

Observationally calibrating snow-on-sea-ice model free parameters and estimating uncertainties using a Markov Chain Monte Carlo method

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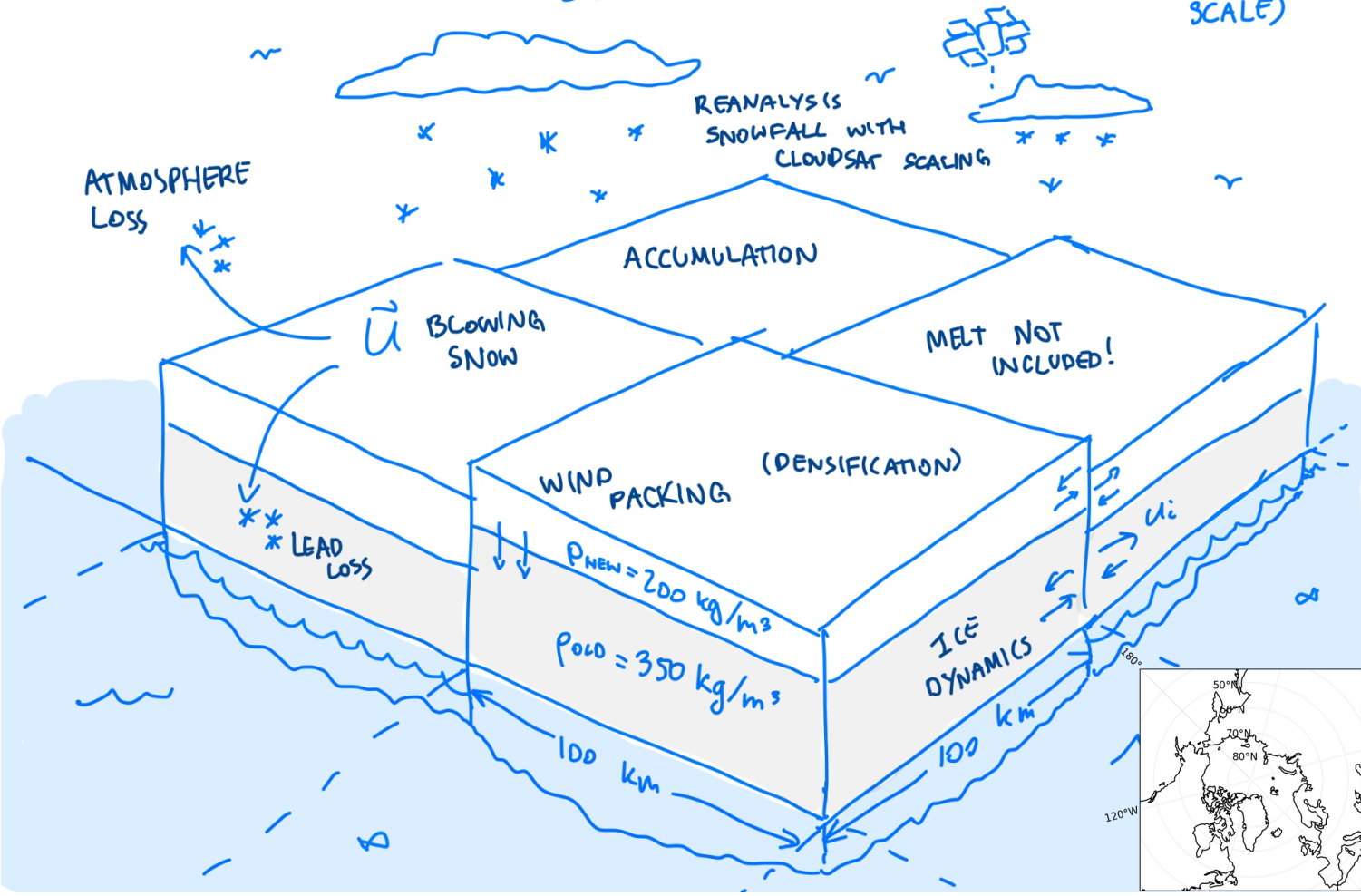
This study was supported by a grant from the Canadian Space Agency's Earth System Sciences Data Analyses fund.

Background: Snow on Arctic sea ice poses many challenges

- Direct, in situ measurements tend to be infrequent and/or sparse
- Remote observations can be highly uncertain
- Warren (1999) climatology still commonly in use, but with a modification over first-year ice
- Models can help address the observation gap, but how can these models be calibrated to existing observations?

