Automatic Picking of Teleseismic P- and S-Phases Using an Autoregressive Prediction Approach

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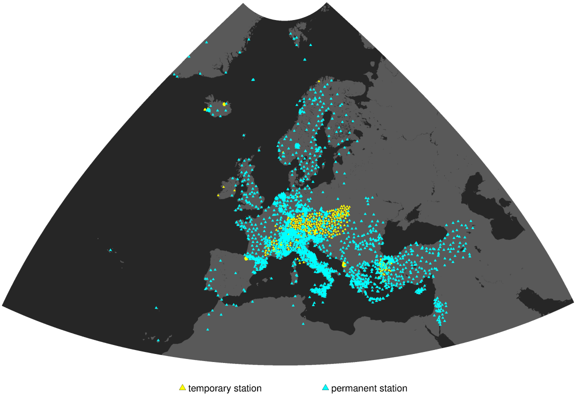
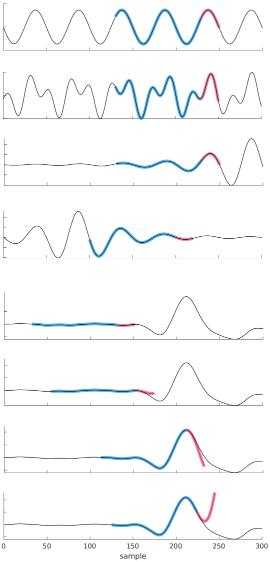
1 Motivation

> 3500 stations available in europe! (in 2020)
> 150 events of magnitude 6 or larger globally per year
$\implies$ Millions of event arrivals to pick!



Consistent picking of first arriving P-phases as well as later phases (e.g. S-phase) requires automated approach for this volume of data.

2 Picking Algorithm



Identification of phases using multi-component, multi-sample autoregressive prediction. Model the data as an autoregressive process of order p, determine its coefficients b_j by minimizing prediction error in the determination window (**det**) in a least-squares sense[a].

$$\hat{y}_{c,i} = \sum_{j=1}^{p} b_j \, y_{c,i-j} \quad \text{(autoregressive model)}$$

$$\mathrm{E} = \sum_{i \in \mathbf{det}} \sum_{c \in \mathbf{C}} (\hat{y}_{c,i} - y_{c,i})^2 \quad \text{(prediction error over Comp.)}$$

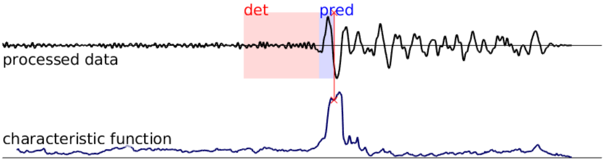
$$\frac{\partial \mathrm{E}}{\partial b_k} = 0\,, \quad \text{for all } k \in [1, p] \quad \text{(minimizing E)}$$

$$\sum_{j=1}^{p} b_j \sum_{i \in \mathbf{det}} \sum_{c \in \mathbf{C}} y_{c,i-j} y_{c,i-k} = \sum_{i \in \mathbf{det}} \sum_{c \in \mathbf{C}} y_{c,i} y_{c,i-k}\,,$$

where $k \in [1, p]$

[a] L. Küperkoch et al. "Automated determination of S-phase arrival times using autoregr application to local and regional distances". In: *Geophysical Journal International* 188.2 (2 10.1111/j.1365-246X.2011.05292.x. eprint: /oup/backfile/content_public/journal/gji/188/2/10.1111/j.1365-246x.2011.0529

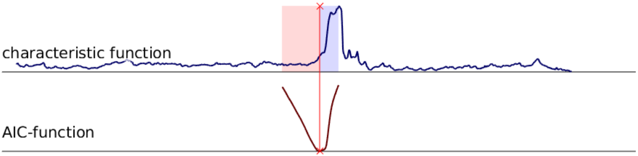
2 Picking Algorithm - Characteristic Function



