Intense Downwelling and Diffuse Upwelling in a Nonlinear Ekman Layer

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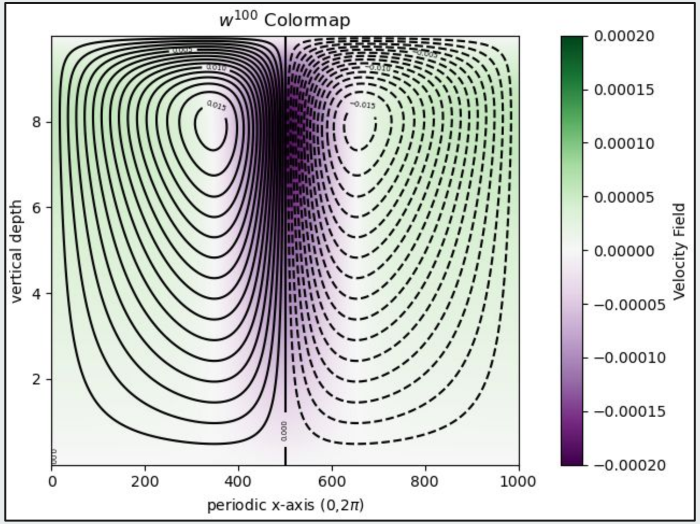


- Nonlinearity includes Ekman-geostrophic and Ekman-Ekman interactions
- Ekman-Ekman interactions can lead to strong asymmetry in the pumping velocities
- Ekman-Ekman nonlinearity can be large when forcing length scales are small (eg, sea ice leads)
- We solve the steady problem in a simple geometry

$$u = u_g + u_e$$

Task of project: Investigate E-E nonlinearity by assuming a z-independent geostrophic flow in order to explicitly solve the x-z nonlinear Ekman layer problem

Steady State Equations

Comes from Navier-Stokes (in x-z)

$$\vec{u}\cdot\nabla\vec{u} + f\hat{z}\times\vec{u} = -\nu\nabla^2\vec{u}$$
$$\nabla\cdot\vec{u} = 0$$
$$\partial_y \longrightarrow 0$$

Curl is taken and put into component form

$$\underbrace{uu_x + wu_z}_{\text{Non-linear}} - fv = -\phi_x + \nu\left(u_{xx} + u_{zz}\right)$$
$$\underbrace{uv_x + wv_z}_{\text{Non-linear}} + fu = \nu\left(v_{xx} + v_{zz}\right)$$
$$\underbrace{uw_x + ww_z}_{\text{Non-linear}} = -\phi_z + \nu\left(w_{xx} + w_{zz}\right)$$

velocity (u, v, w)

relative vorticity ζ:

$$\zeta = \nabla^2\psi = u_z - w_x$$

$$\nu\nabla^2 v - fu = J(\psi,\zeta)$$
$$\nu\nabla^2\zeta + fv_z = \vec{u}\cdot\nabla v$$
$$= J(\psi, v)$$

nonlinear

$$\nu\nabla^2 v - fu = 0$$
$$\nu\nabla^2\zeta + fv_z = 0$$

linear

Note: Superscript denotes order number except for on gradient operator

Method for Solving Equations

Order 0

$$\nu \nabla^2 v - fu = 0$$
$$\nu \nabla^2 \zeta + f v_z = 0$$

Adds fourier mode →

Order 1

$$\nu \nabla^2 v^0 - fu^0 = J(\psi^0, \zeta^0)$$
$$\nu \nabla^2 \zeta^0 + f v_z^0 = J(\psi^0, v^0)$$

Adds fourier mode →

Order 2

$$\nu \nabla^2 v^1 - fu^1 = J(\psi^1, \zeta^0) + J(\psi^0, \zeta^1)$$
$$\nu \nabla^2 \zeta^1 + f v_z^1 = J(\psi^1, v^0) + J(\psi^0, v^1)$$