NESOSIM: NASA EULERIAN SNOW ON SEA ICE MODEL (NOT TO SCALE)

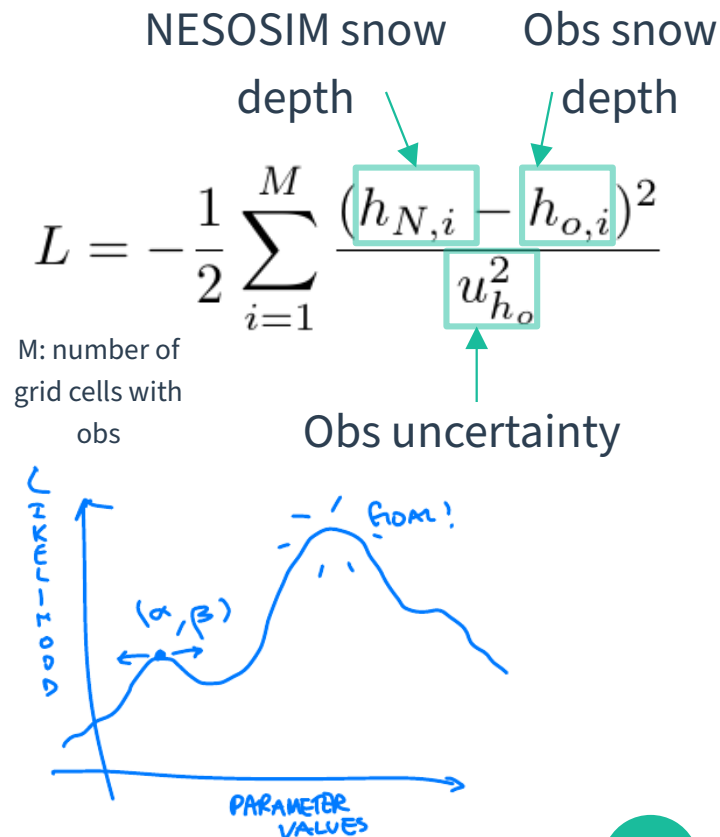
(A. PETTY)



- NESOSIM Version 1.1, <https://zenodo.org/record/4448356>
- Simple 2-layer model, 100x100 km resolution
- Designed for SIT retrievals from ICESat-2 lidar observations (Petty et al., 2020)
- ECMWF ERA5 reanalysis snowfall input constrained to CloudSat monthly climatology (Cabaj et al., 2020)
- Runs from Sept 1st to April 30th
- Goal: Calibrate wind packing and blowing snow parameters

Metropolis Markov Chain Monte Carlo (MCMC) algorithm

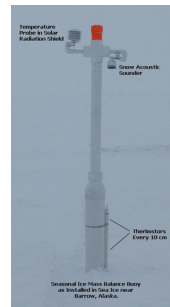
- Goal: find parameter values α which maximize the (log-)likelihood function, L
- Start with prior parameters and corresponding log-likelihood
- For each iteration :
 - Obtain new parameters a small step from previous value
 - Calculate new log-likelihood function L_{i+1}
 - Accept new values if $L_{i+1} - L_i \geq U(0,1)$ (a log-uniform value; helps avoid local maxima)
- The distribution of accepted parameters approximates the posterior distribution of α



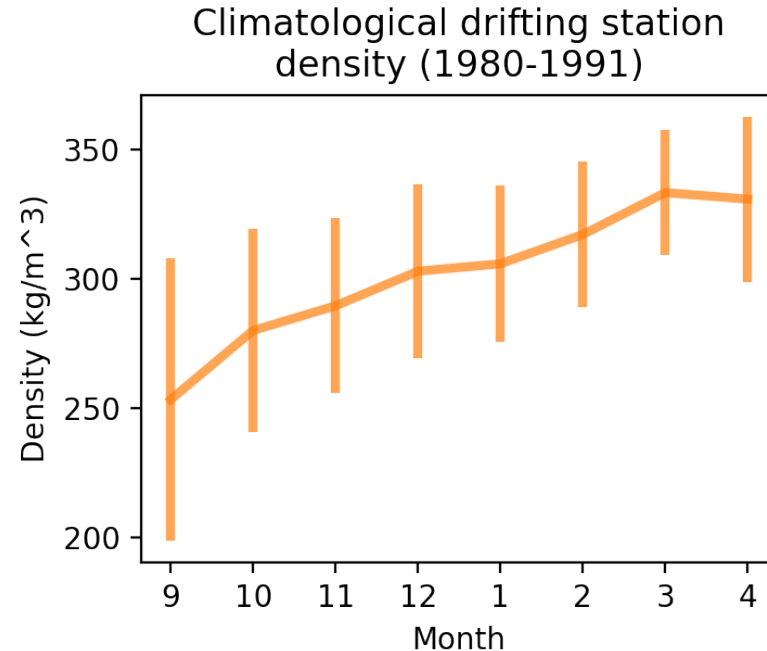
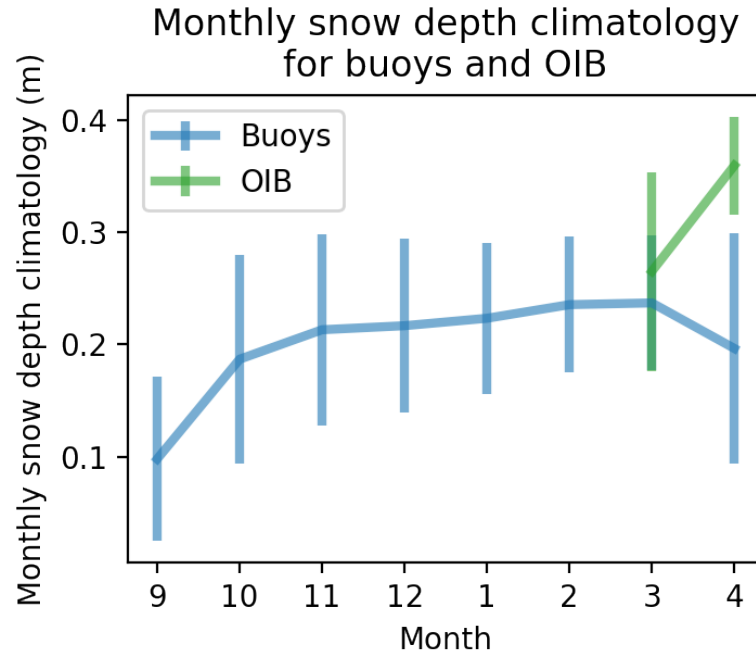
Measurements used for NESOSIM MCMC calibration



