



Bundesamt für  
Kartographie und Geodäsie



# Solution-Level Fast Constraints Transformations with Case Studies for GNSS Networks

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# Constrains transformation

The transformation among different constraints solutions (Minimal Constran - MC to Over Constrain - OC and vice versa) plays crucial role for Terrestrial Reference Frame (TRF) realization, the classical approach:

1. Invert the CoVariance (CV) matrix
2. Remove initial constraints
3. Construct the Helmert matrix
4. Impose new constrain
5. Solve the Normal EQuation (NEQ) system

This approach is rigorous but with heavy computational effort, considering the number of GNSS station is constantly and rapidly increasing.

# Fast Constraints Transformation (FCT)

## 1. MC to OC solution

$$\hat{\mathbf{x}}_{[m,1]}^{\text{OC}} = \left( \mathbf{I} - \underbrace{\mathbf{C}^{\text{MC}} \mathbf{G}^T (\mathbf{C}_{\mathbf{G}} + \mathbf{G} \mathbf{C}^{\text{MC}} \mathbf{G}^T)^{-1} \mathbf{G}}_{[k,k]} \right) \hat{\mathbf{x}}_{[m,1]}^{\text{MC}}$$

$[m, m] \quad [m, k] \quad [k, k] \quad [k, m] [m, m] [m, k] \quad [k, m] \quad [m, 1]$

## 2. OC to MC solution

$$\hat{\mathbf{x}}_{[m,1]}^{\text{MC}} = \left( \mathbf{I} + \mathbf{C}^{\text{OC}} \mathbf{G}^T (\mathbf{C}_{\mathbf{G}} - \mathbf{G} \mathbf{C}^{\text{OC}} \mathbf{G}^T)^{-1} \mathbf{G} \right) \hat{\mathbf{x}}_{[m,1]}^{\text{OC}}$$

$[m, m] \quad [m, k] \quad [k, k] \quad [k, m] [m, m] [m, k] \quad [k, m] \quad [m, 1]$

$\mathbf{C}_{\mathbf{G}}$ : covariance matrix of No-Net Conditions (NNC),  $\mathbf{G}$ : Helmert parameter design matrix,  $m$ : number of parameters;  $k$ : number of constraints ( $k \leq 14$ )

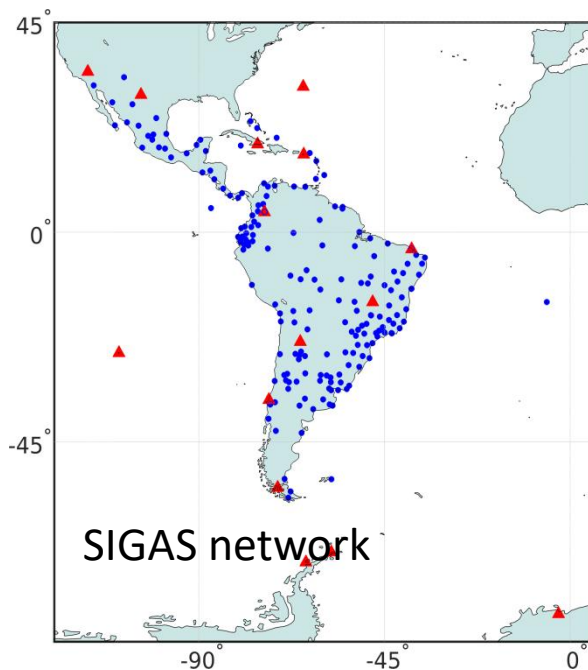
*For more details regarding the FCT, please refer to:*

Ampatzidis D, Wang L, Mouratidis A, Balidakis K (2022) Rigorous and fast constraints transformations at the solution level: case studies for regional and global GNSS networks. GPS Solut 26(2):44. <https://doi.org/10.1007/s10291-022-01225-3>

