# SOME REMARKS ABOUT ORTHOMETRIC AND NORMAL HEIGHT SYSTEMS

## "We don't need in quasigeoid itself"

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# Introduction

## Initial moments

All the height systems in linear measure are non-optimal:

- physical height is not a “vertical distance”
- leveling staff is not in potential measure

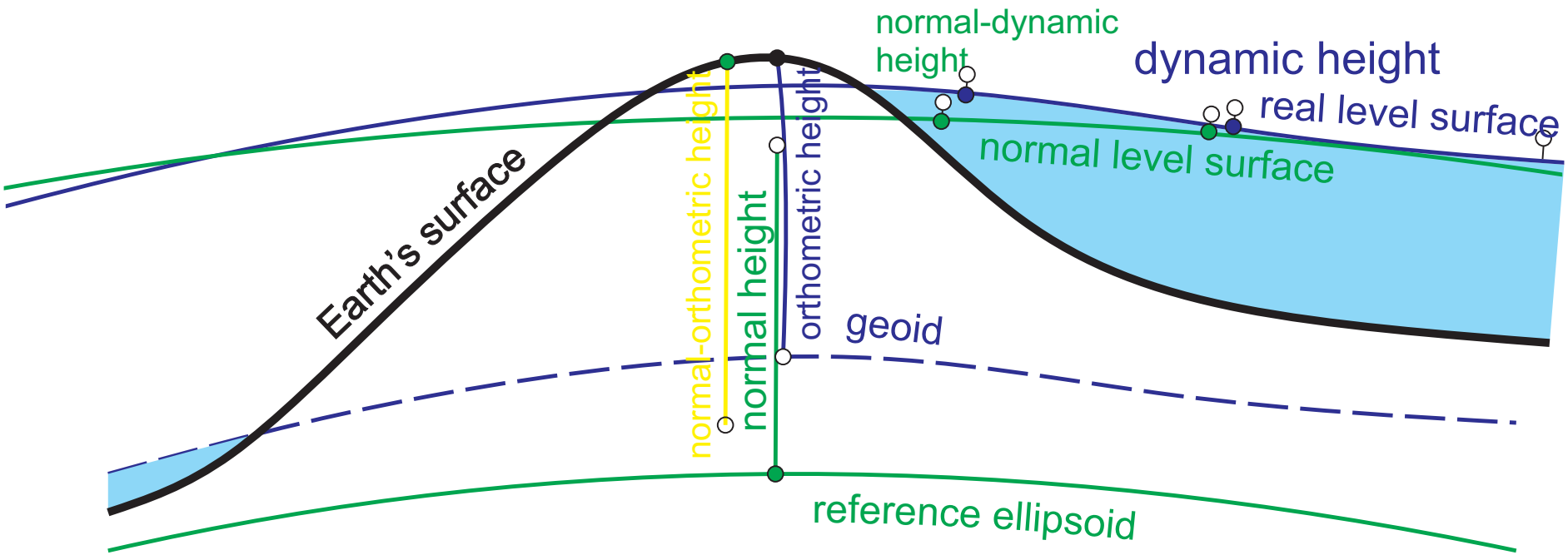
Consequently, the main requirements to the physical heights:

- rigor of calculation from the potential difference (without any hipothesis)
- the maximum proportionality of the potential differences and the measured elevations:
  - small corrections to the measured elevations
  - height marks on the one level surface more closer to each other
- other: connection to some BVP solution (like Molodensky problem)

# Heights classification

(I hate classification, but here it is useful)

| | Orthometric-type | Normal-type |
|---|---|---|
| Above sea level | Orthometric | Normal (Molodensky) |
| Shifted | Dynamic | *Shifted normal-dynamic* |