Data source	Data used	Years used	Notes	References
NASA Operation IceBridge (OIB)	Snow depth on sea ice from airborne snow radar	2010-2015	Median of SRLD, GSFC, and JPL products used Only available in March and April	Koenig et al., 2016; Kurtz et al., 2013; Kwok et al., 2017
Soviet drifting stations	Snow density on sea ice from in situ obs	1980-1991	Historical measurements; monthly climatology used	Radionov et al., 1997
CRREL- Dartmouth snow buoys	Snow depth on sea ice from acoustic sounders	2010-2013	Using monthly climatology of daily-averaged measurements	Perovich et al, 2021



Measurements used for NESOSIM MCMC calibration: plots



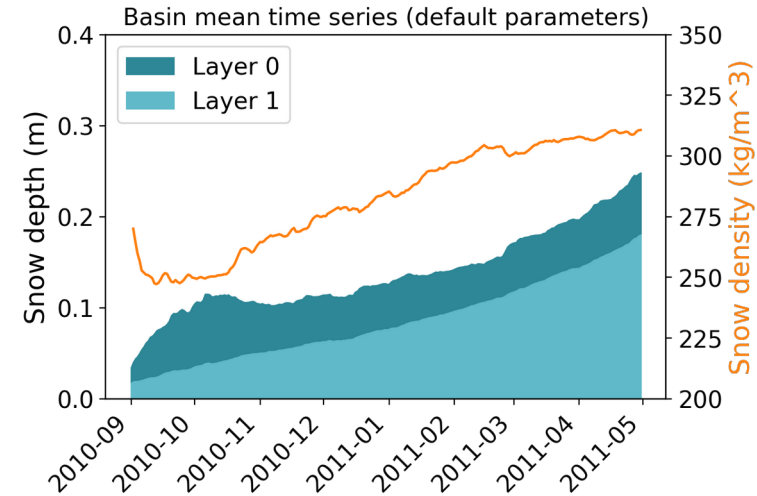
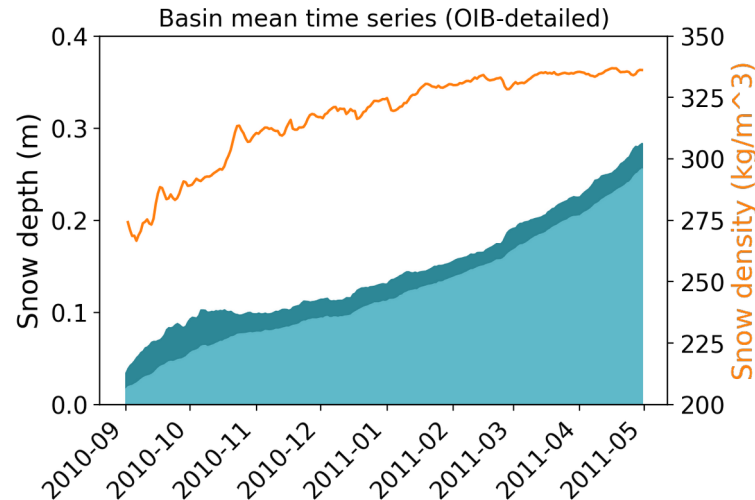
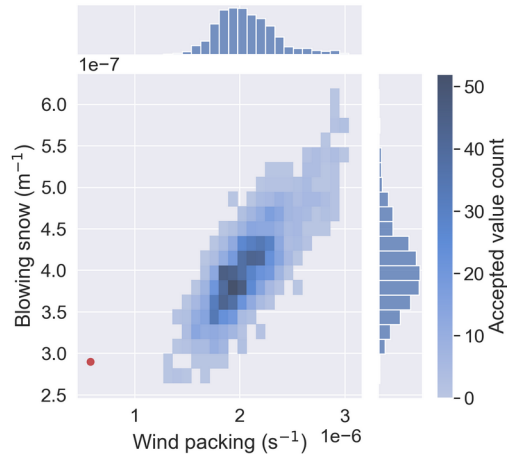
MCMC optimization results: similar depth, higher density