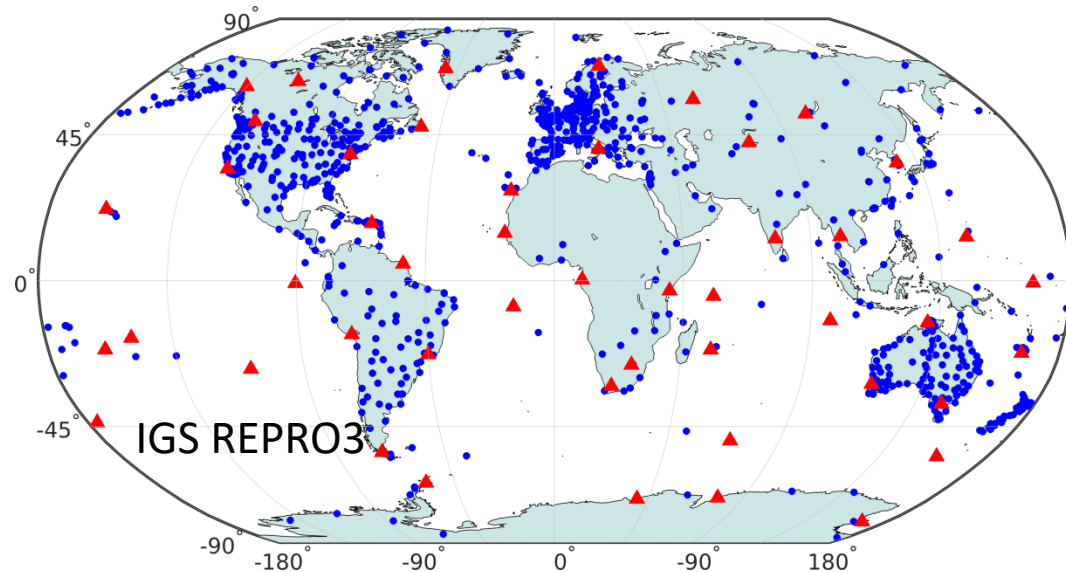
<https://github.com/drlwang/falcons> 

# Numerical test

1. Weekly loosely constrained SIRGAS solution to NNR+NTT (days 134-140, 2012)
2. Weekly MC solution of IGS RERPRO 3 to OC NNR+NTT (week 1930, day 0)
3. Simulated GNSS network of up to 5000 stations



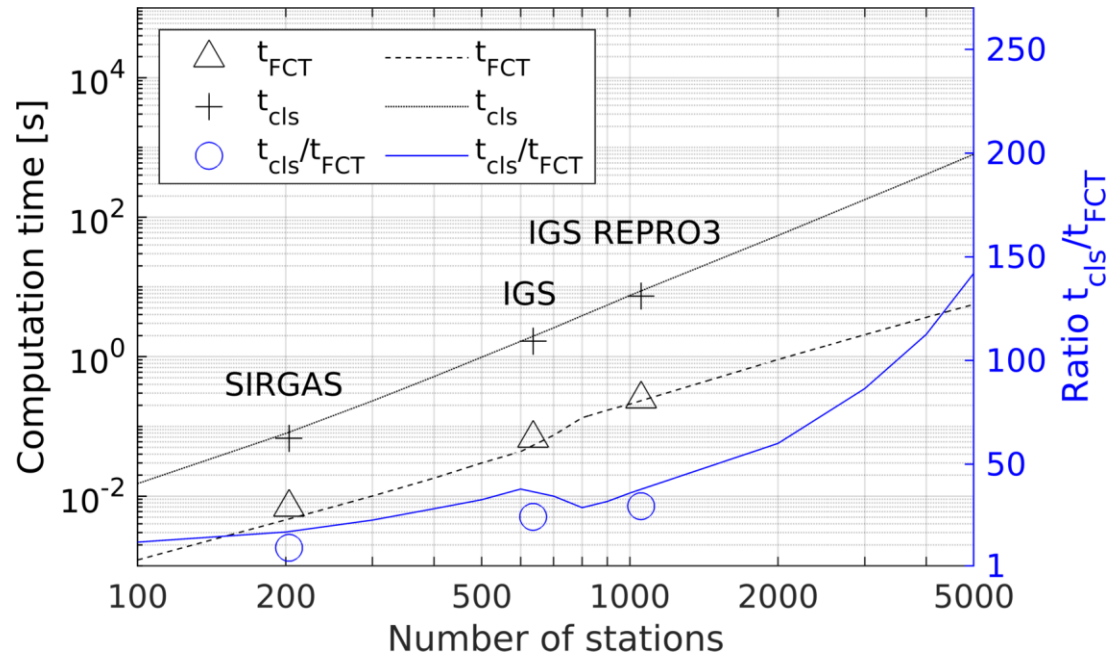
**Fig. 1** SIRGAS network consists of 203 stations. The red triangles are 15 fiducial stations; their coordinates participate in the no-net conditions. Blue dots are the remaining non-fiducial sites.



**Fig 2** IGS REPRO3 network consists of 1053 stations. The red triangles are 48 fiducial stations for NNT and NNR conditions, and blue dots are the non-fiducial stations

# Computation effort

The computation in FCT can be significantly reduced.



**Fig. 3** Computation times for transforming the constraints in classical and FCT method in logarithmic scale (time used for the solutions using the classical approach versus FCT). The markers are computation time taken from the SINEX files, with their network labels written on top of those markers. The right axis presents the ratio between the computation time from two approaches, with a linear scale in blue color.

# Conclusions

1. Significant computational effort reduction to change No-Net Conditions, highly recommend for the large networks.
2. Fast conversions between MC and OC.
3. FCT can be applied at global, regional and local scale.
4. FCT applies to Helmert-type constraints.
5. Updating constraints between MC, the alternative S-transformation is recommend due to no requirement of the a prior constrain information.



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# Thank you for your attention!

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*Or visit us at the github page:*

<https://github.com/drlwang/falcons> 

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