Local energy release by extreme vertical drafts in stratified geophysical flows

Raffaello Foldes[1,2], S.S. Cerri[3], R. Marino[2], F. Feraco[2] and E. Camporeale[4]

[1] University of L'Aquila, *Dept. of Physical and Chemical Sciences*, (Italy),
[2] École Centrale de Lyon - CNRS, *Lab. of Fluid Mechanics*, (France),
[3] *CNRS Laboratoire Lagrange*, Observatoire de la Côte d'Azur (France),
[4] University of Colorado, Boulder (USA)

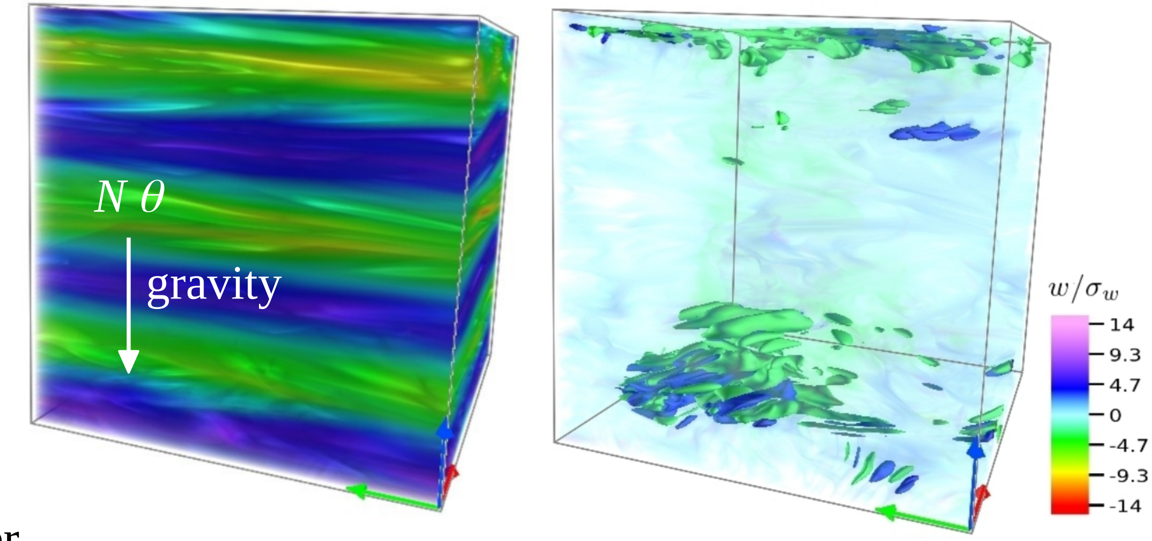
Investigating Geophysical Flows in the Boussinesq Framework

Boussinesq approximation:

$$\partial_t \bar{u} + (\bar{u}\cdot\mathbf{\nabla})\,\bar{u} = -\mathbf{\nabla} p - N\theta\hat{z} + \mathbf{F} + \nu\mathbf{\nabla}^2\bar{u}$$

$$\partial_t \theta + (\bar{u}\cdot\mathbf{\nabla})\,\theta = Nw + \kappa\mathbf{\nabla}^2\theta; \quad \mathbf{\nabla}\cdot\bar{u} = 0$$

Brunt-Väisälä (gravity waves)



512[3] DNS with pseudo-spectral code (**G**eophysical **H**igh-**O**rder **S**uite for **T**urbulence, GHOST)

- Anisotropy (gravity)
- Competition of **Turbulence** and **Waves**

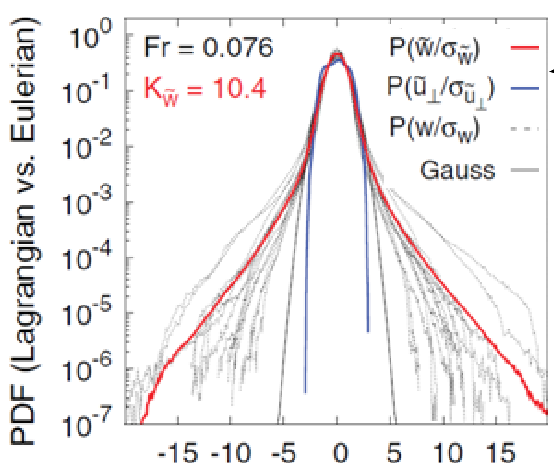
$\mathrm{Re} = \tau_\nu/\tau_{NL} = UL/\nu$ — **Reynolds** number