Define the characteristic function (**CF**) as the rms-error of AR-prediction in the prediction window (**pred**). The **CF** shows a steep rise in amplitude during signal onset. Amplitudes are much larger during the signal than for the preceding noise. This **CF** is sensitive to changes in signal amplitude, as well as signal frequency and phase.

$$\mathbf{CF} = \left(\frac{l_{det} \sum_{c \in \mathbf{C}} \sum_{i \in \mathbf{pred}} \left(\hat{y}_{c,i} - y_{c,i} \right)^2}{n_{comp}\, l_{pred} \sum_{j \in \mathbf{det}} y_{c,j}^2} \right)^{0.5}$$

2 Picking Algorithm - Phase Arrival Estimation with AIC

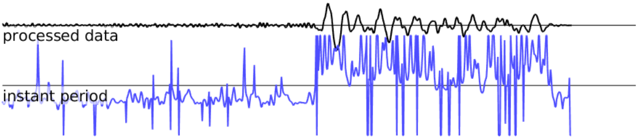


Use the Akaike-Information-Criterion[1] (**AIC**) to find the separation point for noise and signal onset (as the minimum of the **AIC**-function), in a time window before the **CF**-maximum. This is usually still a bit later than we want to pick.

$$\mathbf{AIC}_i = (i - M)\log\left(\frac{1}{i}\sum_{j=0}^{i}\mathbf{CF}_j{}^2\right) + (L - i - M)\log\left(\frac{1}{L-i}\sum_{k=i+1}^{L}\mathbf{CF}_k{}^2\right)$$

[1] Hirotugu Akaike. "On the Likelihood of a Time Series Model". In: *Journal of the Royal Statistical Society. Series D (The Statistician)* 27.3/4 (1978), pp. 217–235. ISSN: 00390526, 14679884. URL: http://www.jstor.org/stable/2988185.

2 Picking Algorithm - Inst. Period of the Analytic Signal



Calculate the analytic signal (Y) using the Hilbert Transform. Signal onset shows as discontinuity in the instantaneous phase, which in turn results in spikes in the instantaneous period. Define the final pick as the last spike (extremal value larger than a threshold in relation to the signal period) in a time window before the **AIC**-minimum.

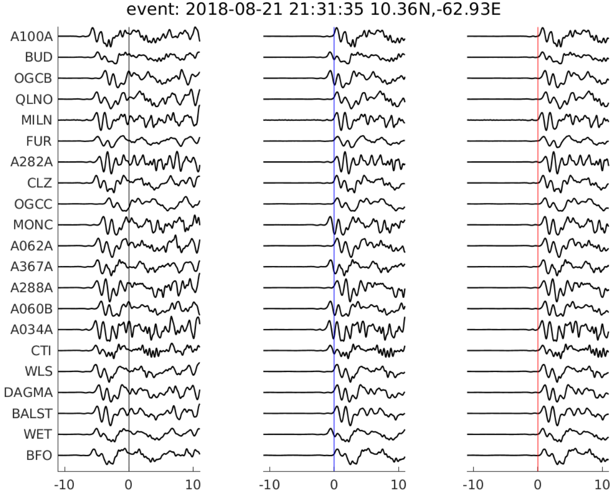
$$Y_i = y_i - i\,\mathcal{H}\{y_i\} \quad \text{(analytic signal)}$$

$$\phi_i = \arctan\left(\frac{\Im(Y_i)}{\Re(Y_i)}\right) \quad \text{(instantaneous phase)}$$