OIB gridded depth

Drifting station density
climatology

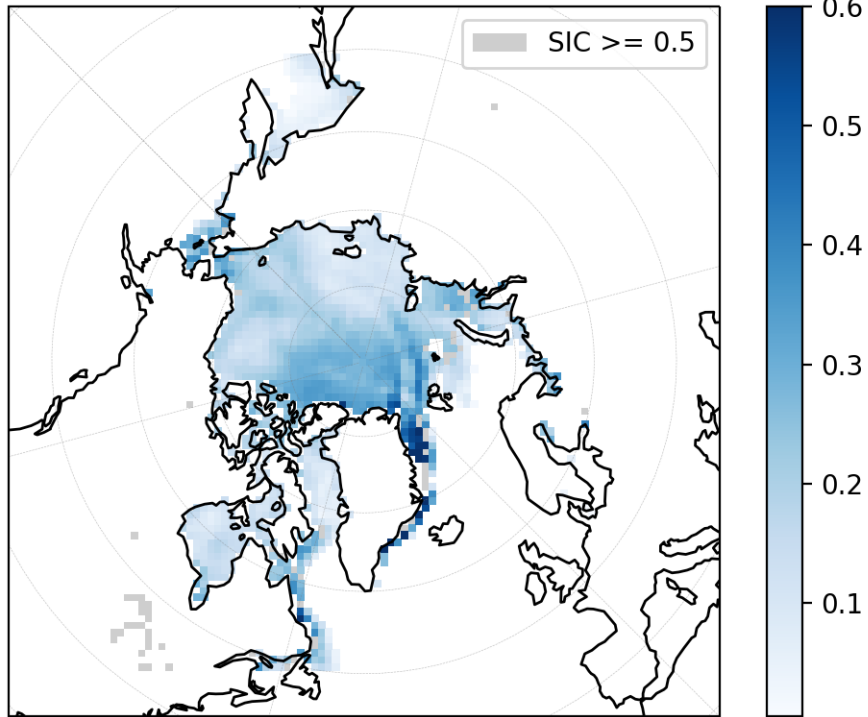
Snow buoy depth
climatology

$$L_{grid} = -\frac{1}{2} \sum_{i=1}^M \frac{(h_{N,i} - h_{o,i})^2}{u_{h_o}^2} - \frac{1}{2} \sum_{j=1}^8 \frac{(\langle \rho_{N,j} \rangle - \langle \rho_{d,j} \rangle)^2}{\langle u_{\rho_{d,j}} \rangle^2} - \frac{1}{2} \sum_{k=1}^8 \frac{(\langle h_{N,k} \rangle - \langle h_{b,k} \rangle)^2}{\langle u_{h_{b,k}} \rangle^2}$$