$Fr = \tau_{W_g}/\tau_{NL} = U/LN$ — **Froude** number

$Fr \ll 1$ values of geophysical interest considered for this analysis

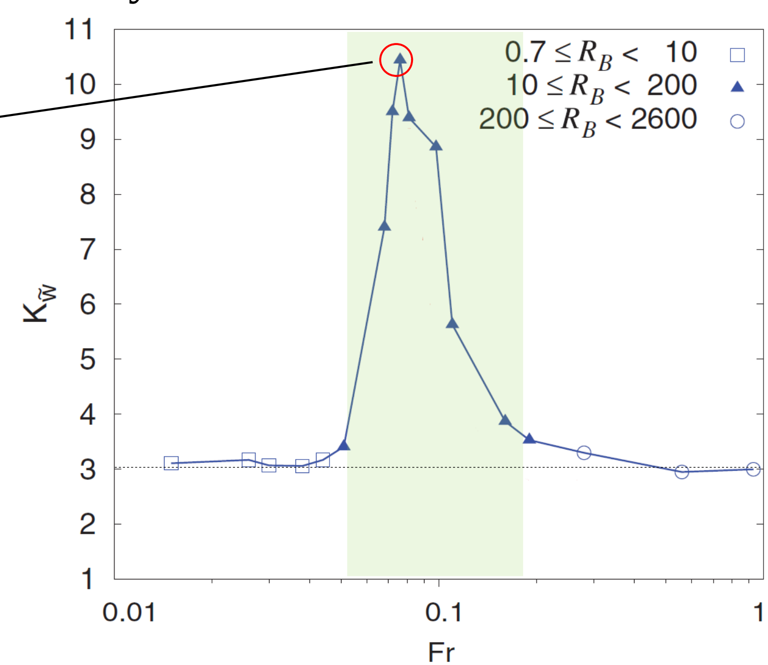
[Feraco et al., *EPL*, 2018]

Vertical Drafts



Resonant Inteprlay between **Turbulence** and **Waves**

$$K_w = \frac{\langle w-\bar{w}\rangle^4}{\langle (w-\bar{w})^2\rangle^2}$$



Filtered Energy Equations

Boussinesq approximation:

$$\partial_t \bar{u} + (\bar{u}\cdot\boldsymbol{\nabla})\,\bar{u} = -\boldsymbol{\nabla} p - N\theta\hat{z} + \mathbf{F} + \nu\boldsymbol{\nabla}^2\bar{u} \qquad (1)$$

$$\partial_t \theta + (\bar{u}\cdot\boldsymbol{\nabla})\,\theta = Nw + \kappa\boldsymbol{\nabla}^2\theta; \quad \boldsymbol{\nabla}\cdot\bar{u} = 0 \qquad (2)$$

Filtering the equations and multiplying eq. 1 by $\widetilde{\mathbf{u}}$ and eq. 2 by $\widetilde{\theta}$

Filtered Kinetic Energy Eq.

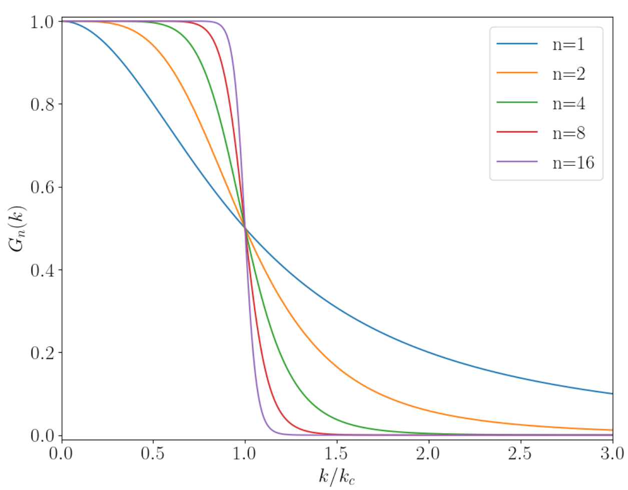
$$\partial_t\langle\widetilde{\mathcal{E}}_u\rangle = -N\langle\widetilde{\theta}\widetilde{w}\rangle + \langle\mathcal{S}_u\rangle + \langle\widetilde{D}_\nu\rangle + \langle\widetilde{\epsilon}_{ext}\rangle$$