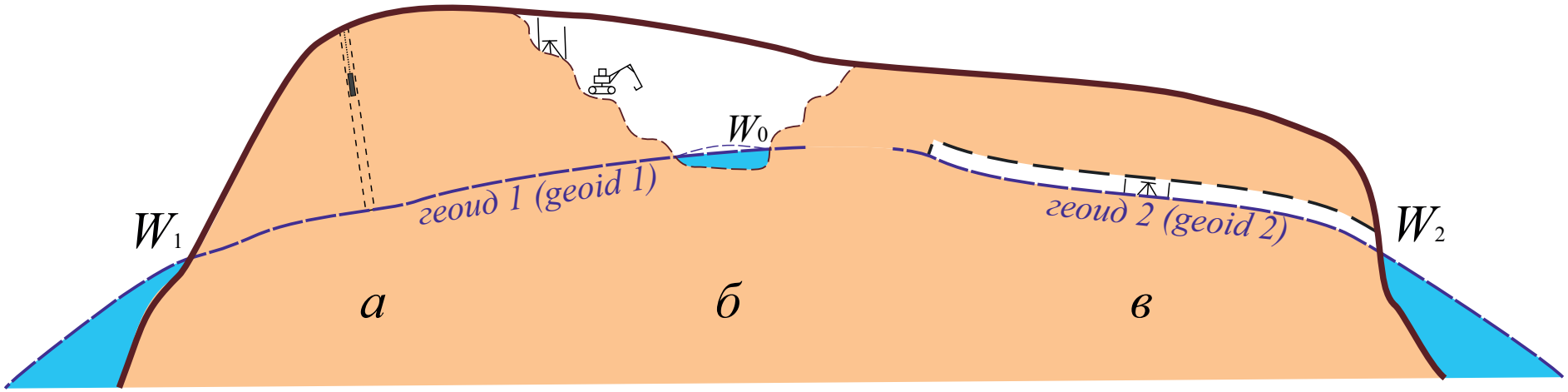

If we select the normal reference surface $U \neq U_0$ near to the [water] level surface under consideration, the **shifted normal-dynamic height** is *practically* constant (the dynamic height is *strictly* constant).

# Orthometric height system

## Common geoid and orthometric height determination

The geoid is an element of the real field ⇒ there are only practical ways to determine geoid:

- "vertical" levelling with the gravimeter on the rope
- quarry with spirit leveling
- strictly horizontal tunnel from the seaside

