One Saddle Point and Two Types of Sensitivities within the Lorenz 1963 and 1969 Models

by

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Outline

- Background & Motivations
  - A Predictability Limit of Two Weeks based on Classical Lorenz Models (LMs)
  - Promising Global 30 days Simulations
- Insightful Analysis of the LMs and Development of Generalized LMs
  - Aggregative Negative Feedback in a Generalized LM (GLM, Shen 2019)
  - Coexistence of Chaos and Order in Weather and Climate (Shen et al. 2021a,b)
  - One Saddle Point and Two Types of Sensitivities (Shen et al. 2022a)
  - Three Kinds of Butterfly Effects Within LMs (Shen et al. 2022b)
- Selected References
- Special Issue: *"50th Anniversary of the Metaphorical Butterfly Effect since Lorenz (1972): Multistability, Multiscale Predictability, and Sensitivity in Numerical Models"*
  https://www.mdpi.com/journal/atmosphere/special_issues/predictability_numerical_models

Backgrounds

- Studies by Lorenz (Lorenz 1963, 1972) laid a foundation for (modern) Chaos theory, that is viewed as the third scientific achievement in the 20th century, after relativity and quantum mechanics.

- Lorenz studies (Lorenz 1963, 1969, 1972) yielded the statement of *"weather is chaotic"* and suggested a limit of predictability of 2 weeks.

- The validity of the above findings is being challenged by recent 30 day global model simulations, including:

  (1) 30-day simulations of multiple African Easterly Waves (AEWs):

  African easterly waves in 30-day high-resolution global simulations: A case study during the 2006 NAMMA period

  Bo-Wen Shen,[1,2] Wei-Kuo Tao,[2] and Man-Li C. Wu[2]

  Received 15 June 2010; revised 23 July 2010; accepted 4 August 2010; published 18 September 2010.

  Shen et al. (2010, GRL)

  (2) 30-day simulations of Madden-Julian Oscillation (MJO):

  Coupling Advanced Modeling and Visualization to Improve High-Impact Tropical Weather Prediction

  Bo-Wen Shen
  University of Maryland, College Park
  Wei-Kuo Tao and Bryan Green
  NASA

  Shen et al. (2011, CiSE)

Classical LMs and a Generalized LM (GLM)

International Journal of Bifurcation and Chaos | Vol. 29, No. 03, 1950037 (2019) | Papers

Aggregated Negative Feedback in a Generalized Lorenz Model

Bo-Wen Shen

March 2019

BAMS In Box

Is Weather Chaotic?

Coexistence of Chaos and Order within a Generalized Lorenz Model

Bo-Wen Shen, Roger A. Pielke Sr., Xubin Zeng, Jong-Jin Baik,
Sara Faghih-Naini, Jialin Cui, and Robert Atlas

Shen et al. (2021, BAMS)
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May 2022

Entry

Three Kinds of Butterfly Effects Within Lorenz Models

Bo-Wen Shen[1*], Roger Pielke, Sr.[2], Xubin Zeng[3], Jialin Cui[4], Sara Faghih-Naini[5], Wei Paxson[1], and Robert Atlas[6]

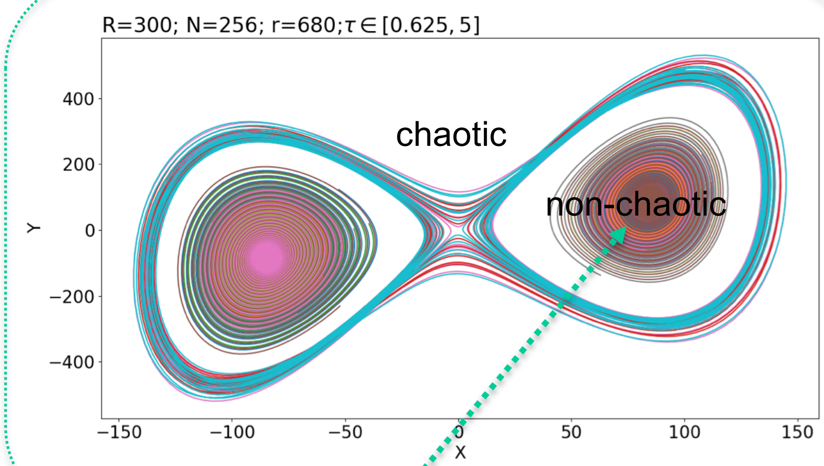
Aggregated Negative Feedback Within the GLM

model	r_c	heating terms	solutions	references
3DLM	24.74	rX	steady, chaotic, or LC	Lorenz (1963)
3D-NLM	n/a	rX	periodic	Shen (2018)
5DLM	42.9	rX	steady, chaotic, or LC/LT	Shen (2014a,2015a,b)
5D-NLM	n/a	rX	quasi-periodic	Faghih-Naini and Shen (2018)
6DLM	41.1	rX, rX_1	steady or chaotic	Shen (2015a,b)
7DLM	116.9	rX	steady, chaotic or LC/LT	Shen (2016, 2017)
7D-NLM	n/a	rX	quasi-periodic	Shen and Faghih-Naini (2017)
8DLM	103.4	rX, rX_1	steady or chaotic	Shen (2017)
9DLM	102.9	rX, rX_1, rX_2	steady or chaotic	Shen (2017)
9DLMr	679.8	rX	steady, chaotic, or LC/LT	Shen (2019a)

- The above critical values are determined by Lyapunov exponents.
- Higher-dimensional LMs require larger heating parameters for the onset of chaos, indicating aggregated negative feedback (Shen, 2019).
- The aggregated negative feedback may lead some unstable (non-trivial) critical points to become stable, yielding attractor coexistence.

Coexistence of Chaos and Order Within the GLM

1. the large-scale time varying forcing (heating);



2. the aggregated negative feedback of small-scale convective processes

seasonal varying currents

"SDIC"

local stability

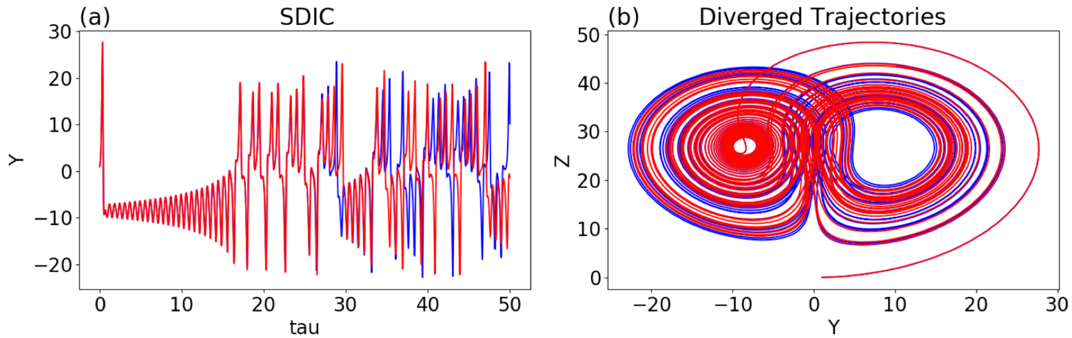
Shen et al. (2021a,b, 2022) proposed a revised view that

"weather possesses chaos and order; it includes emerging organized systems (such as tornadoes) and recurrent seasons",

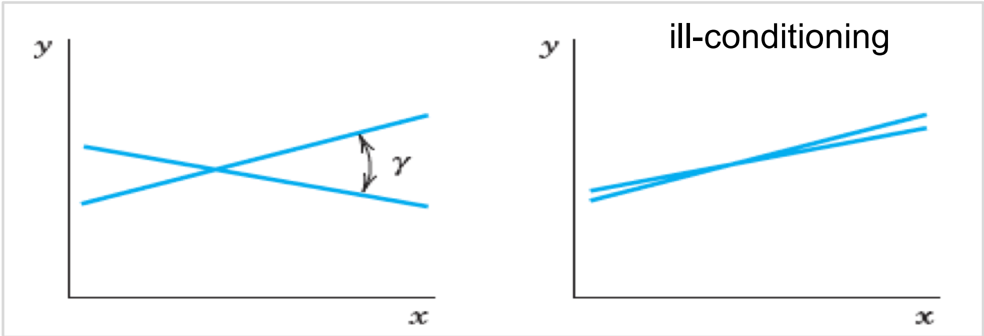
in contrast to the conventional view of "weather is chaotic".

Two Types of Sensitivities Within Lorenz Models

- The Lorenz 1963 model is a nonlinear but limited-scale system with sensitive dependence of solutions on initial conditions (SDIC). It has been applied for understanding the chaotic nature of weather and climate.



- By comparison, major findings of the Lorenz 1969 model were applied for estimating a predictability horizon for the atmosphere. The Lorenz 1969 model is a closure-based, physically multiscale, mathematically linear, and numerically ill-conditioned system (Shen et al. 2022a).



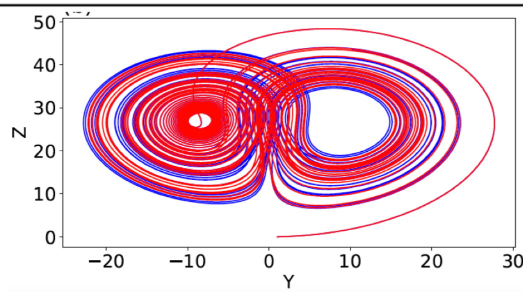
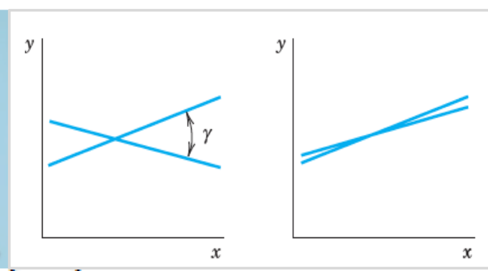
Ill-conditioning Within the Lorenz 1969 Models

	Python	Matlab	Remarks
Table 1 of RS08	8.319352454E5	8.3194e+05	2DV dynamics
Table 2 of RS08	8.446532703E5	8.4465e+05	vs. Lorenz (1969)
Table 3 of RS08	2.791518148E4	2.7915e+04	"unlimited predictability"
Table 4 of RS08	2.146269131E9	2.1463e+09	SQG dynamics
Table A1 of DG14	7.967004925E5	7.9670e+05	vs. Table 1 of RS08
Table A2 of DG14*	9.767672780E6	9.7677e+06	vs. Table 4 of RS08

*There may be a typo in $C_{1,2}$ in Table A2 of DG14. A revised condition # is $O(10^9)$.

- Ill-conditioning is indicated by large condition numbers in all of the matrices in 6 Tables of Rotunno and Snyder (2008, **RS08**) and Durran and Gingrich (**DG14**).
- As discussed earlier, within an ill-conditioned (or ill-posed) system, "small" changes in the data (the input) cause "large" changes in the solution (the output), displaying numerical sensitivity (Kreyszig, 2011)
- On the other hand, within a well-conditioned (or well-posed) system, "small" changes in the data cause only "small" changes in the solution.

Three Kinds of Butterfly Effects

Kind	Features	Consequences	Acceptance	Ref
1st		Finite Predictability	Well accepted	[1] Lorenz (1963)
2nd		Formation of a tornado	As a metaphor	[2] Lorenz (1972)
3rd	 	Finite Predictability	On-going	[3,4,18] Lorenz (1969)

The mechanisms (i.e., sensitivities), which lead to finite predictability within the Lorenz 1963 and 1969 models, are different.

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50th Anniversary of the Metaphorical Butterfly Effect since Lorenz (1972): Multistability, Multiscale Predictability, and Sensitivity in Numerical Models

Guest Editors

Dr. Bo-Wen Shen, Prof. Dr. Roger A. Pielke Sr., Prof. Dr. Xubin Zeng

- Two types of sensitivities appear within the Lorenz 1963 and 1969 models.
- A revised view is proposed as follows: *weather possesses chaos and order; it includes emerging organized systems (such as tornadoes) and recurrent seasons.*

Thanks!