$$\Omega_i = \frac{d\phi_i}{dt} \quad \text{(instantaneous frequency)}$$

$$\Lambda_i = \frac{2\pi}{\Omega_i} \quad \text{(instantaneous period)}$$

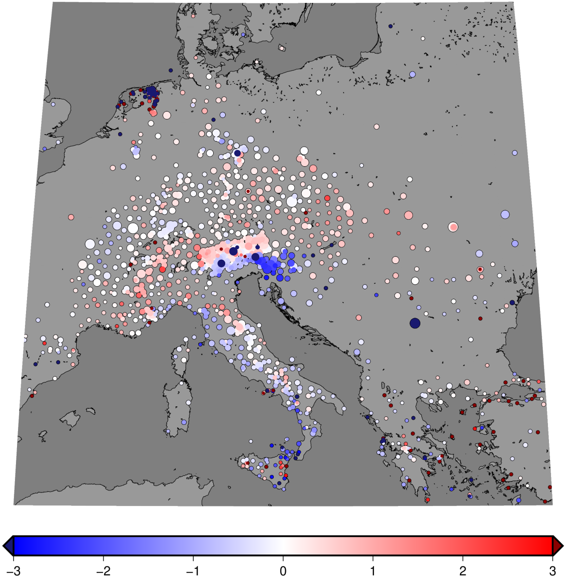
4 Real Data Examples - Seismogram section



Vertical components aligned by theoretical travel times (TauPy, ak135) on the left, by the AIC-minimum in the middle, and by picked P-phase arrival on the right.

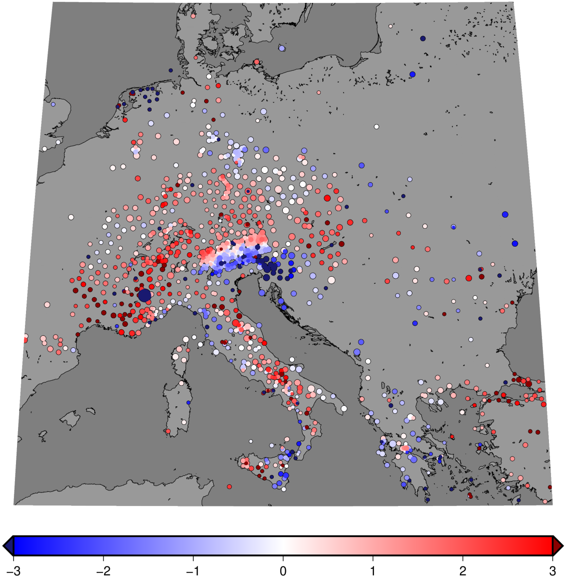
4 Real Data Examples - P-phase residuals

event: 2018-08-12 21:15:02, 69.6°N -144.4°E, magnitude: 6 (ISC)

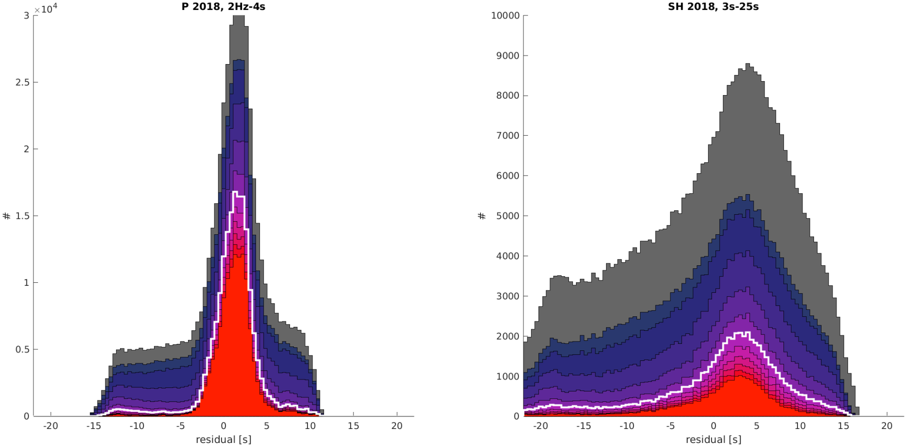


4 Real Data Examples - SH-phase residuals

event: 2018-08-12 21:15:02, 69.6°N -144.4°E, magnitude: 6 (ISC)



4 Real Data Examples - Histograms of Residuals



Summary

- Multi-component, multi-sample AR-prediction in a short moving window using the least-squares approach yields a robust initial pick
- Instantaneous Period enables refinement of this initial pick
- Quality criteria allow for integration of large datasets encompassing a wide range of recording qualities
- Consistent picking of P, SV and SH waves in different frequency and distance ranges possible

$\Longrightarrow$ Thank you for your attention!