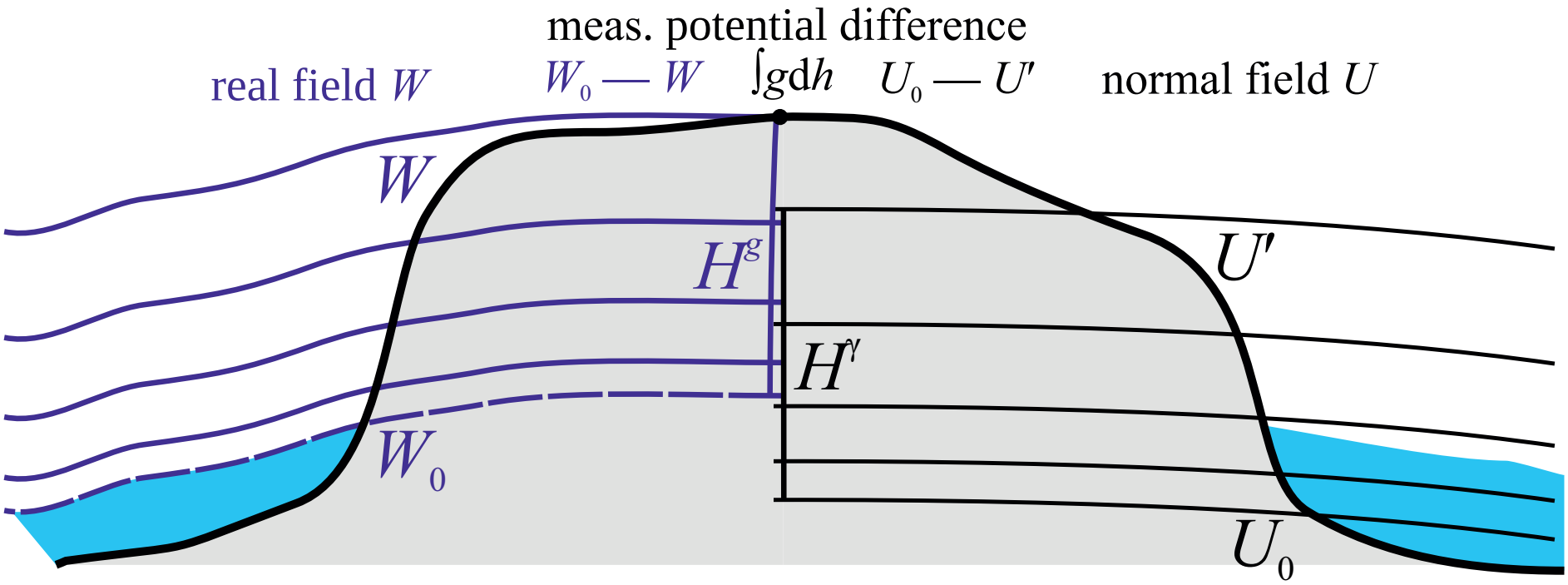



If we know the inner mass distribution within crust and mantle. . . then no BVP has to be solved!

# Normal height system

## Normal height is an "orthometric height in normal field"

"we compute the [curvilinear] coordinate $q$ corresponding to the known potential of the real Earth..., neglecting the disturbing potential and the deflection of the vertical — an obvious first approximation"[1]



