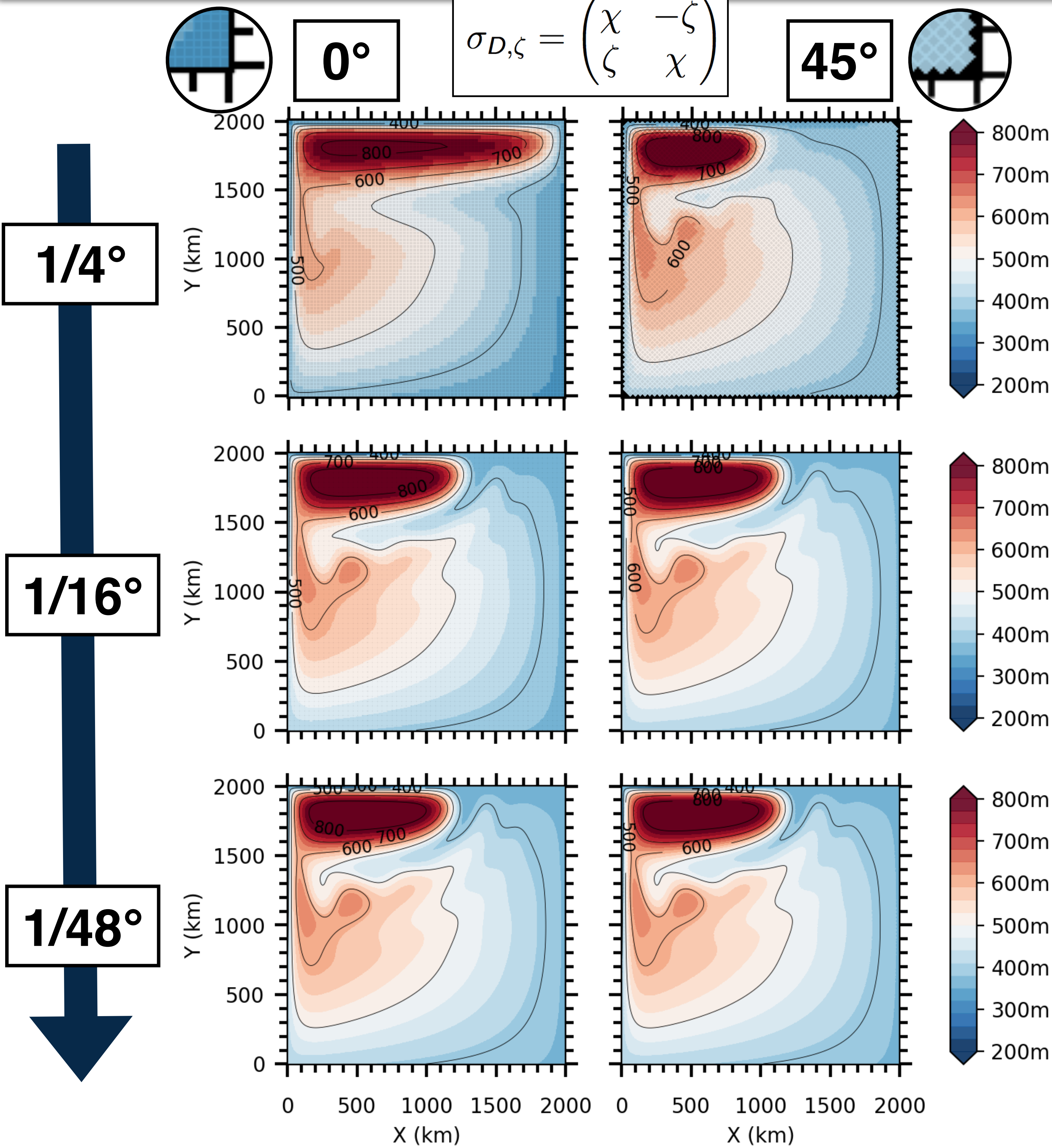

At 1/4° solutions are not converged.

When the model has converged (from 1/16°), solutions are insensitive to steps.

