Causality in long-term predictions, past-value problems and a stochastic-deterministic combination

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Motivation

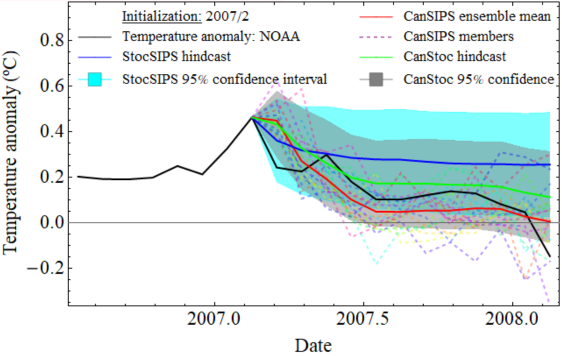
Seasonal forecast

- The atmosphere is a chaotic system

- Conventional ensemble systems

Taken from ECMWF

- Conventional ensemble prediction involves:
  - Many "random" outputs generated with complex deterministic equations
  - Probabilistic interpretation



- **Why not then directly use a stochastic model?**
- **Can we combine the two approaches for improving forecasts?**

Motivation

Scaling

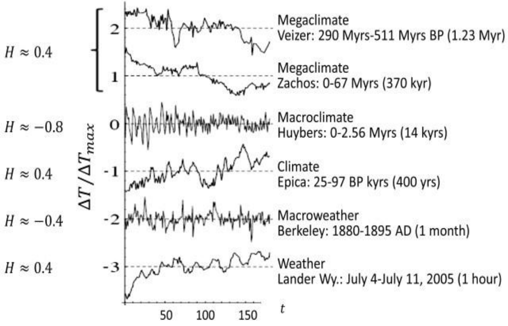
$$\left\langle \left|\Delta T\left(\Delta t\right)\right| \right\rangle \propto \Delta t^{H}$$

Laws of turbulence

Spectrum:

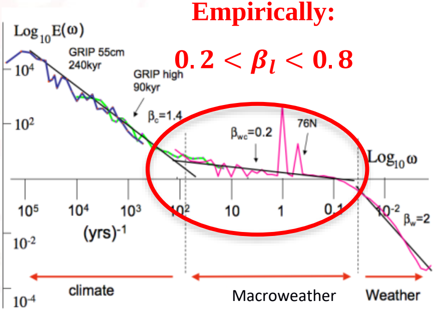
$$E\left(\omega\right) = \left\langle \left|\tilde{T}\left(\omega\right)\right|^{2} \right\rangle \propto \omega^{-\beta}$$

$\beta = 1 + 2H$ (for mono-fractals)



Empirically:

$0.2 < \beta_l < 0.8$



Lovejoy and Schertzer 2011

m-StocSIPS

multivariate Stochastic Seasonal to Interannual Prediction System:

$$T_i(t) = \sum_j \int_{-\infty}^{t} \kappa_{ij}(t - t')\gamma_j(t')dt'$$

$\kappa_{ij}(t)$ is a scaling (fractional Gaussian noise) kernel

The innovations, $\gamma_i(t)$, are normalized Gaussian white noise processes with cross-correlation matrix:

$$\rho_{ij}(t - t') = \langle \gamma_i(t)\gamma_j(t') \rangle = a_{ij}\delta(t - t')$$

This implies no Granger causality = past value problem

A forecast of $T_i(t > 0)$, based on its own past $T_i(s \leq 0)$, is not worst than a forecast of $T_i(t)$ based on both $T_i(s)$ and the information from another pixel, $T_j(s)$.