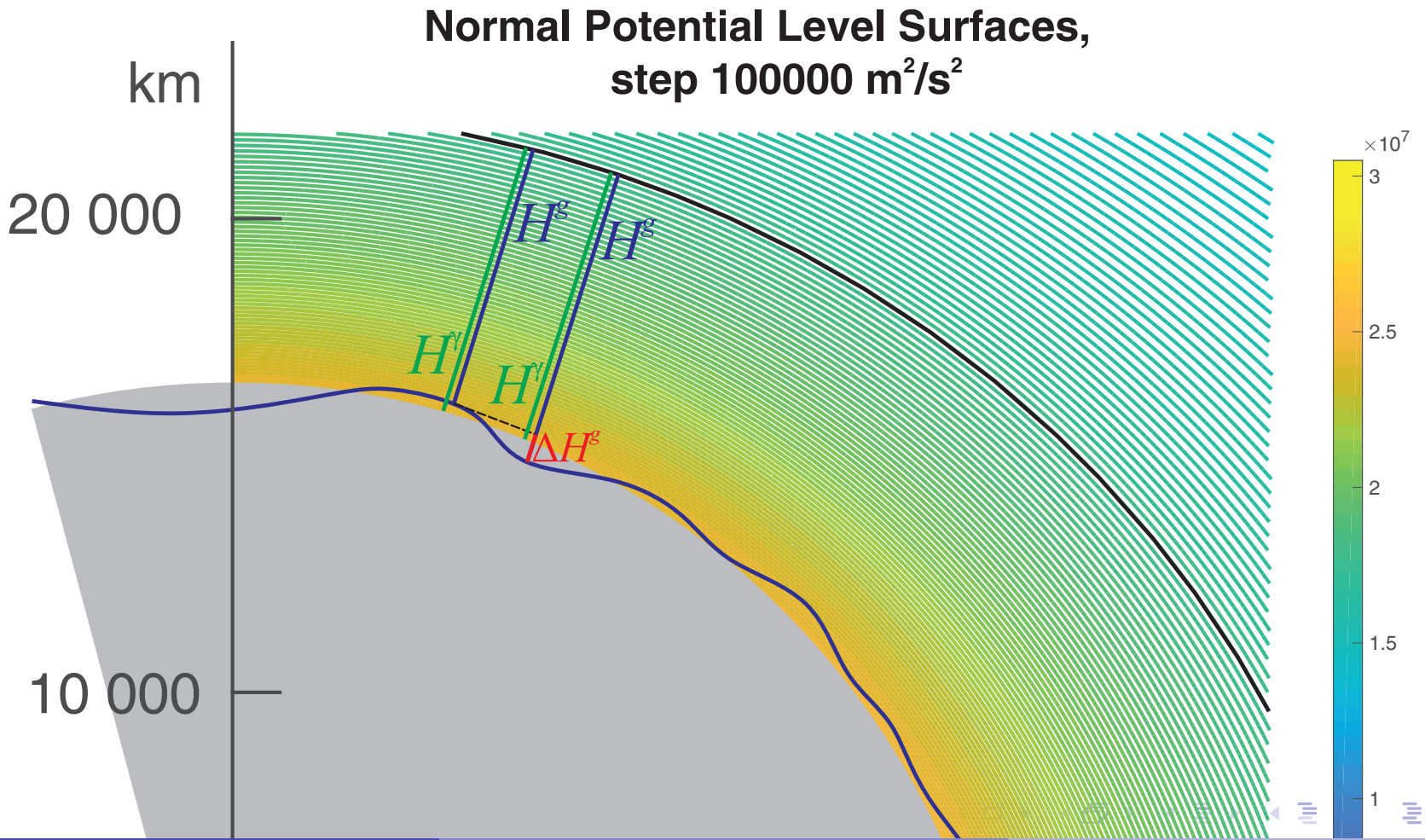

Real geoid in normal field is an ellipsoid, **not quasigeoid**!

[1] see p. 55 of the English translation of [M. S. Molodensky 1945]

# Asymptotic properties...

...at a long distance from the Earth's surface the level surfaces are so smoothed and closer to the normal ones

- normal height tends to geodetic; height anomaly $\to 0$ with $T$
- orthometric height contain a constant part $N = \text{const}$



# Normal height system — 2

Strictly speaking, normal height is a part of the...

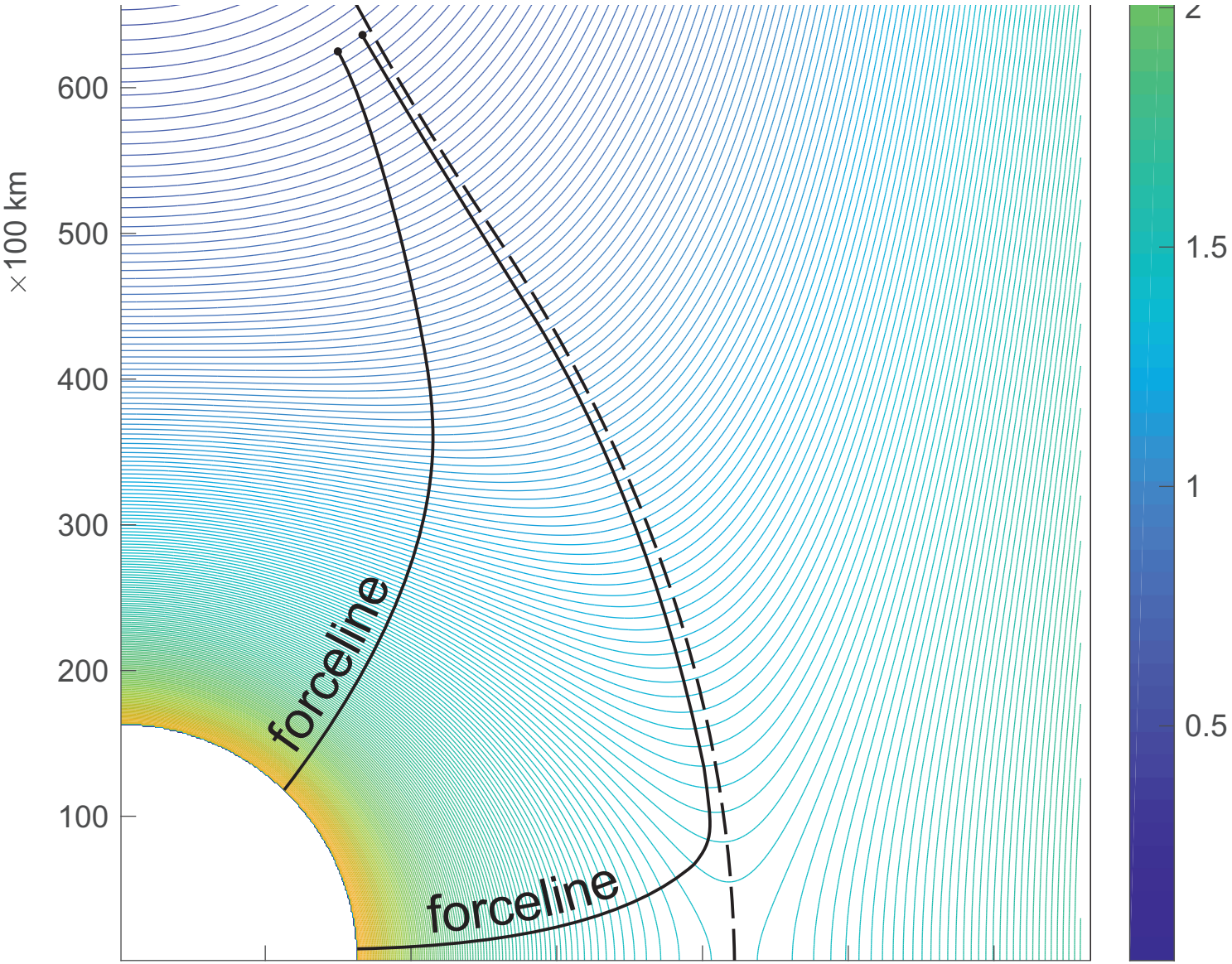
- normal to ellipsoid (it have no physical meaning)
- normal force-line (yes, only physical sense!)
- coordinate line of the spheroidal system (again the coordinate system?)