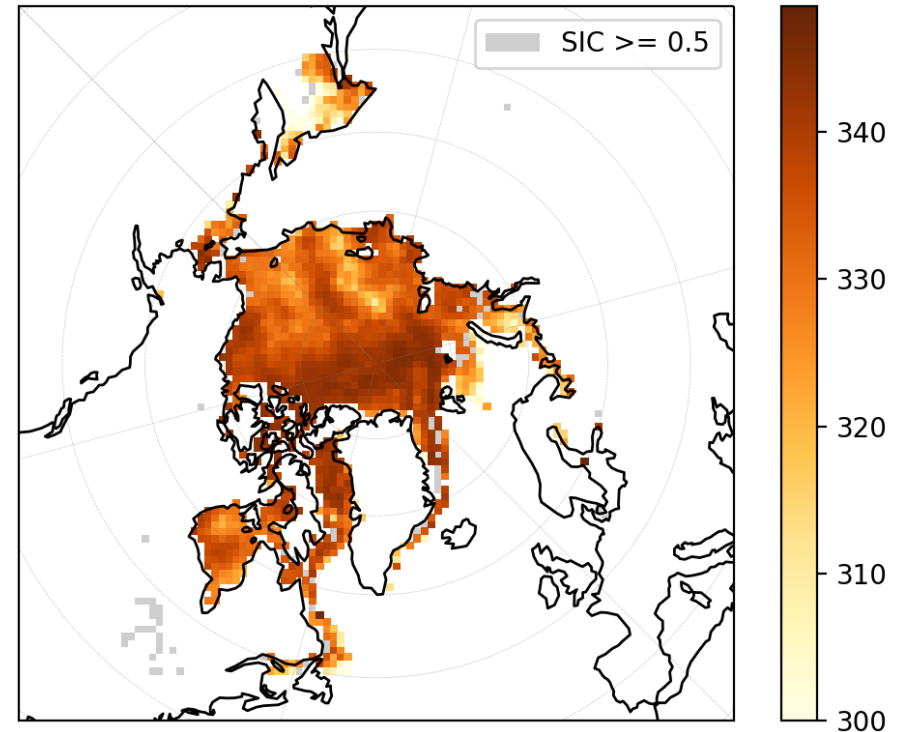


Snow depth and density maps for MCMC-optimized configuration

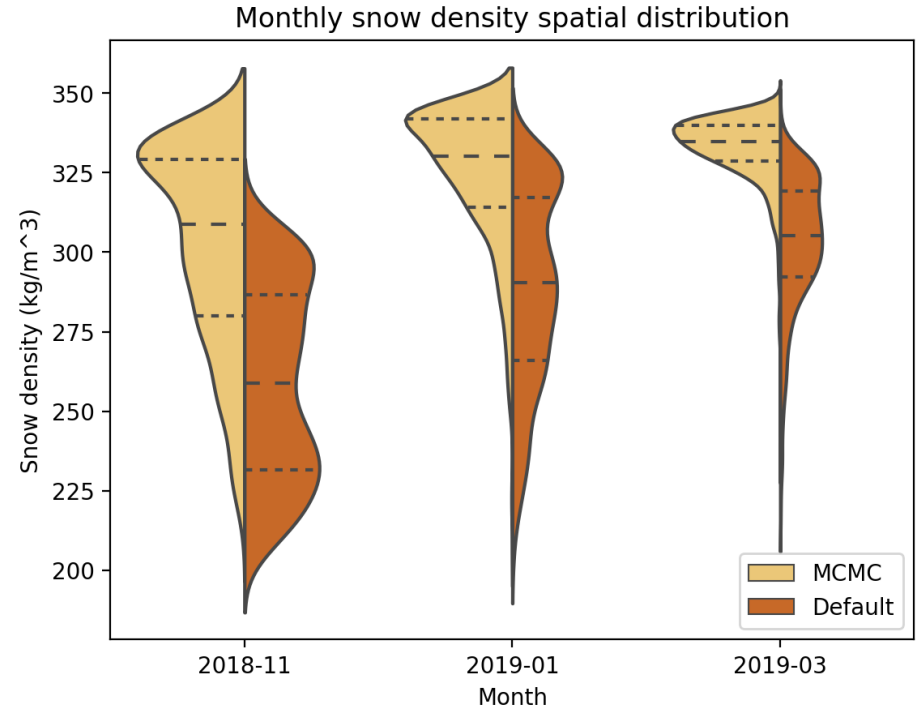
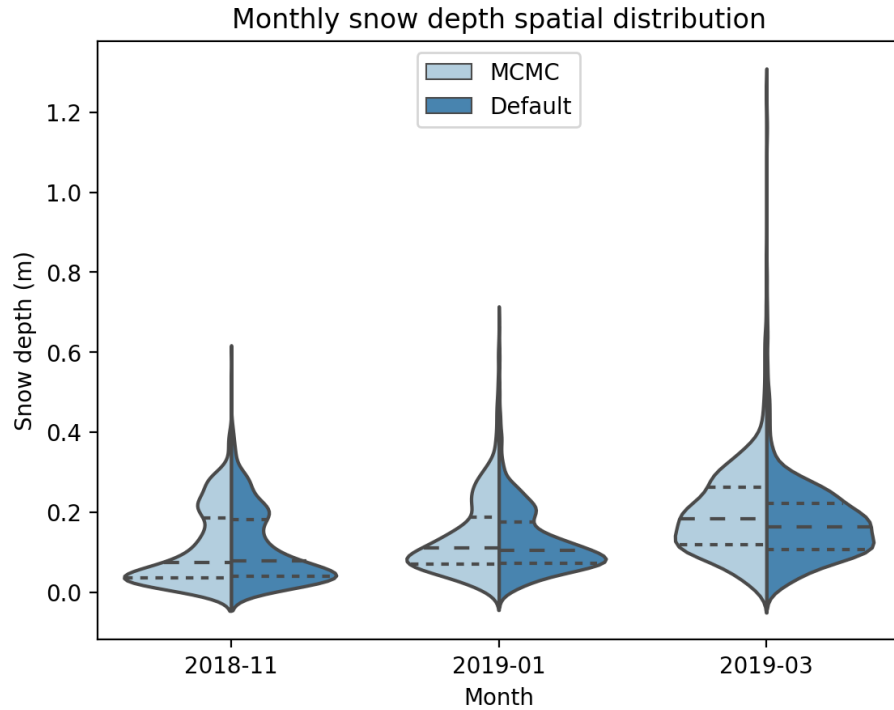
NESOSIM-MCMC snow depth for 2019-03 (m)