Numerical Convergence

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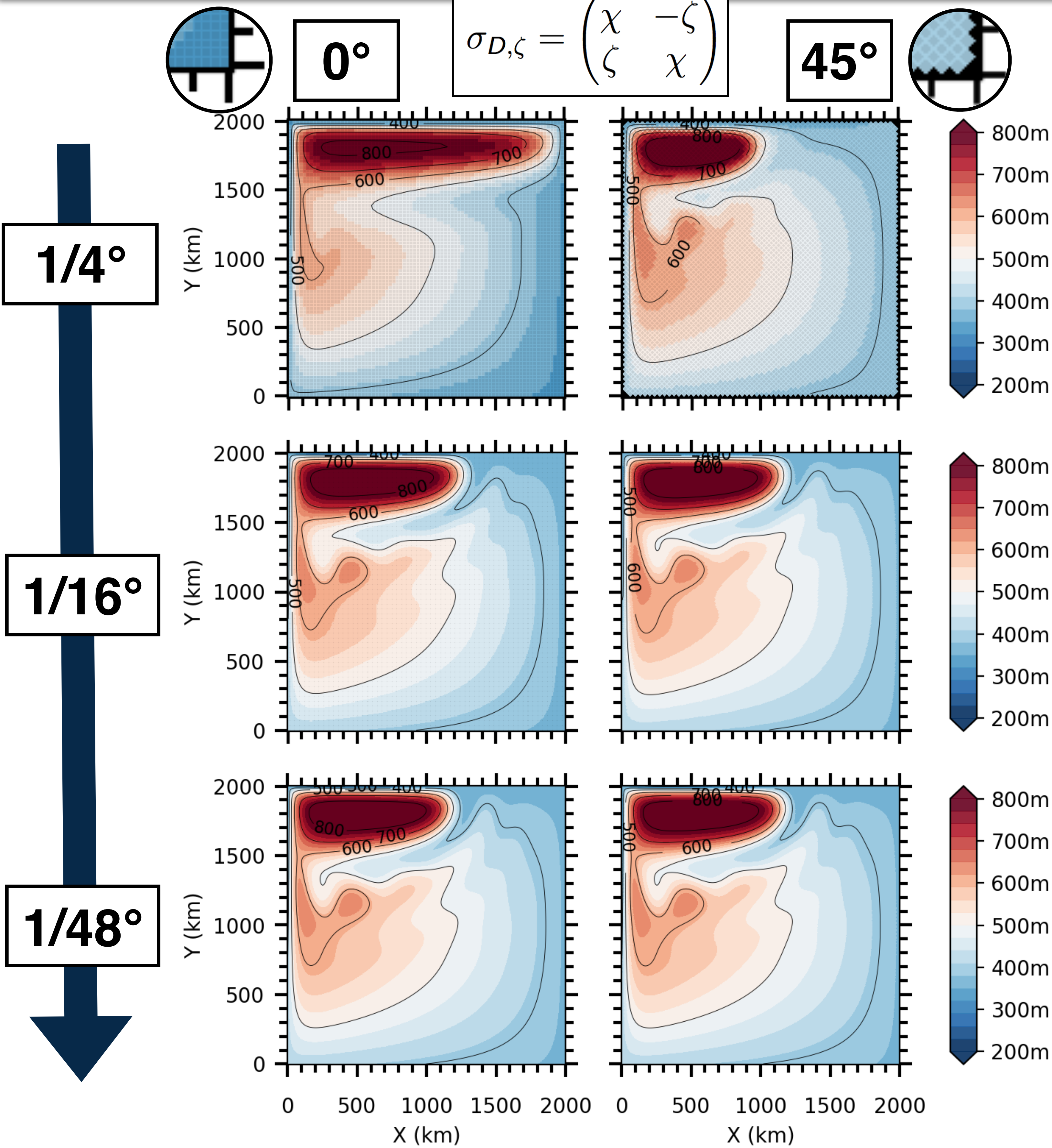

- Right, but the indentation reduce with the step size !
- What is the limiting factor for numerical convergence ?
- Does it stay true with no-slip boundary condition ?
  - Is it true in flux-form ?
  - Does this hold for intermediate orientation ?

- What about the symmetric stress tensor ?

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What about the symmetric stress tensor ?