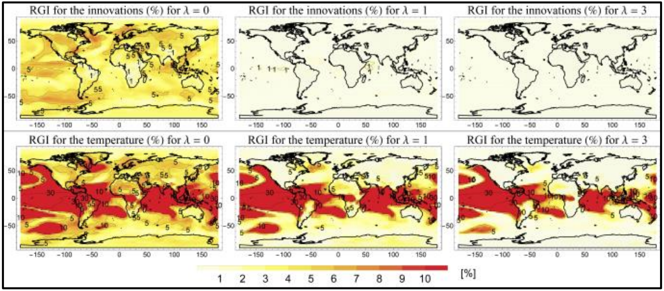
Correlations and connectivity

$$\rho_{ij}(t-t') = \langle \gamma_i(t)\gamma_j(t') \rangle = a_{ij}\delta(t-t')$$

$$R_{ij}(\Delta t) = \langle T_i(t)T_j(t+\Delta t) \rangle = a_{ij} f_{Hi,Hj}(\Delta t)$$

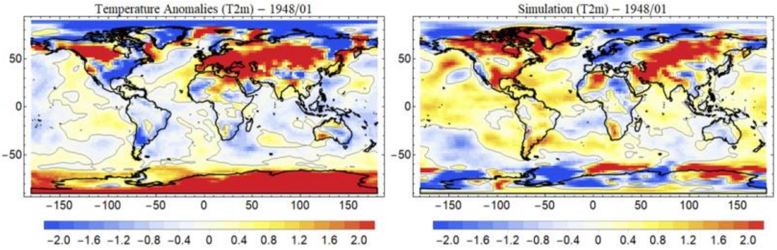
The Ratio of Global Influence (RGI) or connectivity:

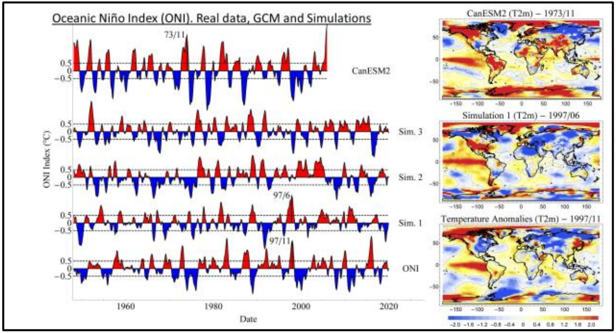
for pixel i is the fraction of the area of the planet for which $|R_{ij}(\lambda)| > 0.2$, averaged over all j (for innovations $|\rho_{ij}(\lambda)| > 0.2$).



For the innovations, almost all the connectivity is lost for $\lambda > 0$.

Simulations





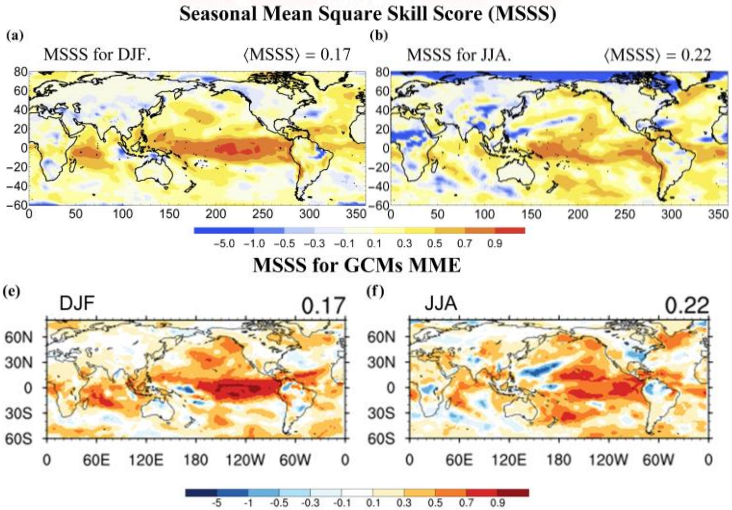
CanESM2 simulation.

StocSIPS simulations.

Observations.

Predictions and comparison with GCMs

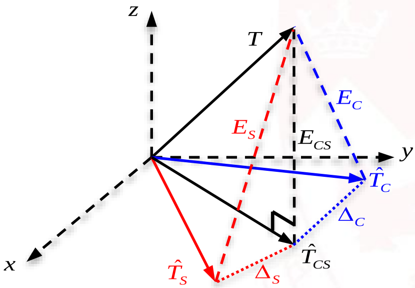
Reference: ERA-Interim; Resolution: 2.5ºx2.5º; Period of verification: 1981-2010.