... →

Order N

$$\nu \nabla^2 v^{N-1} - fu^{N-1} = J^N(\psi, \zeta)$$
$$\nu \nabla^2 \zeta^{N-1} + f v_z^{N-1} = J^N(\psi, v)$$

where

$$J^N := \sum_{i=0}^{N} J\left(\psi^i, \zeta^{N-i}\right)$$

→

Solutions

$$V^N = v^0 + v^1 + \ldots + v^N$$
$$\Psi^N = \psi^0 + \psi^1 + \ldots + \psi^N$$

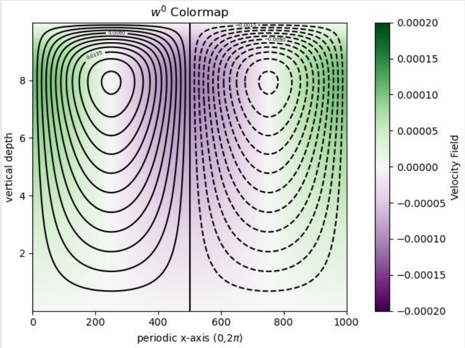
...

(Sum of order N correction terms)

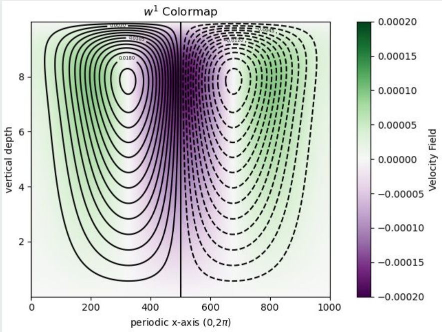
Vertical Velocity in at Ekman-Rossby Number >1

$R_e = U_e / fL$

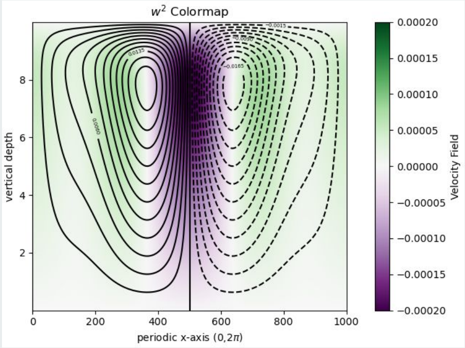
Ekman velocity (U_e), Coriolis (f), Length scale (L)



