A Discontinuous-Galerkin approach to model non-classical nonlinearity observed from lab to global scales

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Purpose

A mathematical model that can account for the change in velocity as elastic wave propagates through rocks or soils.

A numerical scheme that can help predict the effects of nonlinearity.

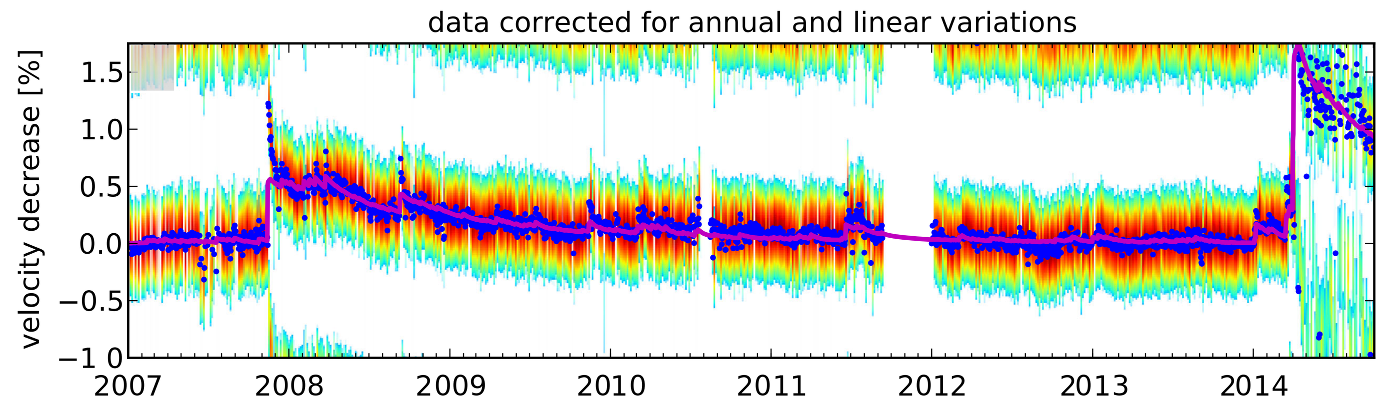


Figure 1: Modeled and observed velocity variations at station PATCX based on 10–15 s lag time in a frequency range of 4–6 Hz, recorded over 8 years in Chile[1].

Methods

Nonlinear model

$$\frac{\partial \varepsilon}{\partial t} + \frac{\partial v}{\partial x} = 0$$

$$\frac{\partial v}{\partial t} + \frac{1}{\rho}\frac{\partial\left((1-g)\cdot\sigma(\varepsilon)\right)}{\partial x} = 0$$

$$\frac{\partial g}{\partial t} = \left(\frac{W(\varepsilon)}{\gamma\cdot\tau_{min}} - \phi_2'(g)\right)$$

$$\sigma(\varepsilon) = E\varepsilon\left(1+\beta\varepsilon+\delta\varepsilon^2+o(\varepsilon^2)\right)$$

$$\phi_2'(g) = \frac{g}{\tau_{max}\left[\frac{g}{g_0}\ln\frac{\tau_{max}}{\tau_{min}}\left(1-\log\left(\frac{g}{g_0}\right)\right)+1\right]e^{-\ln\frac{\tau_{max}}{\tau_{min}}\frac{g}{g_0}} + \tau_{min}\frac{g}{g_0}\cdot\ln\frac{\tau_{max}}{\tau_{min}}}$$

Internal Variable Model[2] with adjustable log scale recovery

Discretization

1D discontinuous Galerkin (DG) method.

2 stage Runge-Kutta method for time integration.

Roe solver for the boudary flux of inner cell.

Extended logrithmic recovery in time

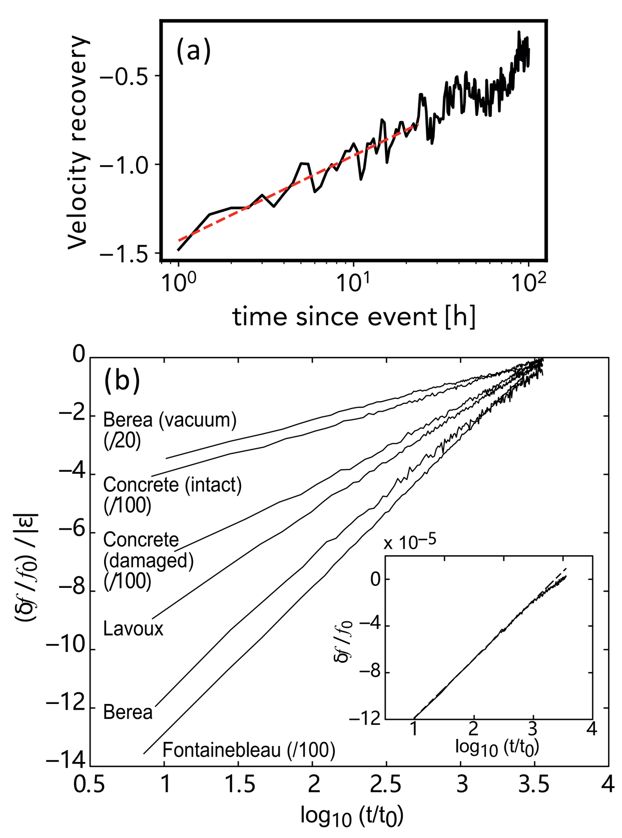


Figure 2: (a) Recovery of the velocity field during the aftershocks of the Gorkha Earthquakes, 2015[3]; (b) Time-dependent shift df of the recovering resonant frequency, normalized by the asymptotic value f0, per unit conditioning strain[4].