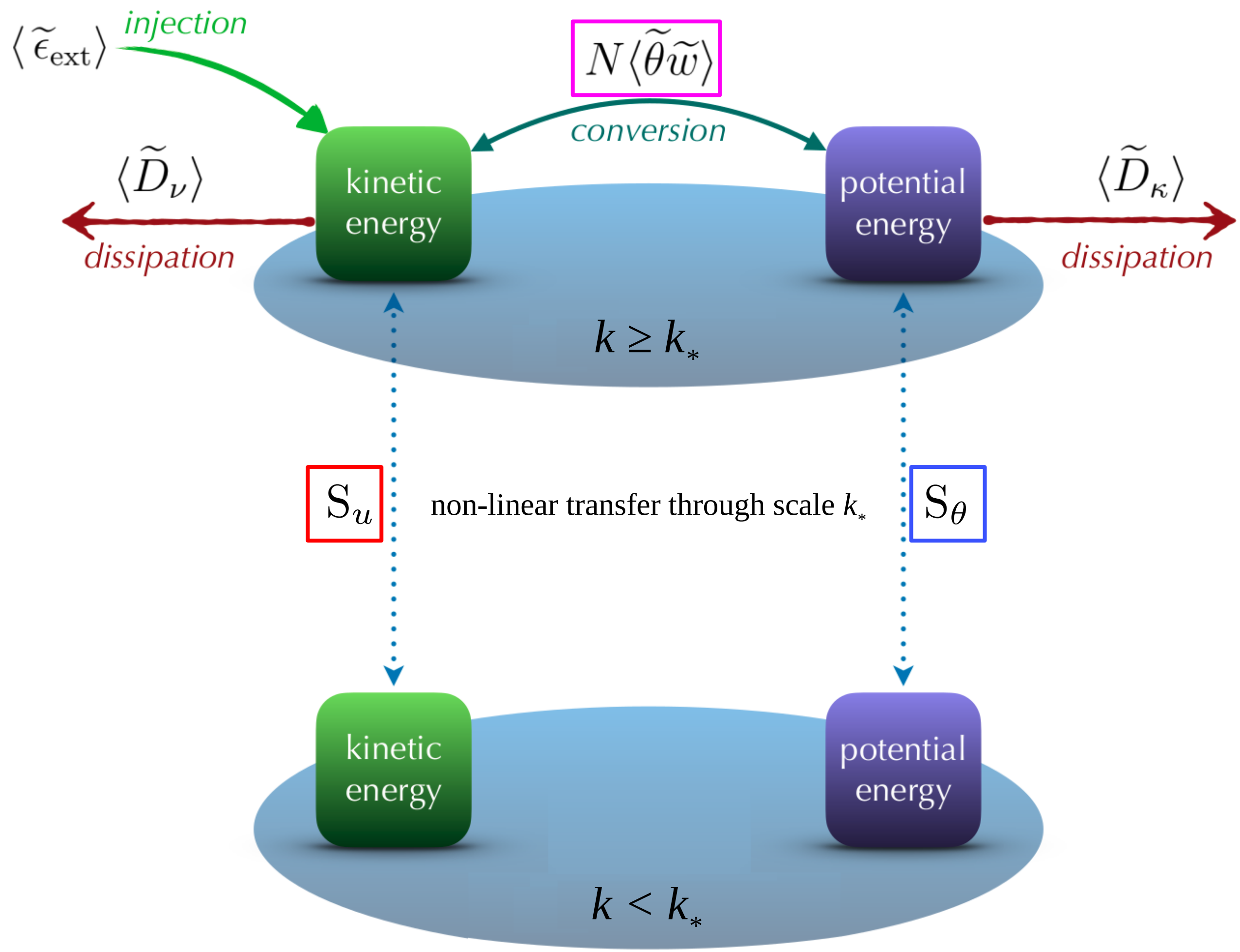
Filtered Potential Energy Eq.

$$\partial_t\langle\widetilde{\mathcal{E}}_\theta\rangle = N\langle\widetilde{\theta}\widetilde{w}\rangle + \langle\mathcal{S}_\theta\rangle + \langle\widetilde{D}_\kappa\rangle$$