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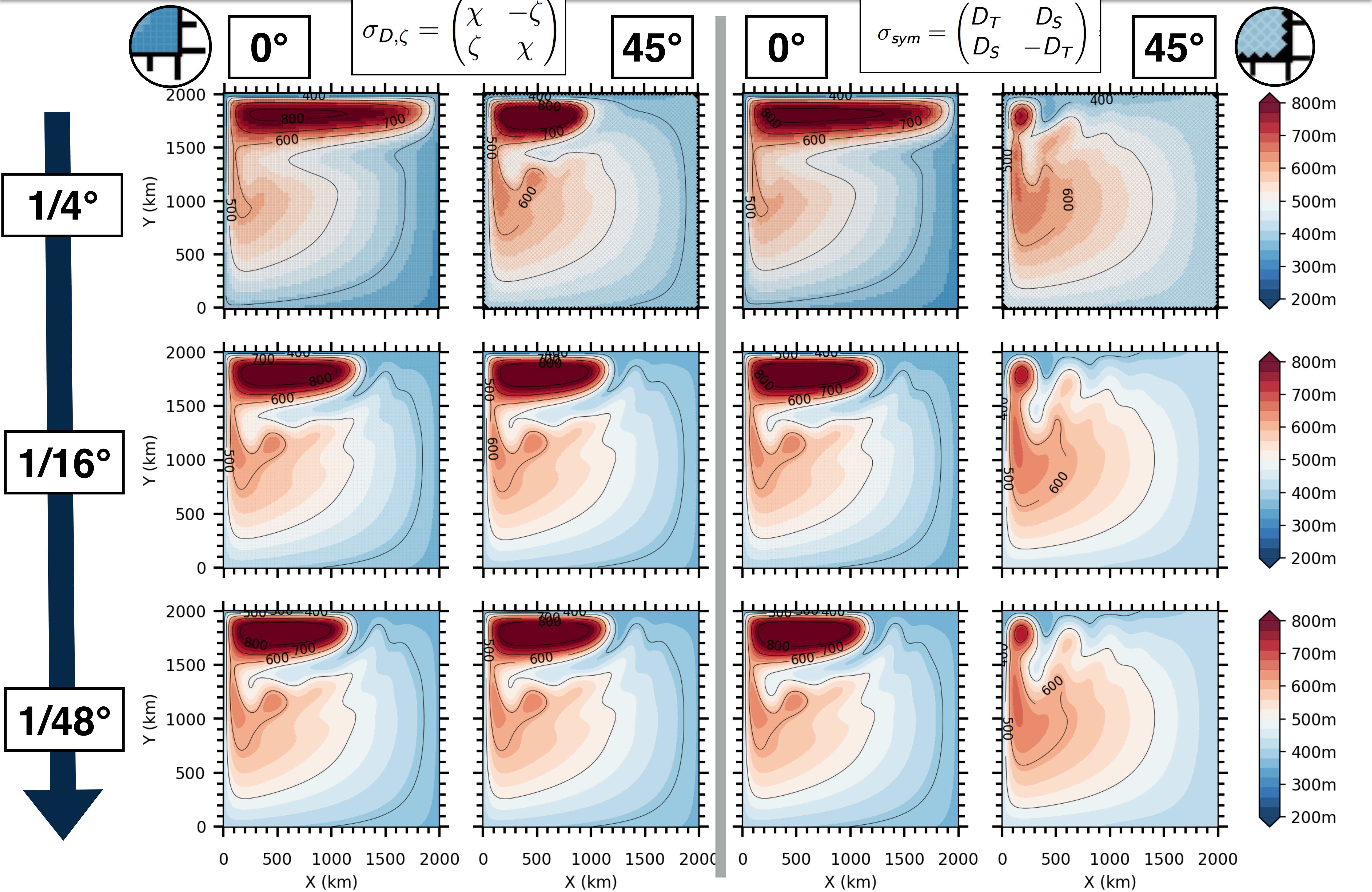
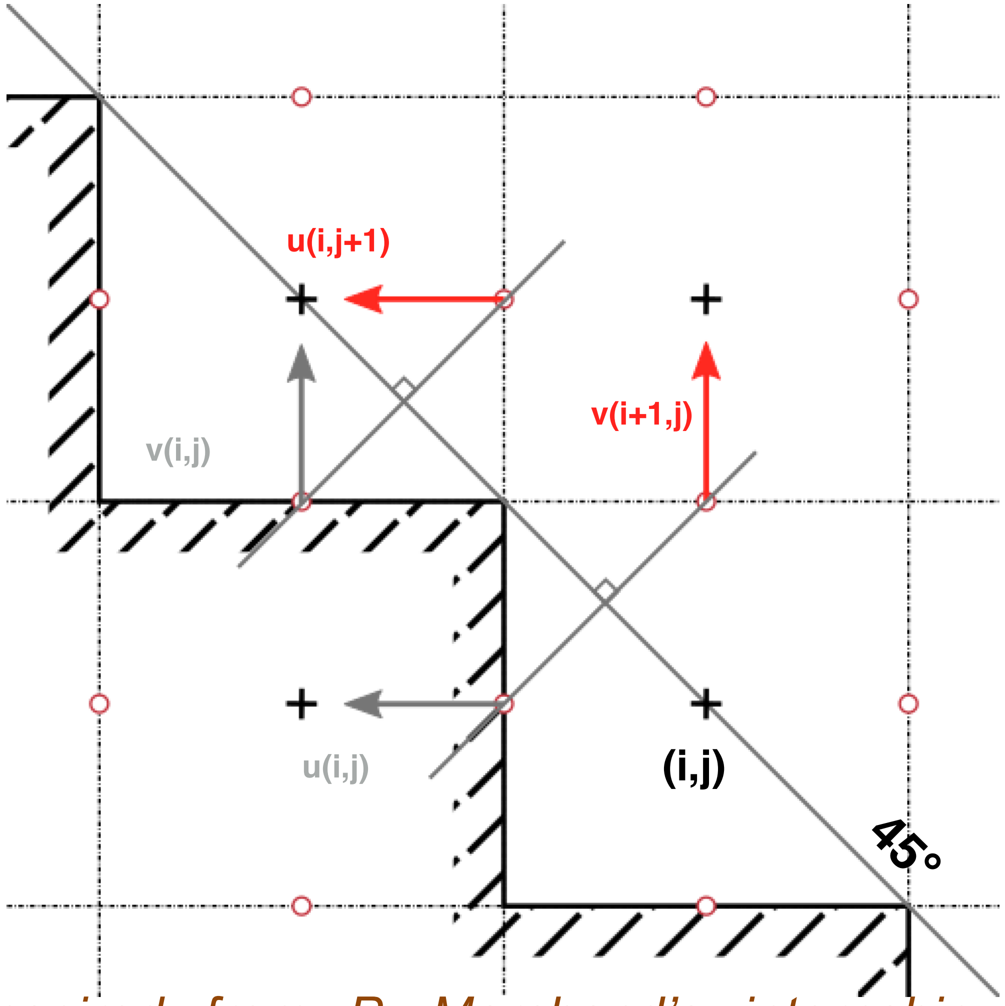
$$\sigma_{sym} = \begin{pmatrix} D_T & D_S \\ D_S & -D_T \end{pmatrix}$$



