# Prediction of evapotranspiration using a nonlinear local approximation approach

IIT Bombay

EGU 2022

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HS3.1: Hydroinformatics: data analytics, machine learning, systems analysis, optimization

# Introduction

- Studies has shown that **evapotranspiration ($ET_0$)** has **chaos properties** (Wang et al., 2014)
- This study **examines chaotic nature** by **predicting** the measured evapotranspiration
- **Phase-Space**, **False Nearest Neighbour** and **Local approximation** chaos methods are used

# Properties of Chaos

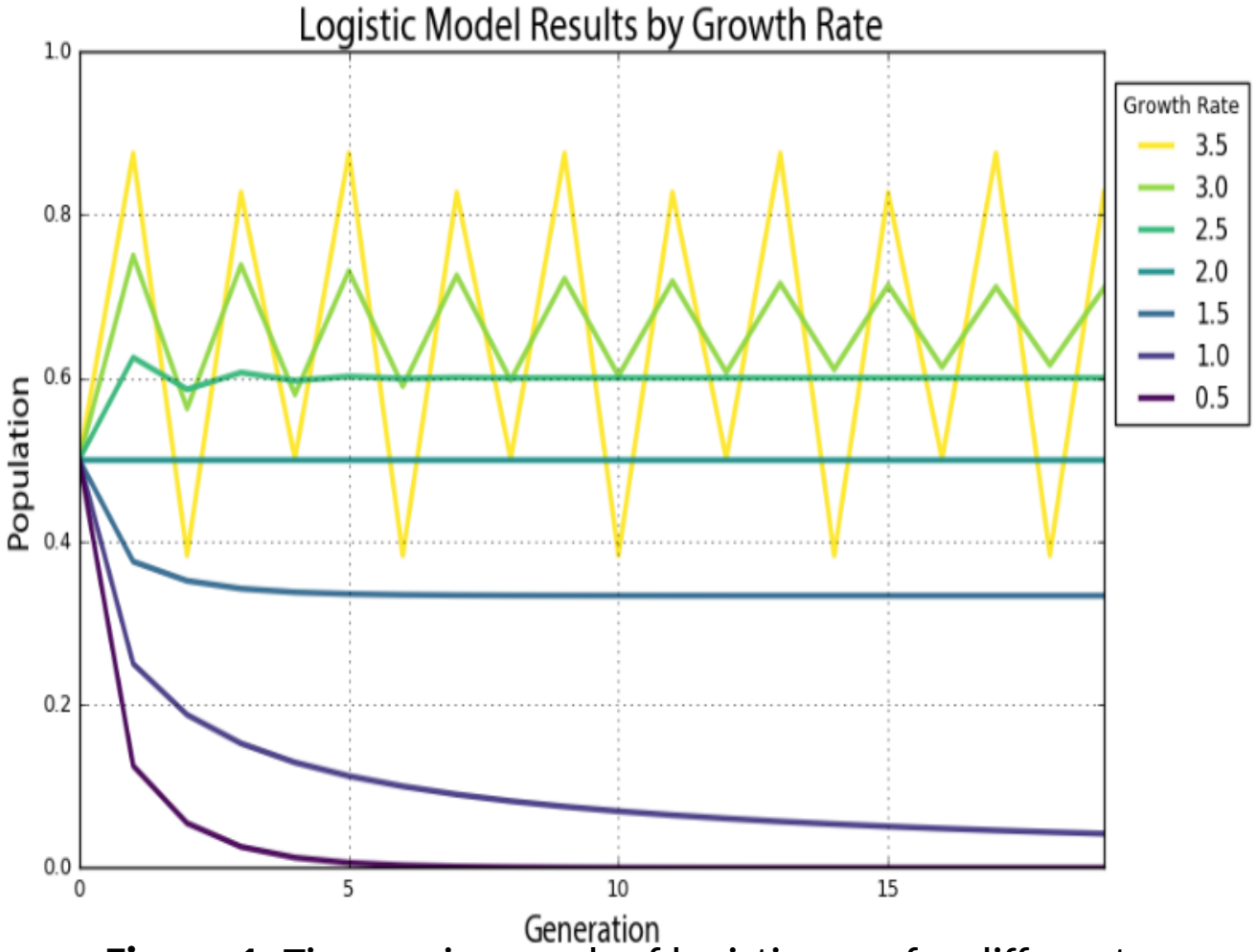



**Figure-1**: Time series graph of logistic map for different population growth rates (Boeing, 2016)

- Sensitive dependence on initial conditions
- Nonlinear dependency
- Hidden determinism

# Phase-Space Reconstruction

- Uses **delay time** to reconstruct time series

| Time series |
|---|
| X1 |
| X2 |
| X3 |
| X4 |
| X5 |
| X6 |
| X7 |
| X8 |
| X9 |
| X10 |

Dimension (1)

$Z_1 = (X_1)$

$Z_2 = (X_2)$

$Z_3 = (X_3)$

$Z_4 = (X_4)$

$Z_5 = (X_5)$

$Z_6 = (X_6)$

$Z_7 = (X_7)$

$Z_8 = (X_8)$

$Z_9 = (X_9)$

$Z_{10} = (X_{10})$

Dimension (2)