Butterworth (low-pass) filter

$$\mathrm{G}_n(k) = 1/\left[1 + (k/k_c)^{2n}\right]$$





$$\widetilde{\mathbf{v}}(\mathbf{x},t) \doteq \int_{\mathcal{V}} G(\mathbf{x}-\xi)\mathbf{v}(\xi,t)\mathrm{d}^3\xi$$

$k \doteq k_\perp = \sqrt{k_x^2 + k_y^2}$ *Axisymmetric*

k_* is the Filtering Scale

<u>Sub-grid terms are defined in (real) **physical space**.</u>

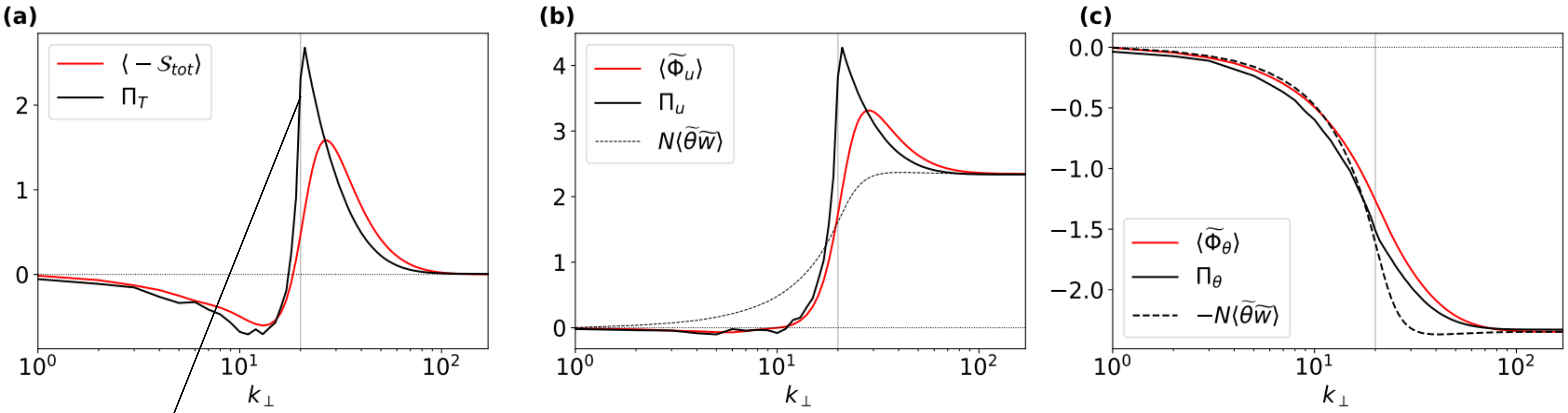
[Foldes et al., (in preparation), 2022]

Sub-Grid terms as a Proxy for the Perpendicular Fourier Flux

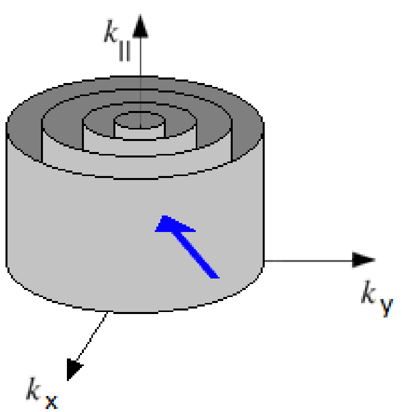
Run I: 512^3, $L_0 = 2\pi$ (triply periodic), Re = 97, $k_F = 20$, $Fr = 0.128$ (*no drafts*)

Total Kinetic/Potential Energy Transfer: $\Phi_{u,\theta} = \langle -\mathcal{S}_{u,\theta} \rangle \pm N \langle \widetilde{\theta} \widetilde{w} \rangle$

"Classical" Fourier Energy Flux: $\Pi_{u,\theta,T}$



[Foldes et al., (in preparation), 2022]



(a) Sub-grid term is able to recover the major features of the classical Fourier flux