Observations → Physical Model → Implementation in Wave Equations

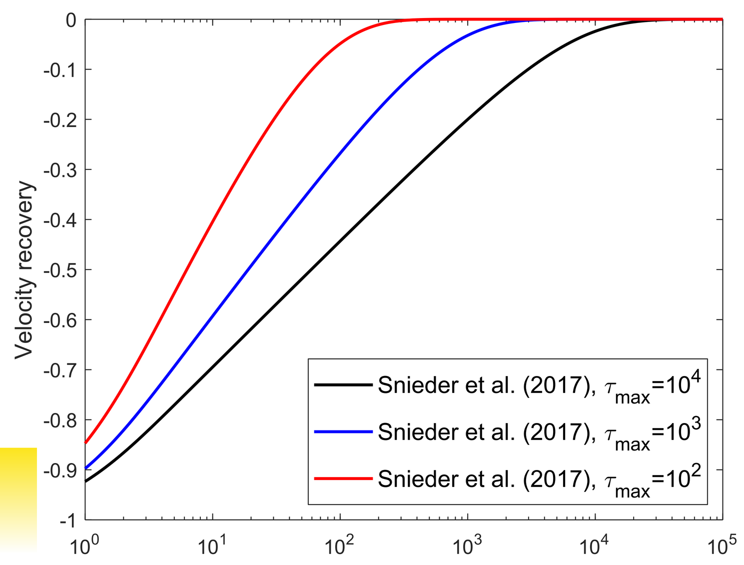


Figure 3: A model for log-time relaxation proposed by Snieder et al. (2017)[5], which has the ability of accounting for the variation in the time scale of the relaxation.

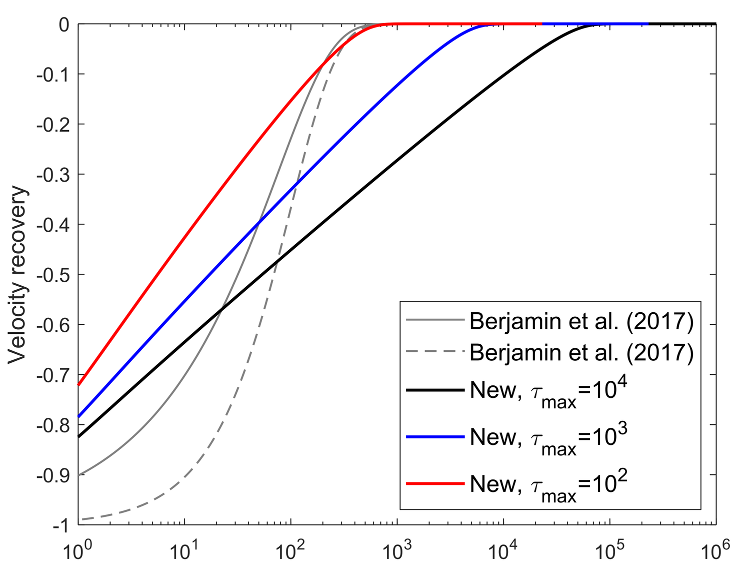


Figure 4: Simulation results of the nonlinear models proposed in this poster based on the equations shown in the method section.

Model classical nonlinearity, hysteresis and slow dynamics

Simulate the damage and healing of the materials as wave propagate through the materials.

Tunable logrithmic time scale recovery of the damage.

Effects of nonlinear parameters

Amplitudes of high-order harmonics increase as wave propagates further away from the source.

2nd order nonlinearity and slow dynamics only generate odd harmonics; while 1st order nonlinearity generates all harmonics.

Downward shifting of the frequencies due to slow dynamics.

Counteractive effects of hysteresis in attenuation and harmonic generation.