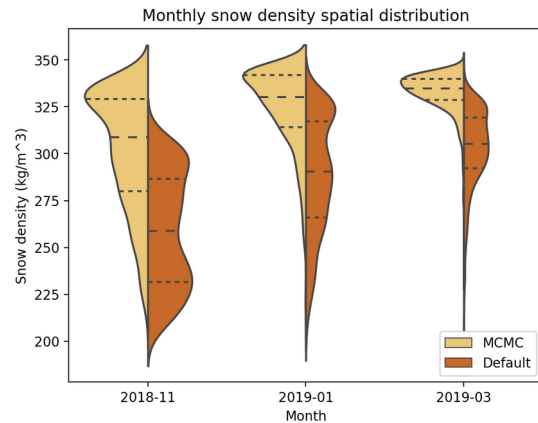
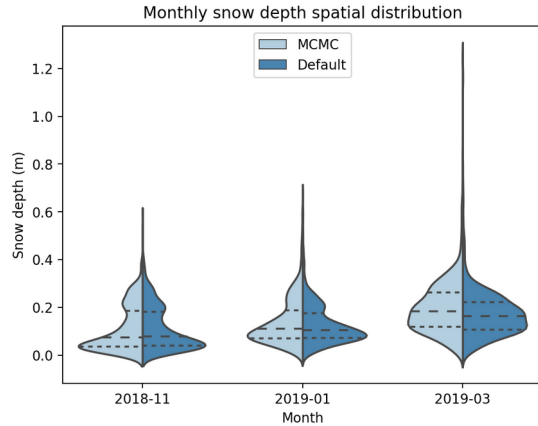
NESOSIM-MCMC snow density for 2019-03 (kg/m^3)



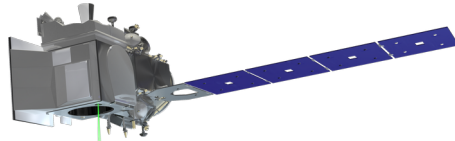
Some differences in snow depth, large differences in density relative to model default



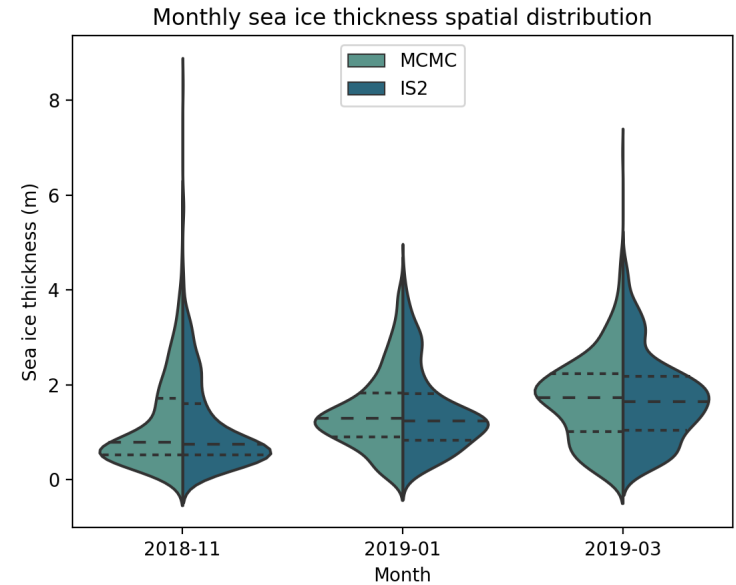
Despite differences in snow loading, resulting ice thickness distributions broadly agree