Forecasts from Kim et al. 2020

Merging CanSIPS and StocSIPS into CanStoc

Geometric Interpretation



Errors

$$E_S(t) = T(t) - \hat{T}_S(t)$$

$$E_C(t) = T(t) - \hat{T}_C(t)$$

$$E_{CS}(t) = T(t) - \hat{T}_{CS}(t)$$

C – CanSIPS
S – StocSIPS
CS – CanStoc

Combined predictor

$$\hat{T}_{CS} = \alpha \hat{T}_S + \beta \hat{T}_C$$

Minimum Square Framework - Orthogonality Principle

$$\left\langle \hat{T}_{CS} E_{CS} \right\rangle = 0$$

Solving for the coefficients

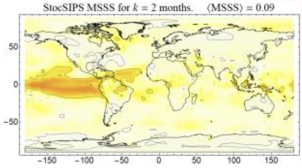
$$\begin{pmatrix} \left\langle \hat{T}_S T \right\rangle \\ \left\langle \hat{T}_C T \right\rangle \end{pmatrix} = \begin{pmatrix} \left\langle \hat{T}_S^{\,2} \right\rangle & \left\langle \hat{T}_S \hat{T}_C \right\rangle \\ \left\langle \hat{T}_S \hat{T}_C \right\rangle & \left\langle \hat{T}_C^{\,2} \right\rangle \end{pmatrix} \begin{pmatrix} \alpha \\ \beta \end{pmatrix}$$

Mean Square Skill Score comparison

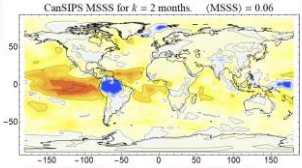
Reference: ERA-Interim; Resolution: 2.5ºx2.5º; Period of verification: 1981-2010.

1-months lead ($k = 2$)

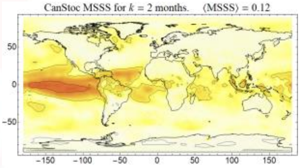
MSSS StocSIPS

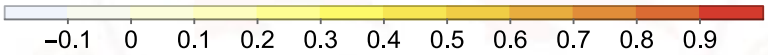


MSSS CanSIPS

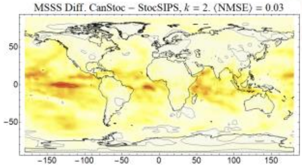


MSSS CanStoc

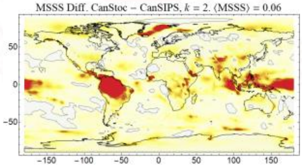




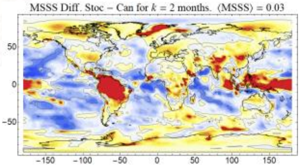
MSSS Diff. CanStoc - StocSIPS

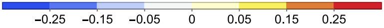


MSSS Diff. CanStoc - CanSIPS

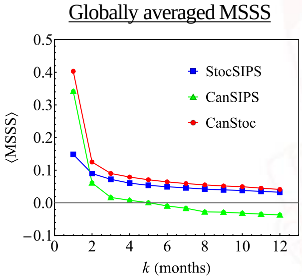


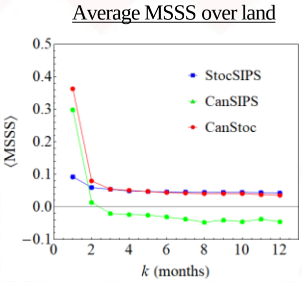
MSSS Diff. StocSIPS - CanSIPS

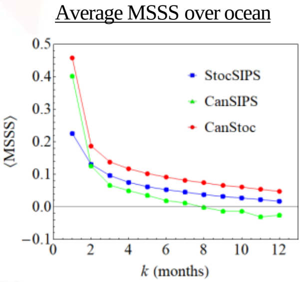




Average MSSS







Conclusions

- This difference in approach (initial vs. past value problems), opens the possibility of combining the two bringing the best of both worlds.
- The skill on the prediction of temperature was improved at global and regional levels.
- In places where both CanSIPS and StocSIPS show low skill (even negative), its combination in CanStoc could have high quality of prediction.
- The theoretical framework is general and can be used to include other copredictors.
- StocSIPS could be used as an advanced postprocessing technique to improve the skill of GCMs in places where they do not have deterministic predictability.

Thank You!

References

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