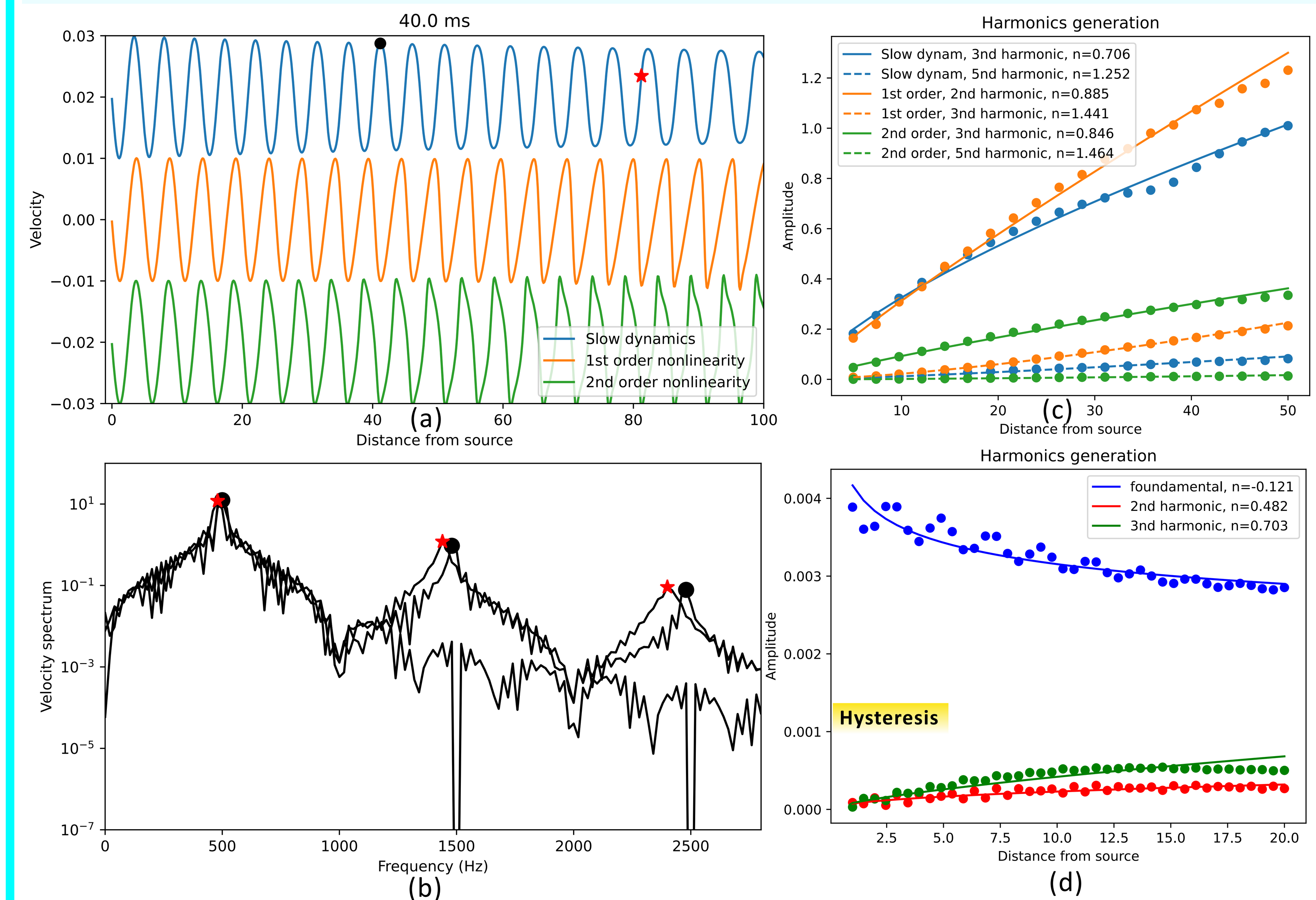


Figure 5: 1D simulation results of the nonlinear model. (a) Changes in the shape of the waveforms as the wave propagates further away from the source due to different forms of nonlinearity; (b) Generation of harmonics from a single frequency source due to nonlinearity; (c) Magnitudes of different orders of harmonics as a function of propagation of distance. The dots are directly derived from simulation with curves displaying polynomial fitting results; (d) Generation of harmonics with the MPII model[6] for comparison. Magnitudes of different orders of harmonics as a function of propagation of distance. The dots are directly derived from simulation with curves displaying polynomial fitting results.

Conclusions

Developed a 1D DG solver for nonlinear wave propagation.

Proposed a method to implement tunable logrithmic time scale recovery of damage in wave propagation.

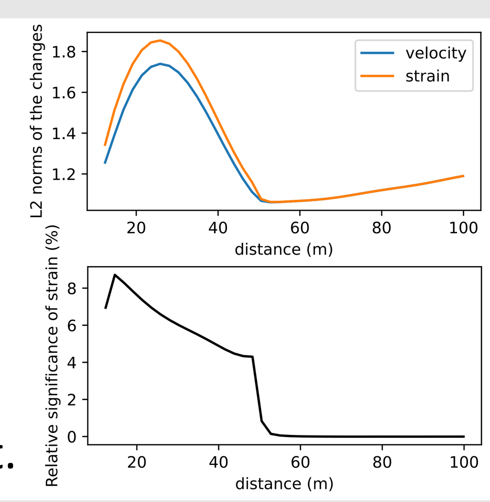
Studied the effect of different nonlinear components on the waveforms in 1D setting.

Outlook

Which part of the Earth will nonlinear strain become important?

Simulate the aftershock wave propagation in the perturbed rheology by the main shock.

Invert for the nonlinear parameters with 12C measurement.



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Funded by the European Union's Horizon 2020 research and innovation programme