NASA ICESat-2

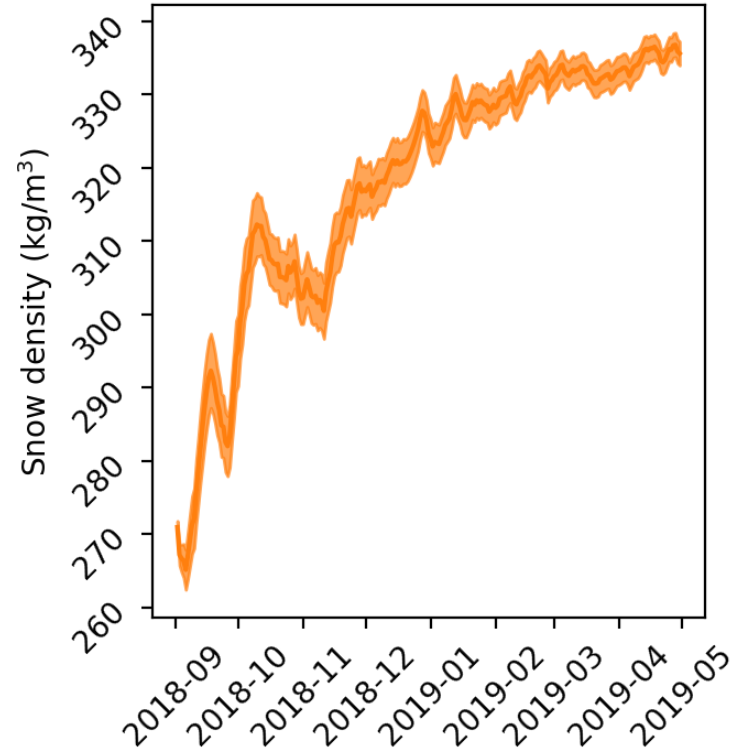
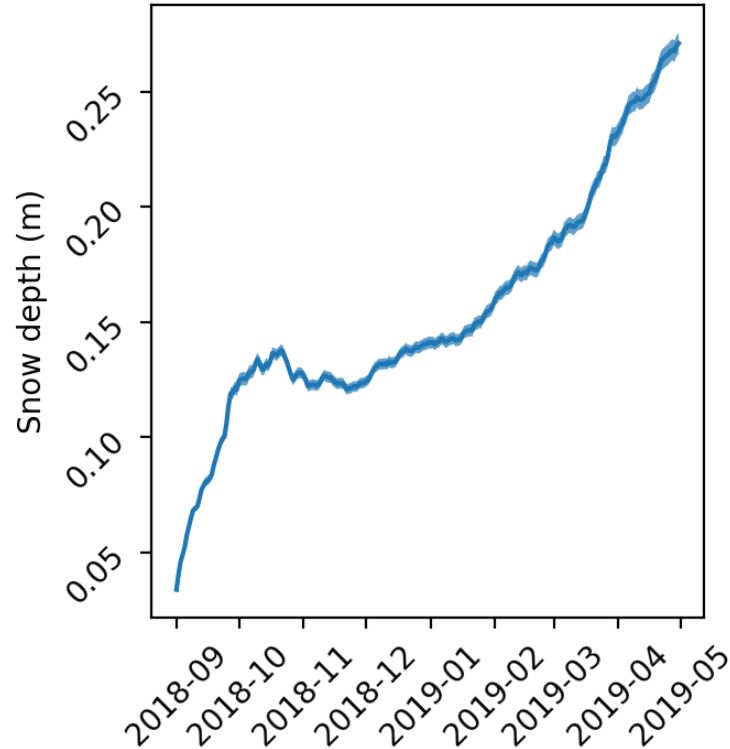


$$h_i = \frac{h_f \rho_w}{\rho_w - \rho_i} + \frac{h_s (\rho_s - \rho_w)}{\rho_w - \rho_i}$$

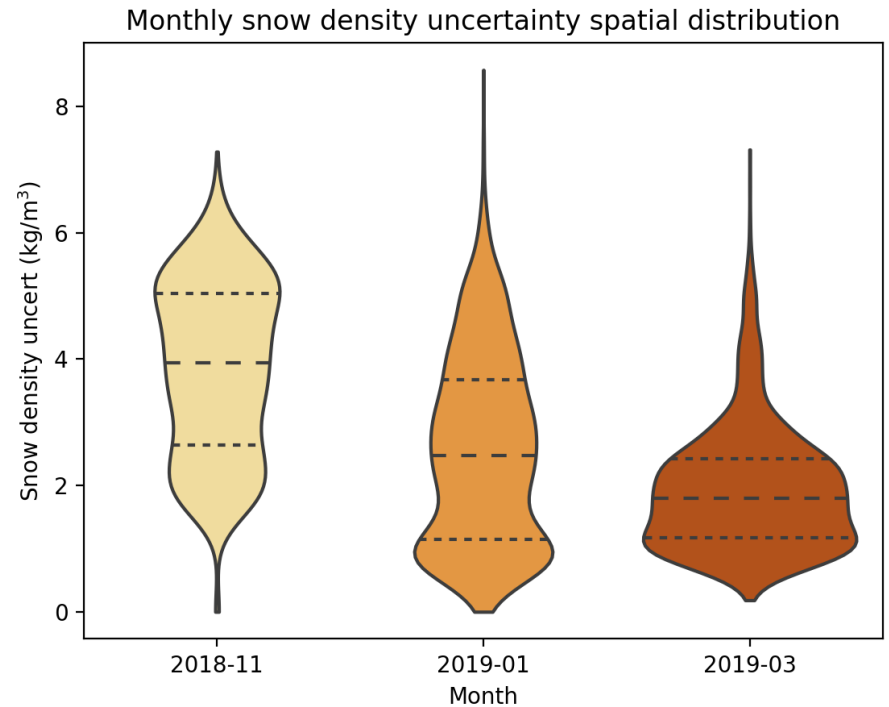
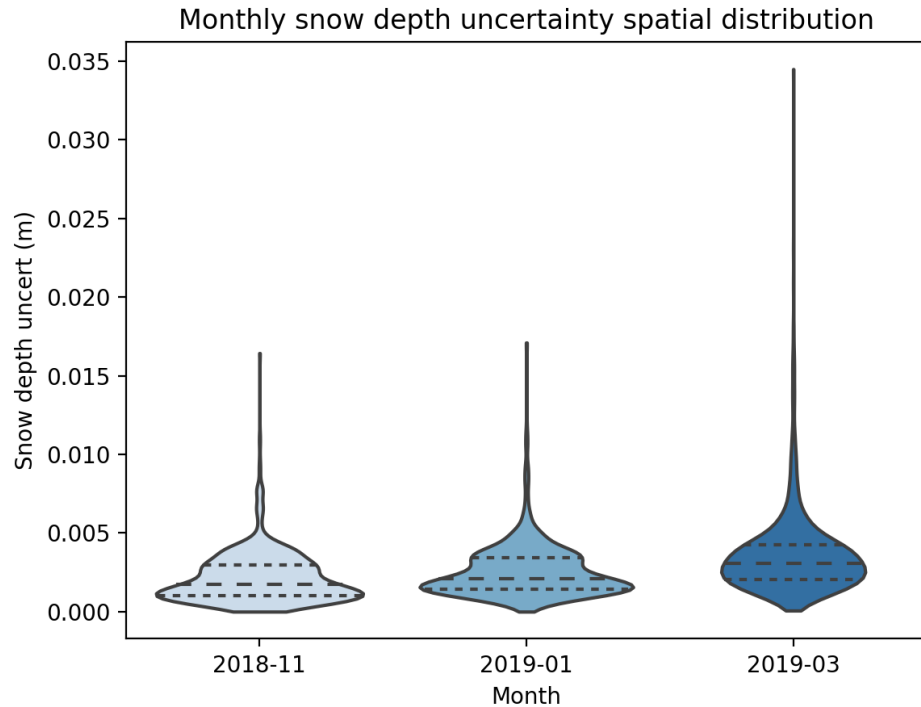