(b) Kinetic energy transfer is always from large to small scales

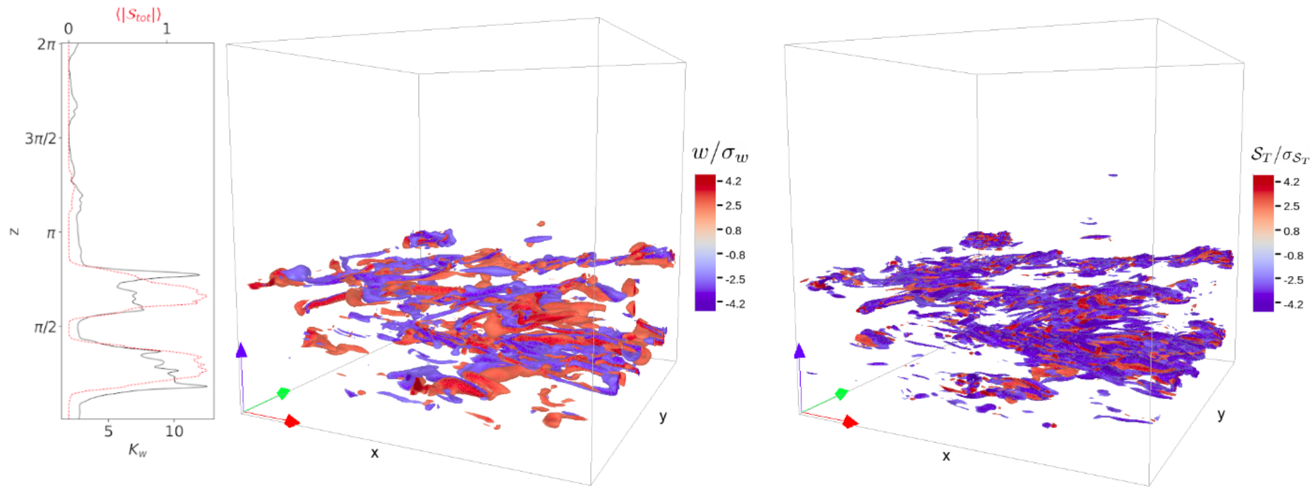
(c) Potential energy transfer is always negative *but* dominated by the conversion of energy from kinetic to potential

➔ Perpendicular Fourier Flux ⟷ Axisymmetric Sub-grid terms

Vertical Drafts as an Energy Injection Mechanism I

Run II: 512^3, $L_0 = 2\pi$ (triply periodic), Re = 3800, $k_F = 2/3$, $Fr = 0.076$ (***drafts***)

- Net total (and kinetic) energy transfer from large to small scales with a wide peak at $7 \le k \le 30$
- In the same range there is a significant conversion of energy from kinetic to potential
- Potential energy see *drafts* as an external energy injection mechanism