Linear solution

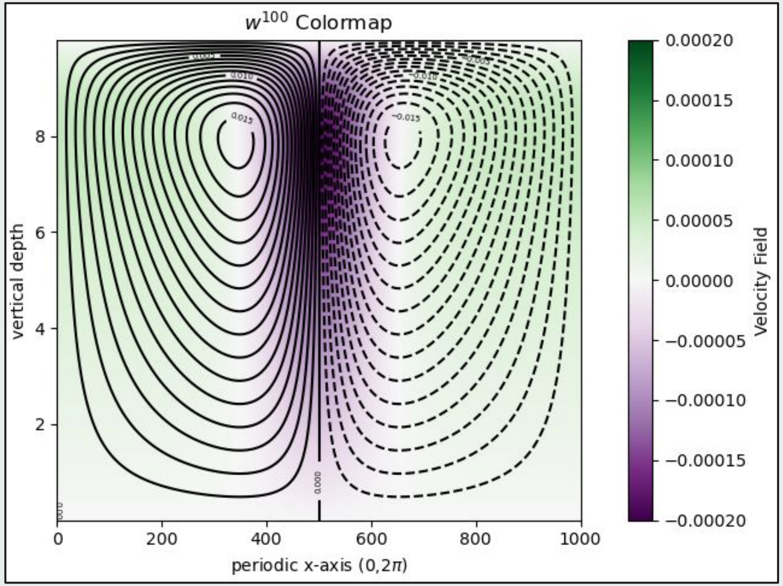


First order correction



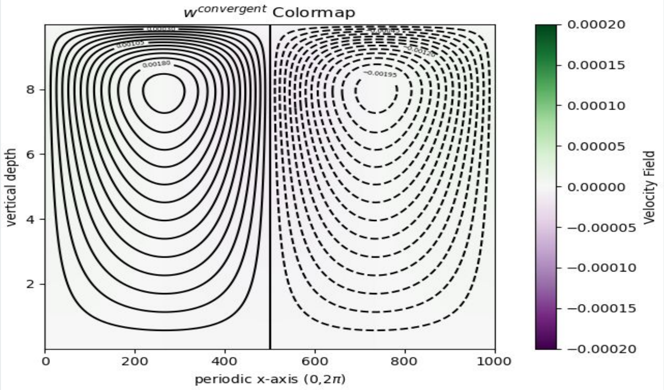
Second order correction

...

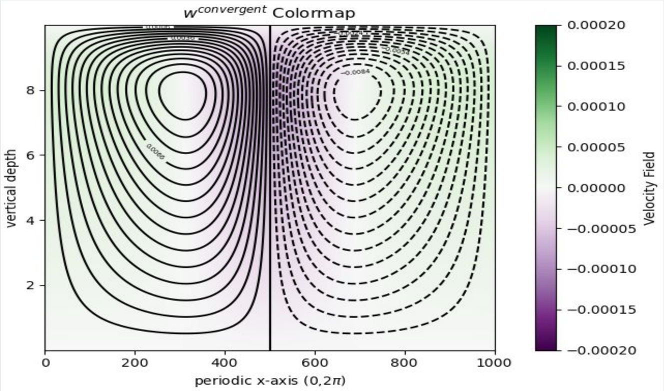


100th order correction (convergent solution)

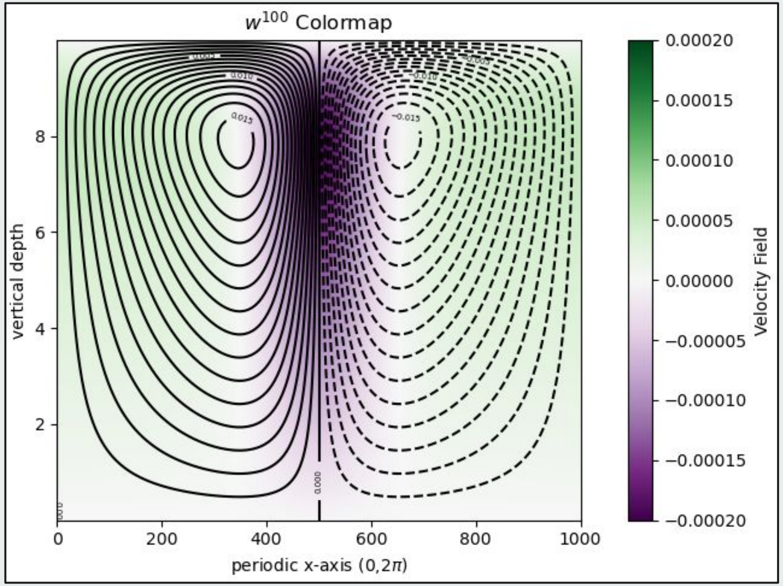
Vertical Velocity under different Nonlinearity Regimes $R_e = U_e / fL$



Ekman-Rossby number <<1



Ekman-Rossby number close to but <1



Ekman-Rossby number >1

Summary of key findings

- Finding: To an Nth order approximation the Ekman-Ekman nonlinear interactions are a main contributor to the intense downwelling upwelling phenomena when Rossby numbers approach and go higher than 1

- Application: Ice-ocean stress field across sharp gradients such as leads in the sea ice cover Rossby number based on the Ekman velocity and horizontal length scale approaches order one values

- Other research: counter-intuitively turbulent fluxes under active leads are higher when we have downwelling than upwelling (Bourgault et al).