Uncertainties in snow depth and density due to parameter uncertainties are relatively small

Basin averages with uncertainty for NESOSIM-MCMC snow

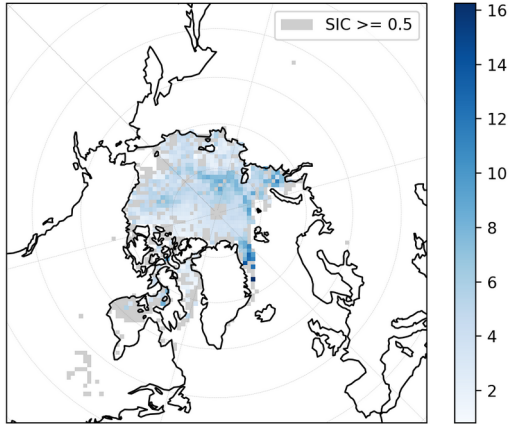


Over the course of a season: increasing snow depth uncertainty, decreasing snow density uncertainty

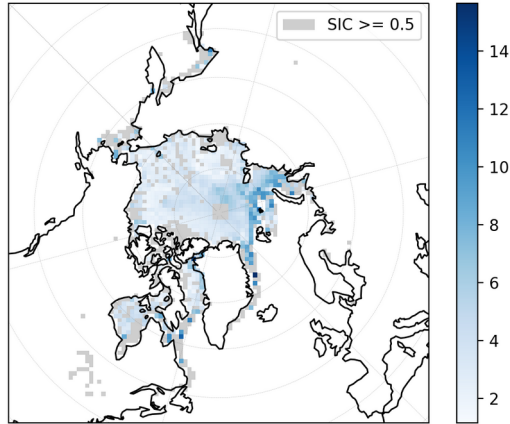


The MCMC snow uncertainty contribution to ice thickness uncertainty is up to 16% of total ice thickness uncertainty

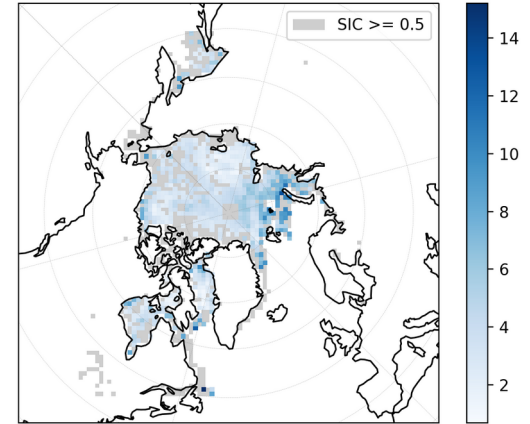
Snow uncertainty contribution to total uncertainty (%) for 2018-11



Snow uncertainty contribution to total uncertainty (%) for 2019-01



Snow uncertainty contribution to total uncertainty (%) for 2019-03



(Basin average: 3-4%)

Summary

- NESOSIM free parameters were calibrated to airborne and in situ measurements using a Markov Chain Monte Carlo process
- Produces estimates of snow depth and density uncertainties due to parameter uncertainty
- Small snow uncertainties can contribute to a considerable fraction of ice thickness uncertainty
- More widespread snow-on-sea-ice measurements needed, especially for density

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