What about the symmetric stress tensor ?

NEW CONDITION	FREESLIP
$\sigma_{D,\zeta}$	$\zeta \times 0$ $\chi \times 2$
σ_{sym}	$D_S \times 2$ $D_T \times 0$



Inspired from P. Marchand's internship results supervised by Eric Blayo and Florian Lemarié (2015)

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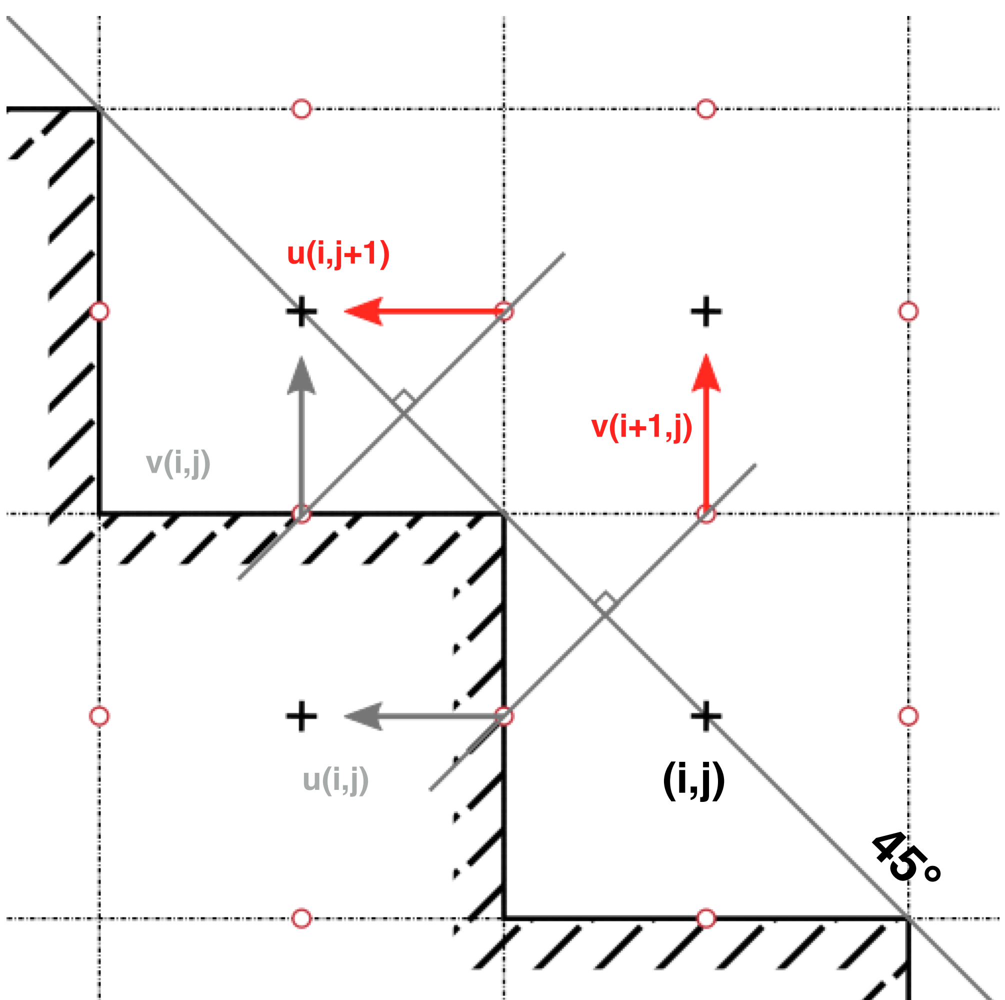