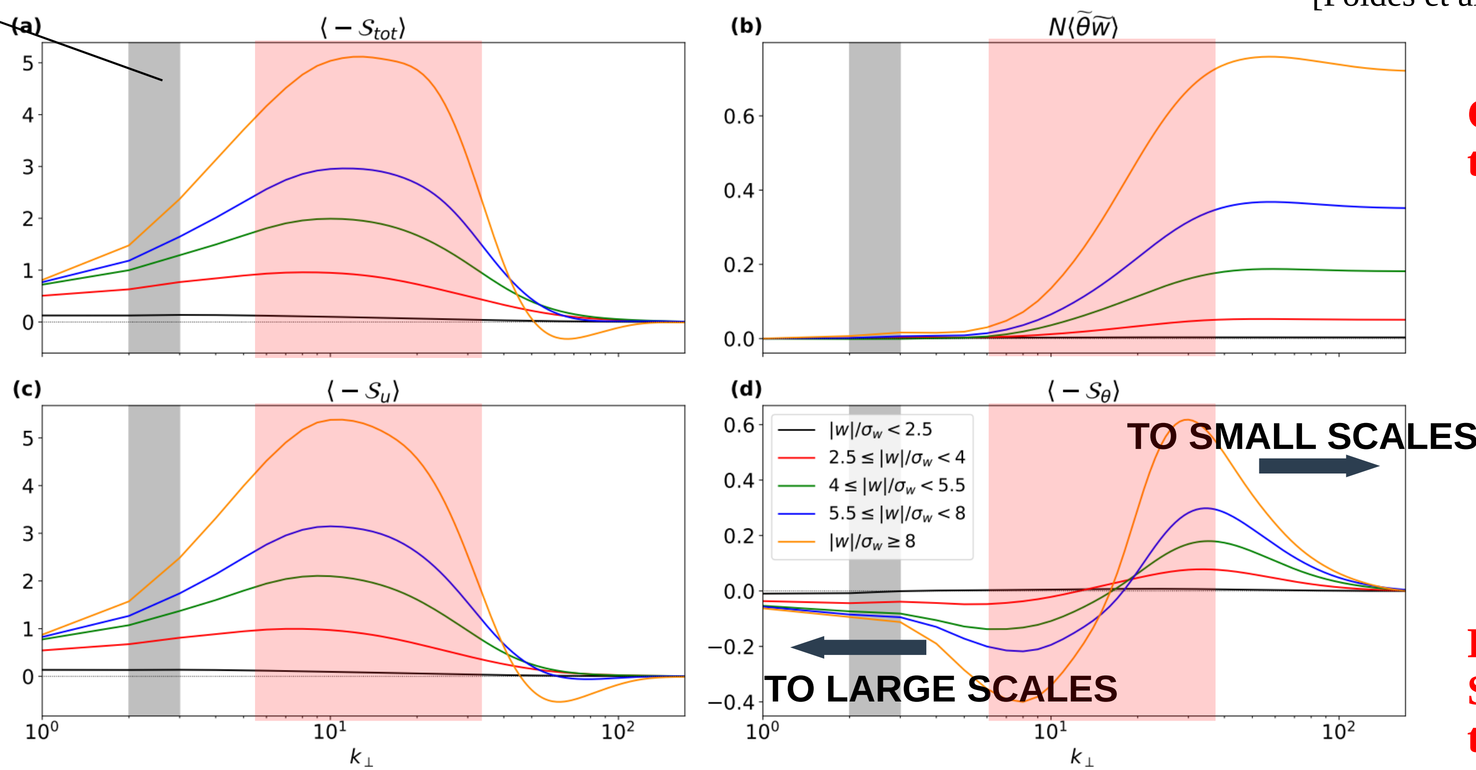
[Foldes et al., (in preparation), 2022]

Forcing (random & isotropic)

Filtered Total Energy

Conversion term

Energy locally injected by vertical drafts?