But at a **very** long distance the normal level surfaces are closer to the equipotential surfaces of the rotating sphere

- the two points on the same equipotential surface have a very different length of the force-line
- the coordinate line of the spheroidal system is more suitable:
- the spheroidal coord. lines have the same properties as the gravitational potential:
- the confocal ellipsoids degradate into spheres as the gravitational potential!

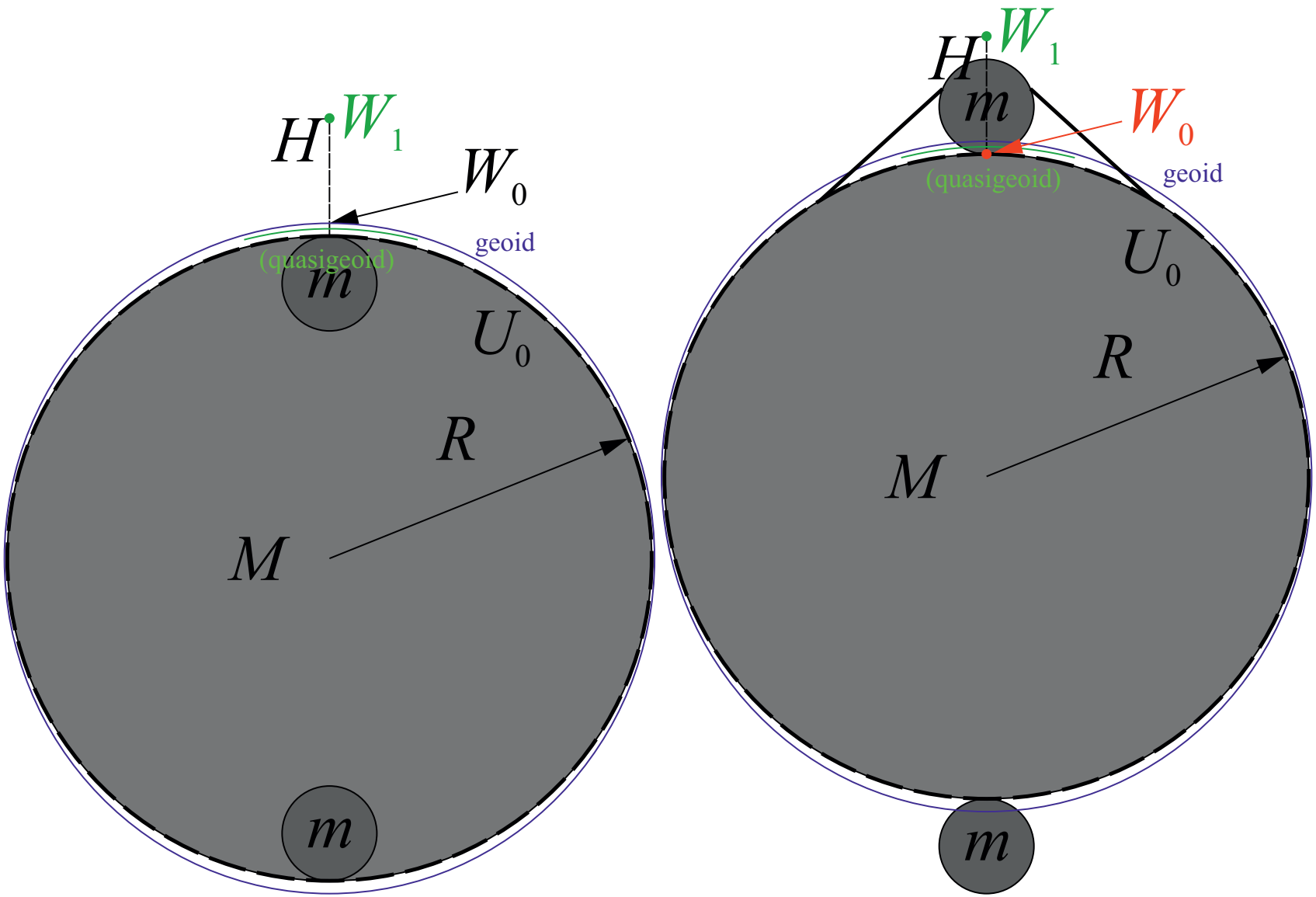
# Asymptotic properties 2

## The forceline is not suitable for the normal height...



# Orthometric and normal height — what is bigger?

Any estimation is possible on the simple models with known mass distribution!



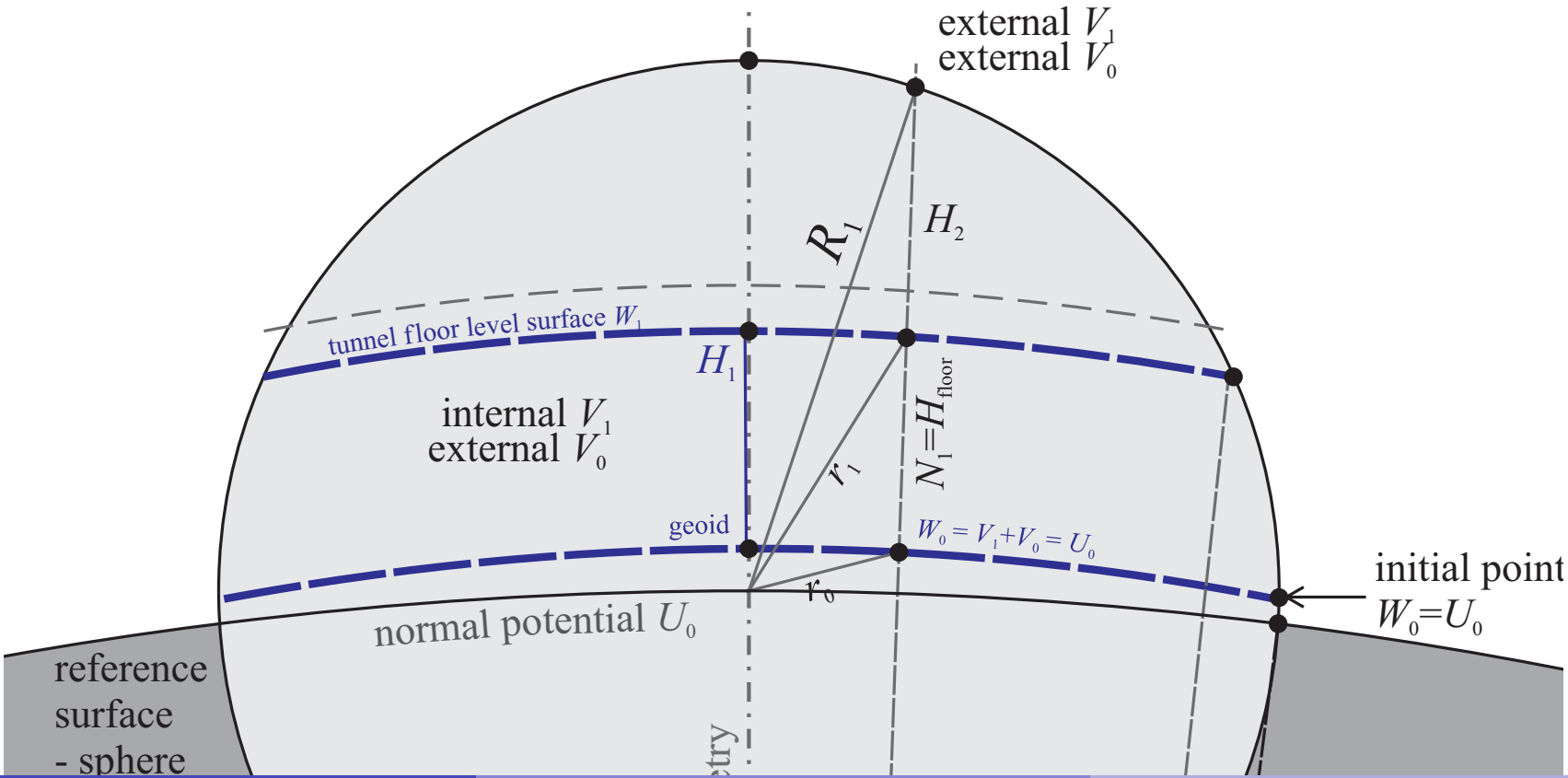
# Orthometric and normal height — what is bigger?