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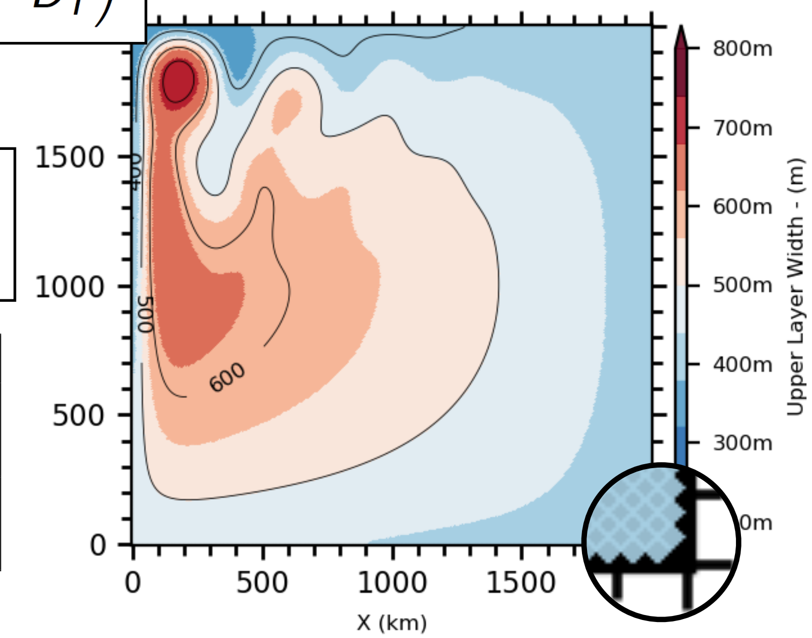
1/16°

actual condition

	FREESLIP
$\sigma_{D,\zeta}$	$\zeta \times 0$ $\chi \times 1$
σ_{sym}	$D_S \times 0$ $D_T \times 1$



1/16°

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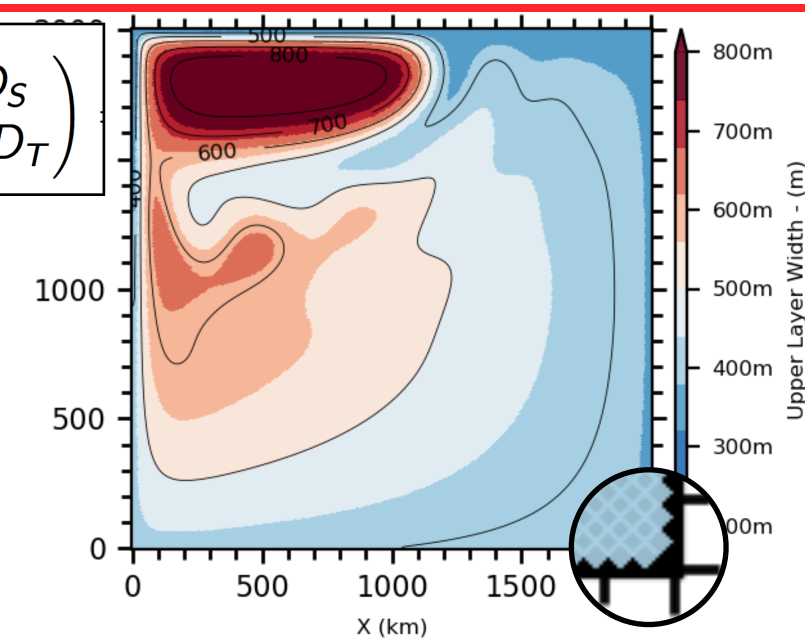
NEW CONDITION



Conclusions

- **The “staircase problem” is an ill-posed problem since it used un-converged solutions that leads to misleading conclusions. When comparing converged solutions, we observe that there is no spurious effect from steps.**

- **A boundary condition must be written on a grid accordingly to the coastline that is represented.**

Thank you !