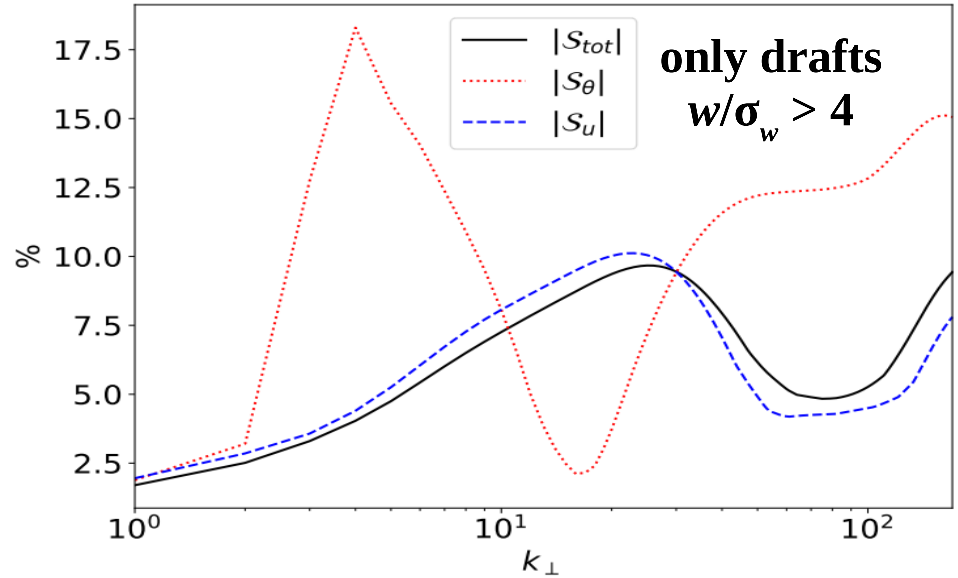


Kinetic Sub-grid term

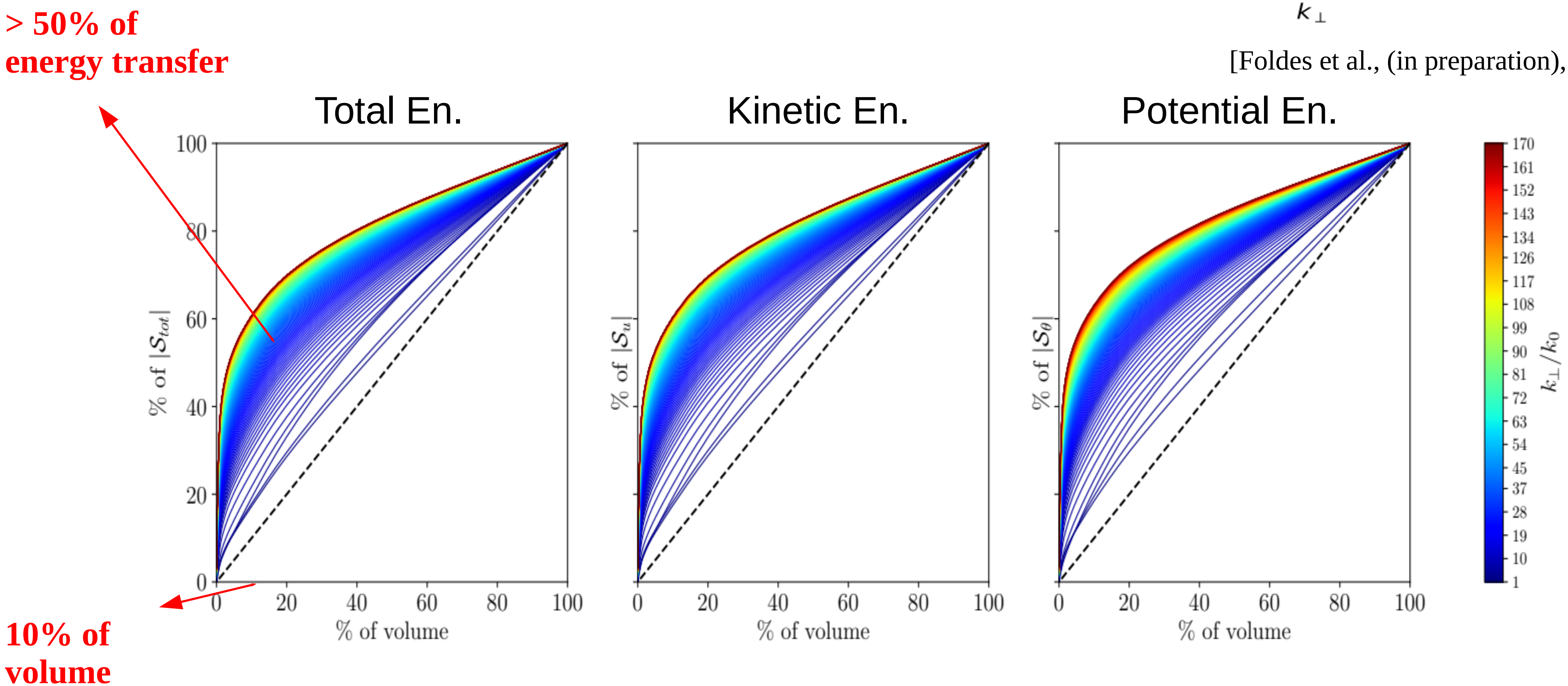
Potential Sub-grid term

Vertical Drafts as an Energy Injection Mechanism II

- At $k > 30$, less than 10% of volume transfers more than 50% of energy
- In particular a tiny percentage of the volume (~ 0.02%, points with $w/\sigma_w > 4$) transfers 5-10% of kinetic energy and up to ~15% of potential



[Foldes et al., (in preparation), 2022]



Thank you for your attention!

Conclusions

- Stratified flows develop extreme vertical drafts in a range of parameters of geophysical interest, making the flow inhomogeneous
- To investigate the feedback of these extreme events on the dynamics and the energetics of the flow we implemented a space-filtering approach starting from the Boussnesq equations.
- We identified a way to obtain a proxy of the energy flux which is local in the physical space and provides information of the same type to those of the classical perpendicular energy flux integrated in cylindrical shells in the Fourier space
- We found that an enhanced energy transfer is associated to a specific range of wave vector which is likely to correspond with the scale of the emerging extreme drafts
- A strong coupling occurs in presence of extreme drafts between kinetic and potential energy, representing a forcing mechanism for the potential temperature field
- This coupling produces a dual transfer of potential energy to small and large scale around $k = 20$

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