On the simple models we can make sure that normal height is usually bigger!

The usual formula for **geoid-quasigeoid separation is incorrect in sign**

$$\zeta - N = H^g - H^\gamma = -\frac{g^m - \gamma^m}{g^m} H^\gamma \not\approx -\frac{\Delta g_{\mathrm{B}}}{\gamma} H$$

The quasigeoid is usually under the geoid.



# Conclusions

## Results 1

1. **Geoid** as an element of the real field is observable, and could be determined only practically by the mechanic way with the **orthometric heights**.
2. The calculation of them is impossible due to the ignorance of the inner Earth's mass distribution and the difficulties in volume integrals computation.
3. The precisely calculated orthometric height is non-optimal on the true level surfaces, they differs mich more than the normal ones. It could be checked on the very simple Earth's models.
4. The *geoid-quasigeoid separation* formula must have another sign:

$$\zeta - N = H^g - H^\gamma \not\approx -\frac{\Delta g_{\mathrm{B}}}{\gamma} H$$

5. **Quasigeoid is not the analogue of the geoid**, Molodensky used the reference ellipsoid as a "normal geoid". The quasigeoid term was used only for illustration.

# Conclusions

## Results 2

1. The **quasigeoid term is excessive** in the theoretical aspect, since we don't have any reason to count the height anomaly from the ellipsoid or any other reference surface.
2. **Normal height** and **height anomaly** could not be measured but only calculated from the observations on the physical surface.
3. The normal-type heights must be related to the **coordinate line of the curvilinear system (spheroidal)**, not with the length of the **normal force-line**.
4. On the real level surface (when the dynamic height is constant) the **normal-dynamic height** is practically constant, if we do select the normal reference surface near to the level surface under consideration.
5. Normal heights are strictly related to the solution of the geodetic BVP in various interpretations.
6. **A quasigeoid is not a "vertical reference surface"**, what with the usual heights being counted from the real points.

# Thank you very much for your attention!

P. S.

A quasigeoid is not a “vertical reference surface”
“We don’t need in quasigeoid itself”
Some possible visual interpretation of the quasigeoid: