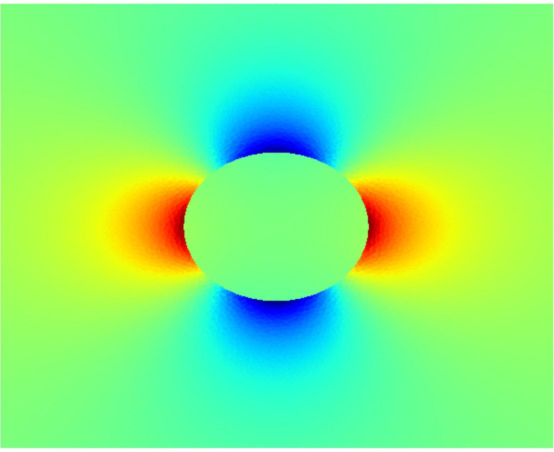
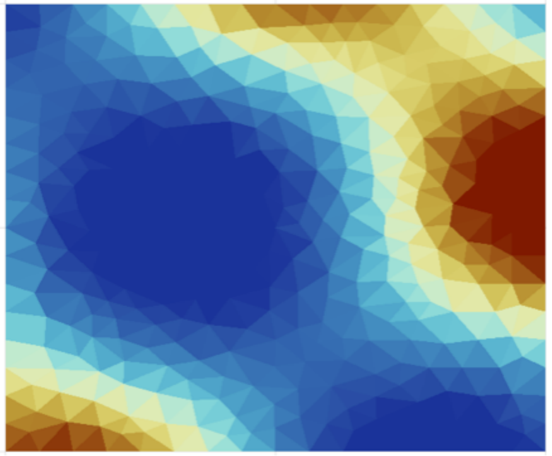
The Face-Centered Finite Volume method for Geodynamic Modelling

T. Duretz, L. Räss and R. Sevilla





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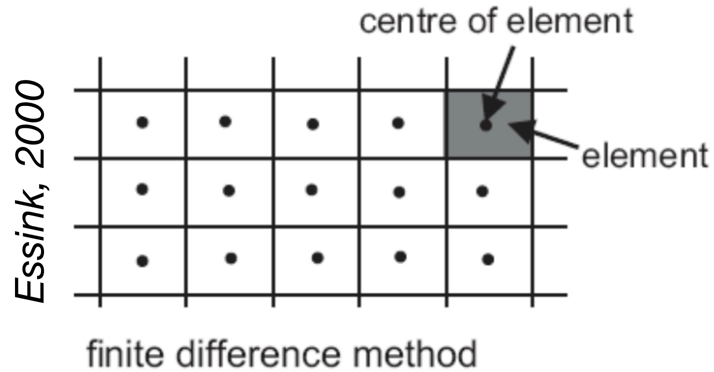
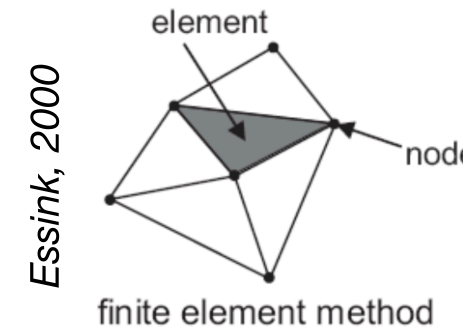
Most common methods in GD modelling

Staggered
FDM



FEM



Pros

Smooth learning curve

Light stencils

Stable pressure

Cons

Poor geometric flexibility

Spurious oscillations at interfaces

Interpolations

Difficult linearisation

Pros

Full geometric flexibility

Straightforward linearisations

Natural free surface BC

Cons

Spurious pressure modes

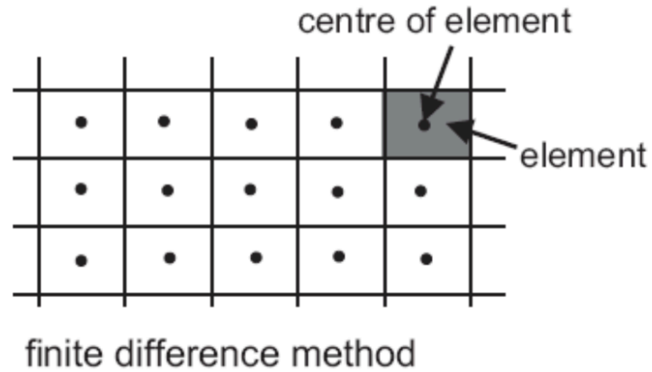
Heavy discretisation

Remeshing

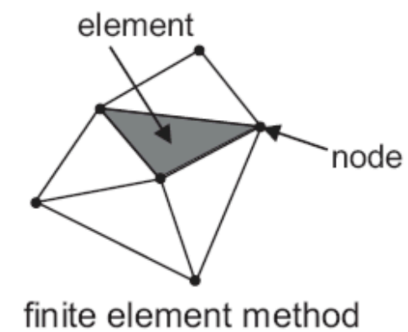
Steep learning curve

Most common methods in GD modelling

Essink, 2000



Essink, 2000



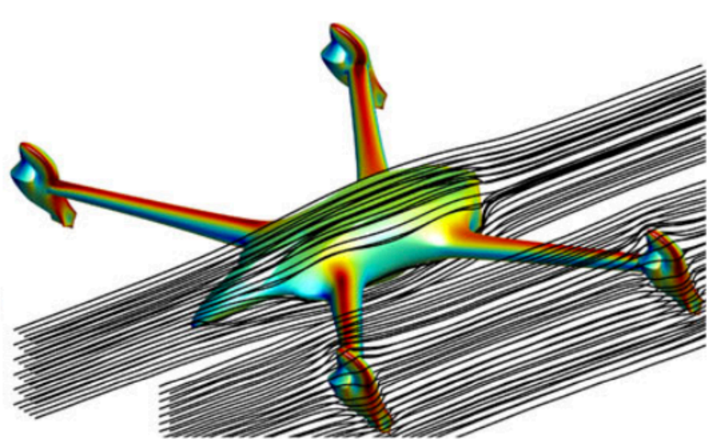
<u>A ideal scheme would combine pros of FEM and FDM</u>

Light stencils / Stable pressure / Full geometric flexibility / Clean interfaces

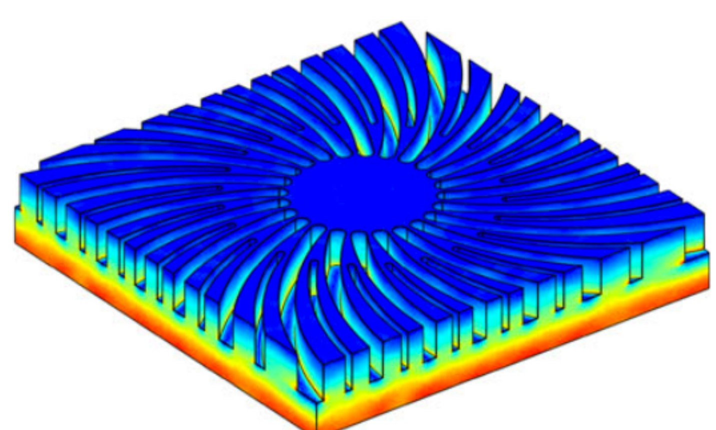
Straightforward linearisations / Natural free surface BC

<u>FCFV: Face-Centered Finite Volume Method</u>

Hybridisable discontinuous Galerkin formulation



(B)



(C)

Sevilla et al. (2018), Giacomini et al., (2020), Vieira et al., (2020)

How does FCFV perform on typical geodynamic problems?

Poisson: FCFV discretisation

Poisson: local problem
Element-by-element

$$\begin{cases} \boldsymbol{q}_e + \boldsymbol{\nabla} u_e = \boldsymbol{0} & \text{in } \Omega_e, \\ \boldsymbol{\nabla} \cdot \boldsymbol{q}_e = s & \text{in } \Omega_e, \\ u_e = u_D & \text{on } \partial\Omega_e \cap \Gamma_D, \\ u_e = \hat{u} & \text{on } \partial\Omega_e \backslash \Gamma_D, \end{cases}$$

Poisson: interfaces
Transmission conditions

$$\begin{cases} [\![u\boldsymbol{n}]\!] = \boldsymbol{0} & \text{on } \Gamma \\ [\![\boldsymbol{n} \cdot \boldsymbol{q}]\!] = 0 & \text{on } \Gamma \\ \boldsymbol{n} \cdot \boldsymbol{q} = -t & \text{on } \Gamma_N. \end{cases}$$

Sevilla et al. (2018)

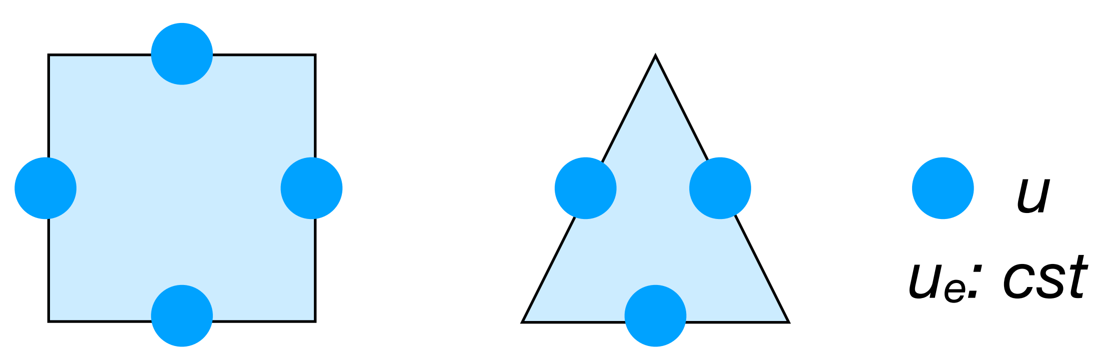
Poisson: Discrete weak problem

$$\sum_{e=1}^{\mathtt{n_{el}}} \left\{ \left\langle \hat{v}, \boldsymbol{n}_e \cdot \boldsymbol{q}_e^h \right\rangle_{\partial\Omega_e \backslash \Gamma_D} + \left\langle \hat{v}, \tau_e u_e^h \right\rangle_{\partial\Omega_e \backslash \Gamma_D} - \left\langle \hat{v}, \tau_e \hat{u}^h \right\rangle_{\partial\Omega_e \backslash \Gamma_D} \right\} = -\sum_{e=1}^{\mathtt{n_{el}}} \langle \hat{v}, t \rangle_{\partial\Omega_e \cap \Gamma_N}.$$

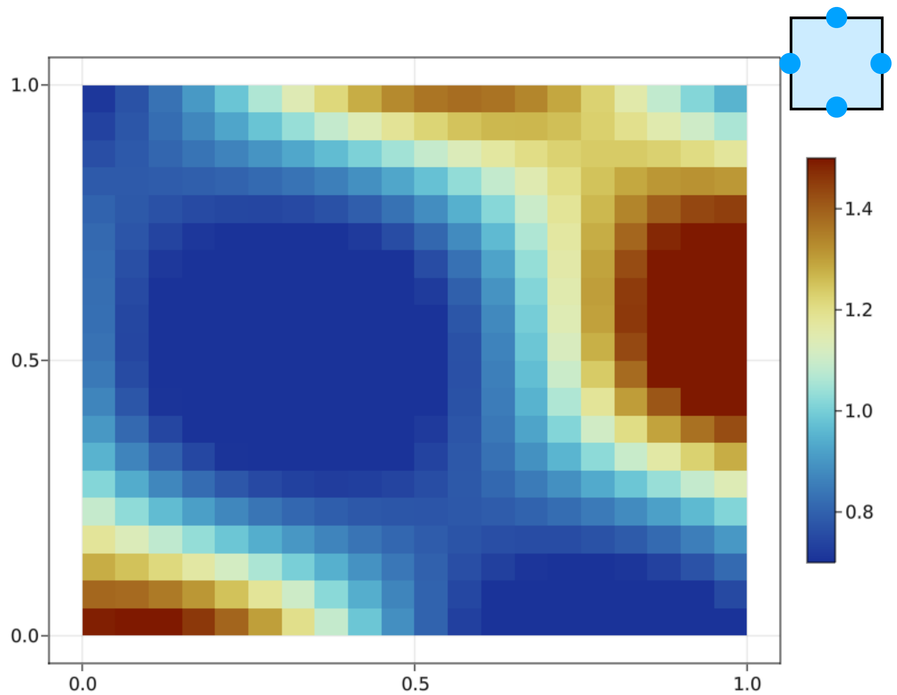
Accuracy: First or second order for primitive variables *(temperature/velocity)*

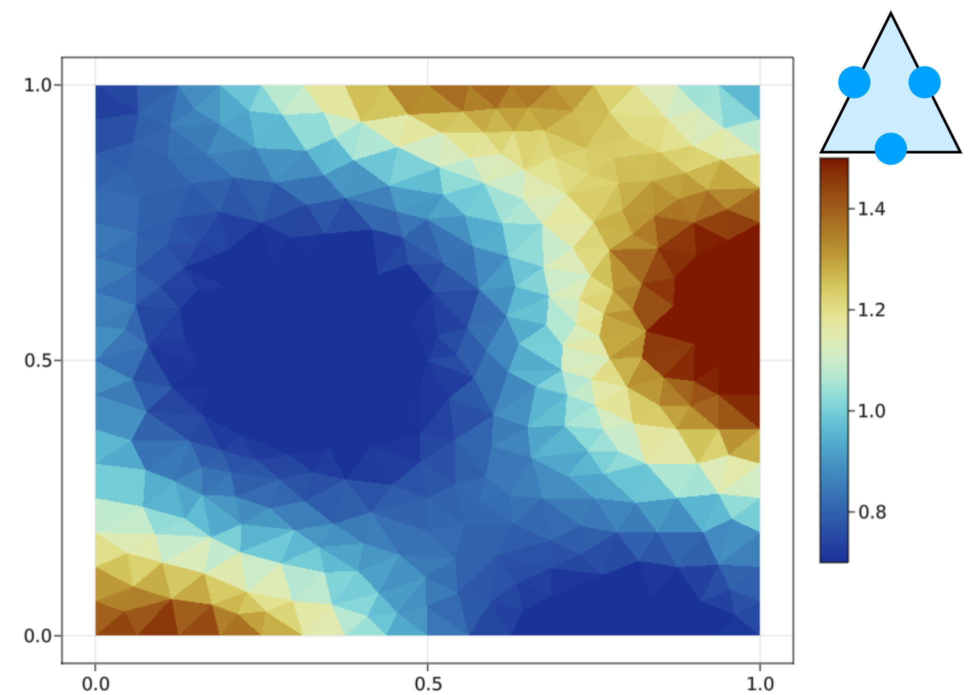
Meshes: 2D models with either quad elements or triangles

Solvers: Direct(-iterative) of fully iterative matrix-free

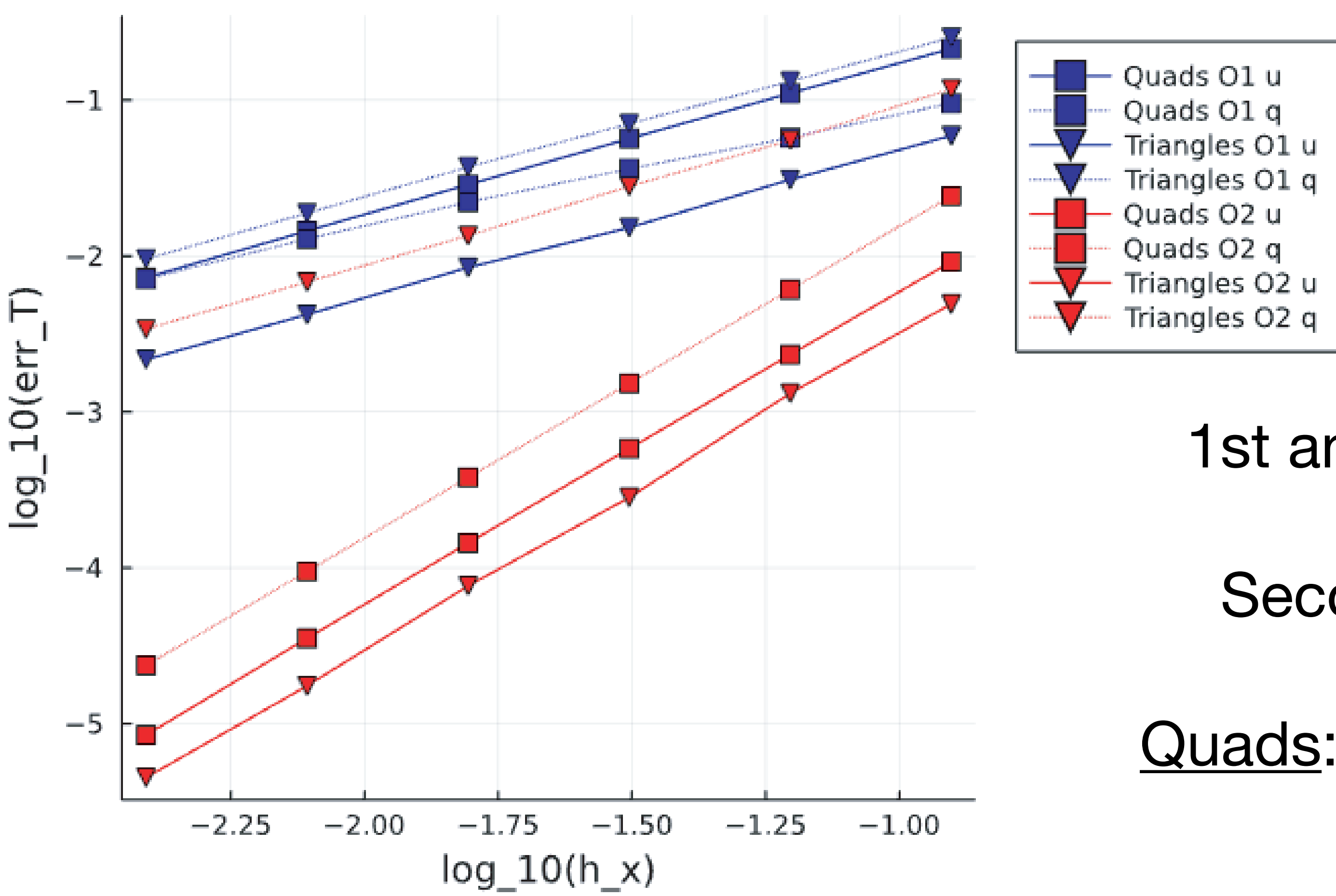


Poisson: FCFV discretisation





Poisson - constant coefficient



julia code

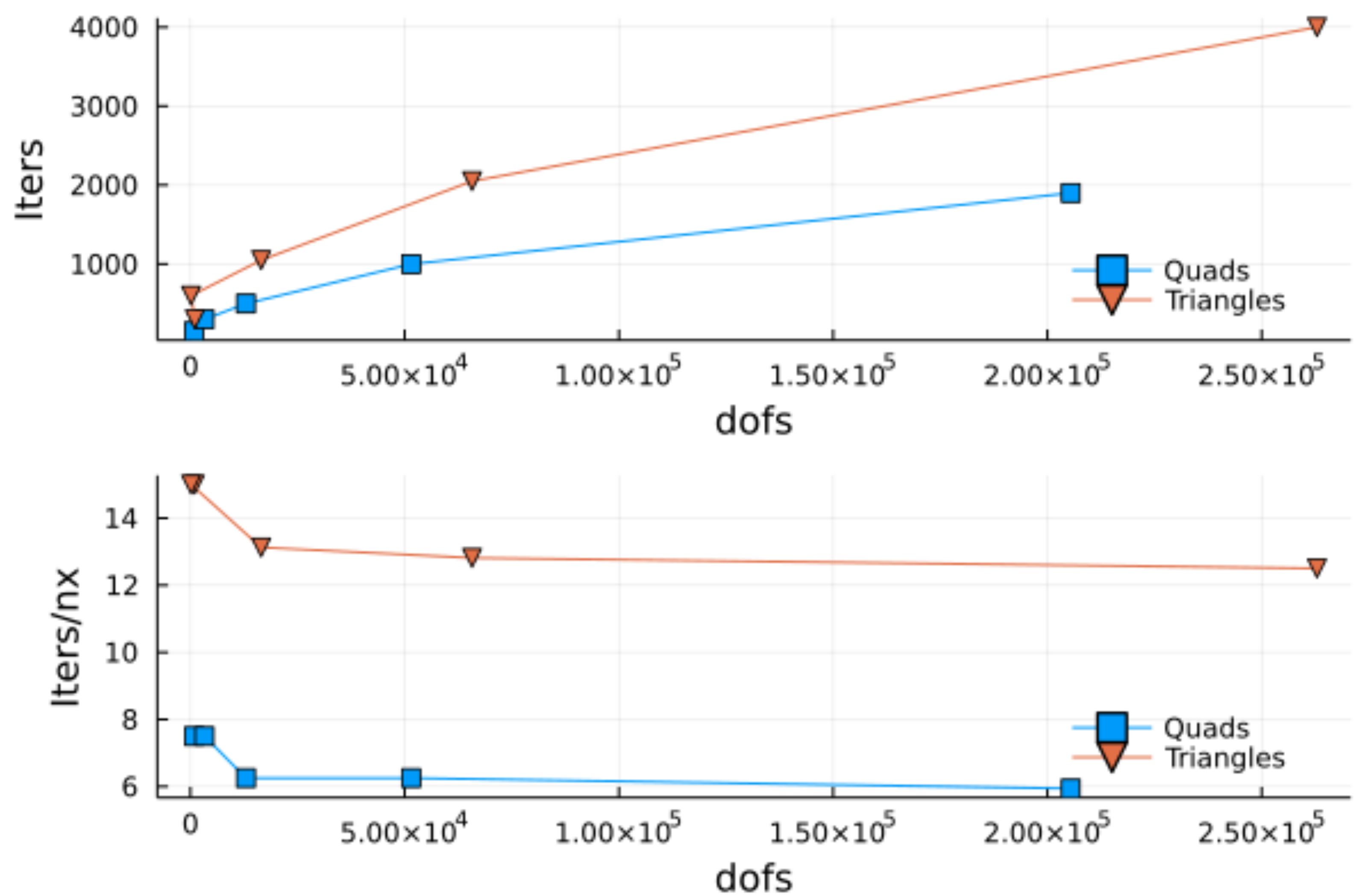
1st and 2nd order on quads and triangles

Second order without additional nodes

Quads: also flux (q) reaches second order (?)

Poisson: Solvers

A pseudo-transient solver has been tested:
The residual needs to be evaluated at vertices *(no lumping needed)*

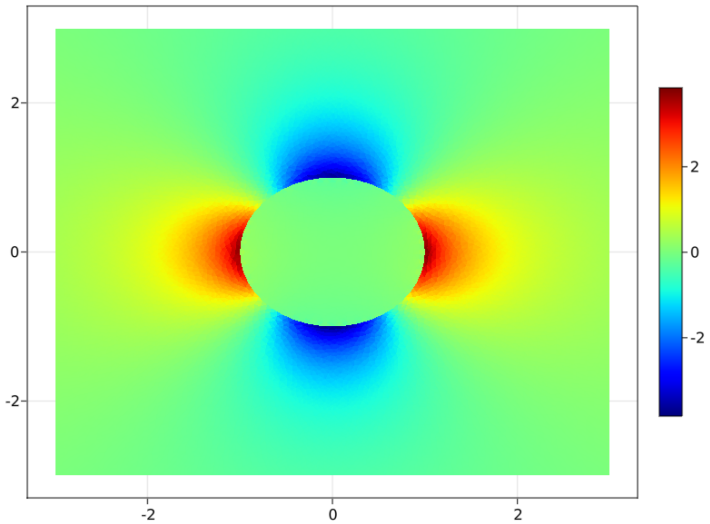


Good scaling of the PT solver, even on an unstructured mesh

Poisson code already works on single GPU

Need to optimise memory accesses for unstructured meshes

Viscous inclusion test - #1

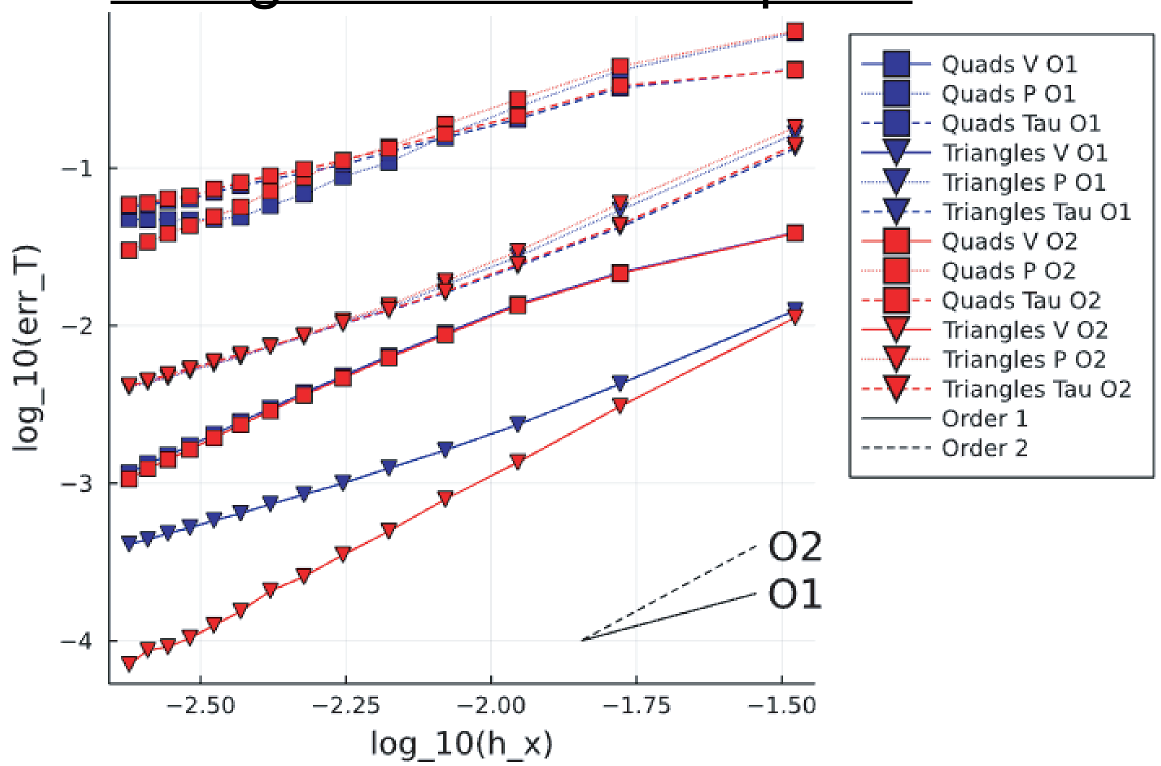


Most useful test (and tough) test for geodynamic modelling

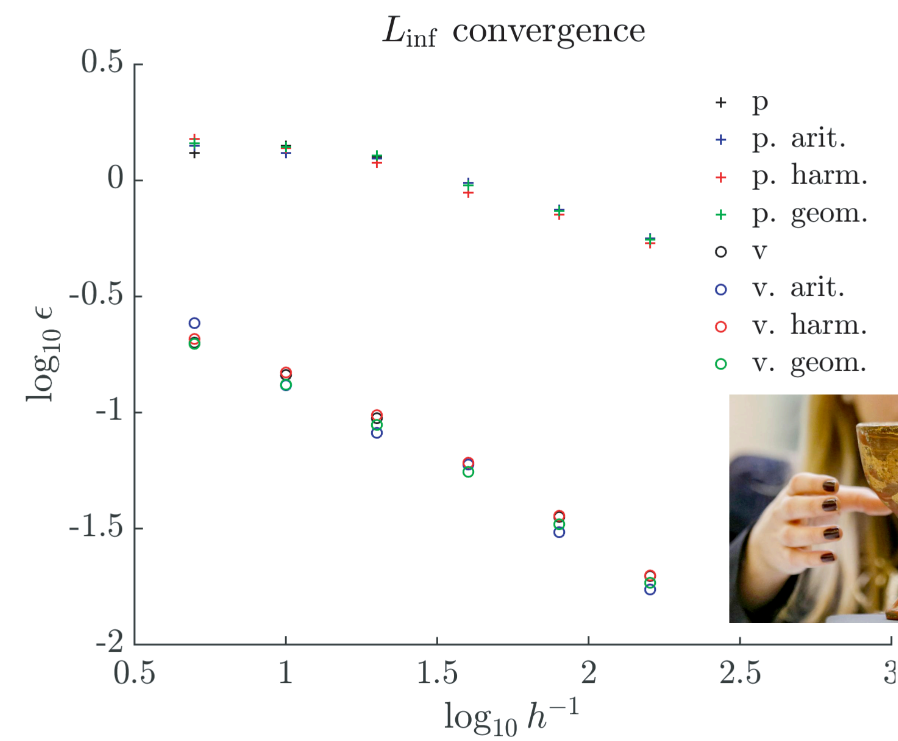
FCFV is a <u>stable</u> discretisation: no need for pressure projection

Need for a <u>jump condition</u> at the inclusion interface

<u>Triangles + non-fitted quads</u>



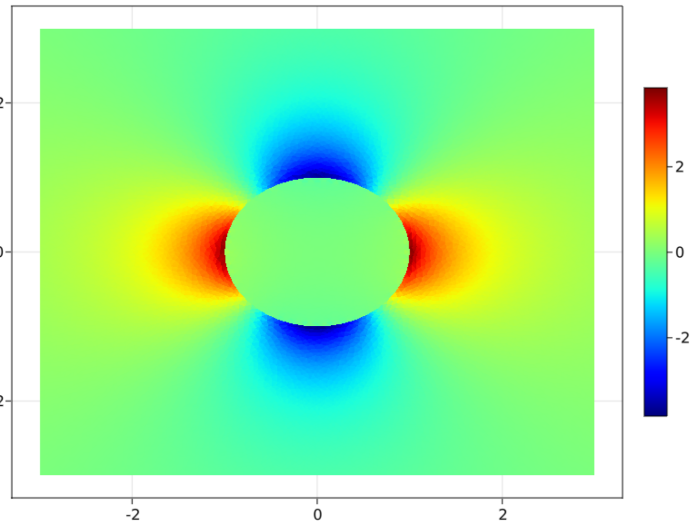
Stokes - problem 13 - Quad mesh - order 1



<u>TEST #1</u>: Jump condition evaluated using the analytical solution: *great results*!

Pressure L_inf convergence using <u>non-fitting mesh</u> + marker-in-cell (!)

Viscous inclusion test - #2



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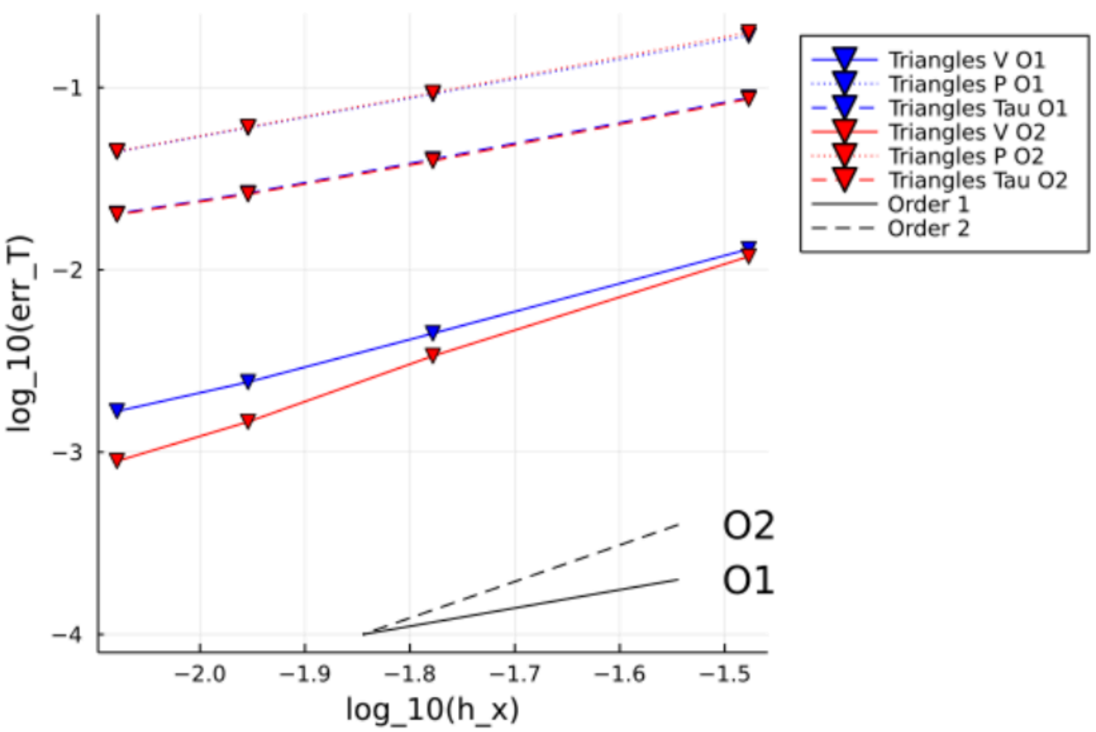
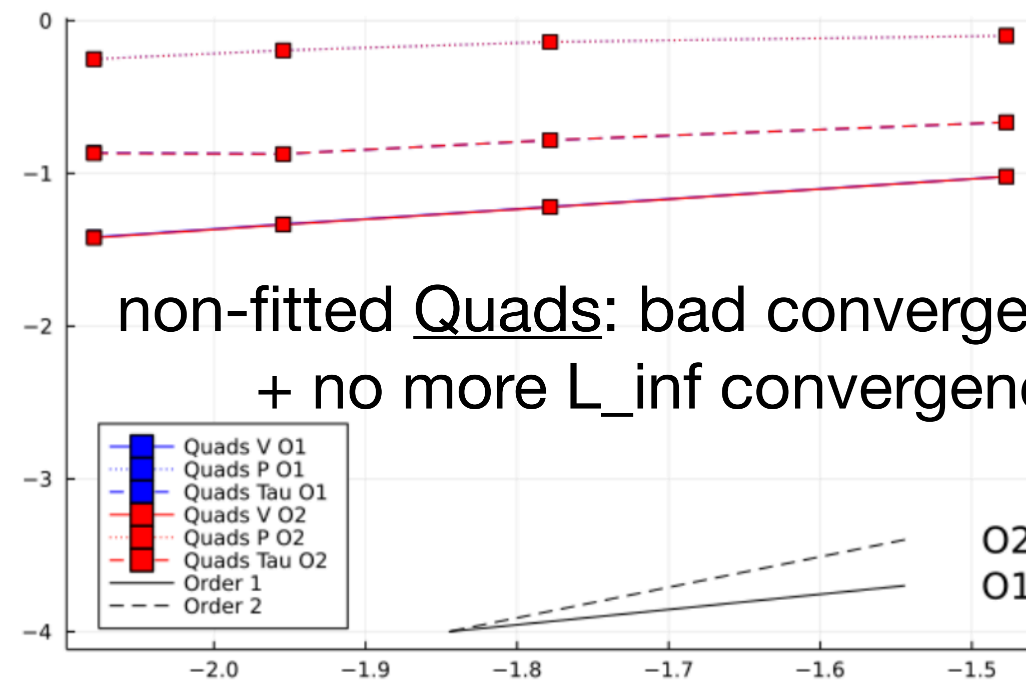


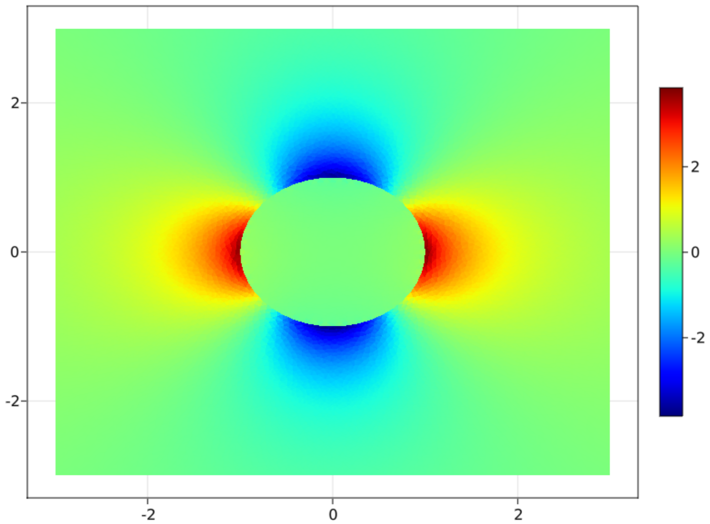


non-fitted Quads: bad convergence in L1
+ no more L_inf convergence…

TEST #2: generic implicit jump condition based on viscosity jump

Some important benefits are currently lost… (for the moment!)

Viscous inclusion test - #2



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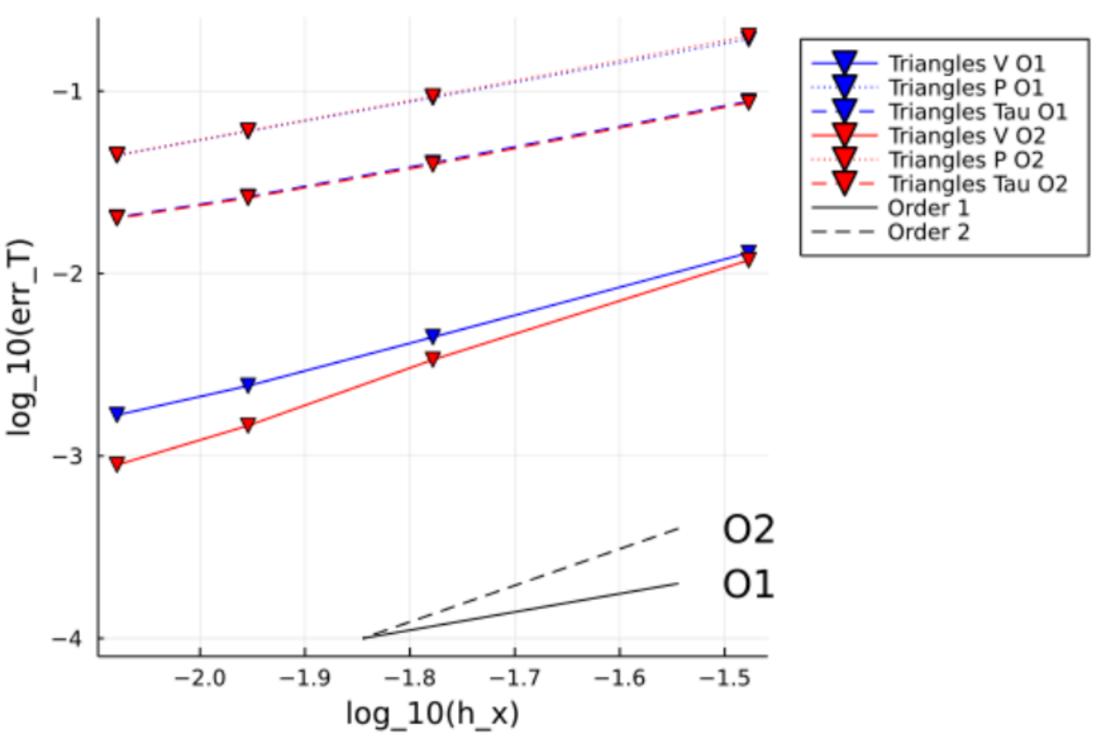

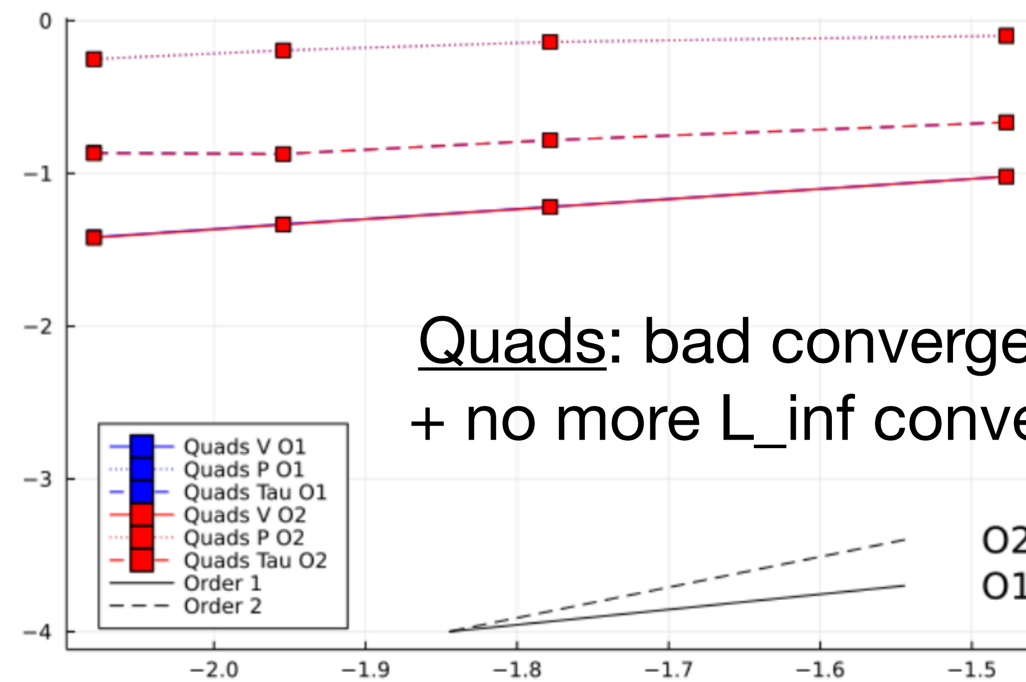

Quads: bad convergence in L1
+ no more L_inf convergence…

The FCFV method is currently under development and already promising

For heterogeneous materials, better stick to conformal meshes (up to now!)

Future developments will potentially make quads great again!