$Z_1 = (X_1, X_2)$

$Z_2 = (X_2, X_3)$

$Z_3 = (X_3, X_4)$

$Z_4 = (X_4, X_5)$

$Z_5 = (X_5, X_6)$

$Z_6 = (X_6, X_7)$

$Z_7 = (X_7, X_8)$

$Z_8 = (X_8, X_9)$

$Z_9 = (X_9, X_{10})$

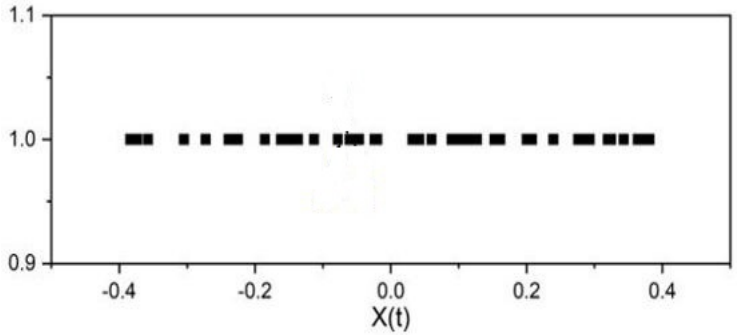



**Figure-2**: Phase space plot for Dimension (1)

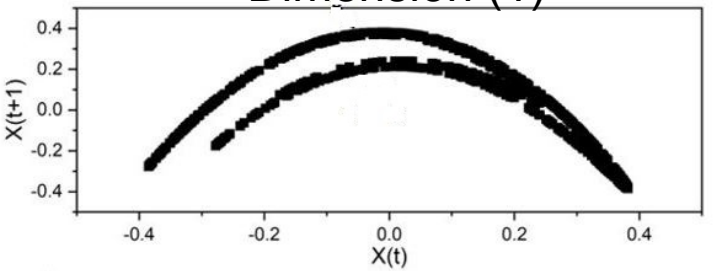



**Figure-3**: Phase space plot for Dimension (2)

# False Nearest Neighbour

- Identifies the number of dominant processes (dimension)
- Uses phase-space diagram to **identify false neighbours**
- Identification of false neighbours based on **distance** and **loneliness** criteria
- **Minimum** percentage of **false neighbours** is the optimum dimension

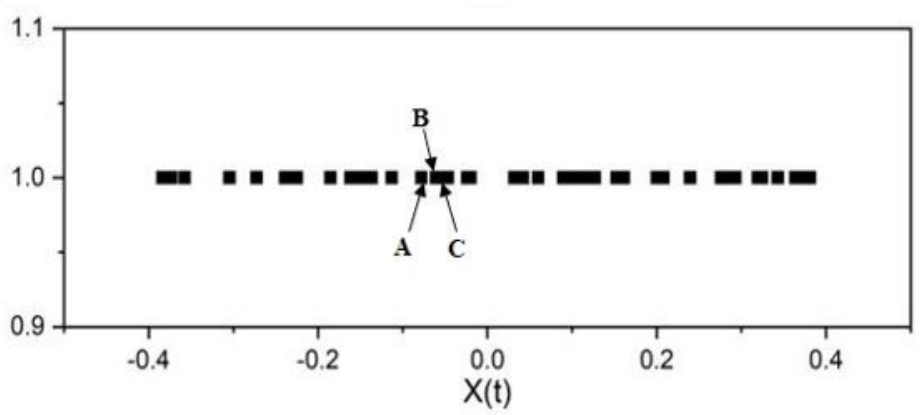



**Figure-4**: Phase space plot for Dimension (1) showing point A, B, and C

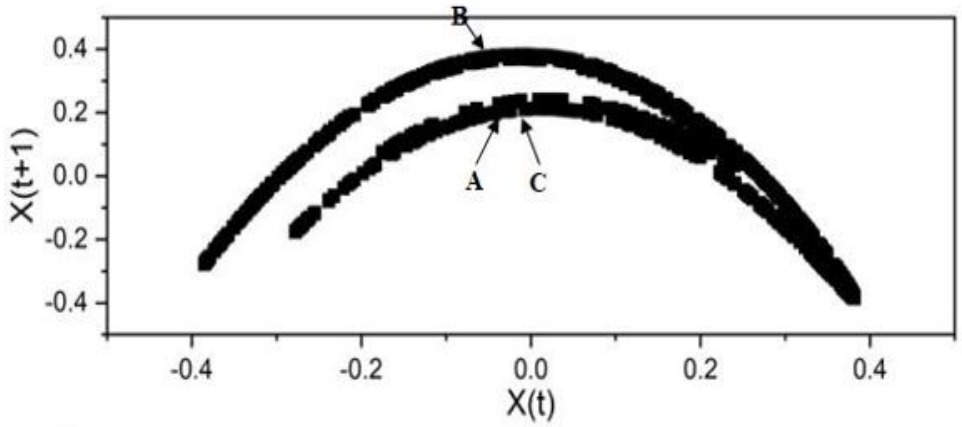



**Figure-5**: Phase space plot for Dimension (2) showing point A, B, and C

# Local approximation

- This method uses phase-space plot to predict the future
- From the **last observed data** (point C), identify **neighbours to C**
- **Prediction** value is considered as the **point A**, is **close** to **point C**

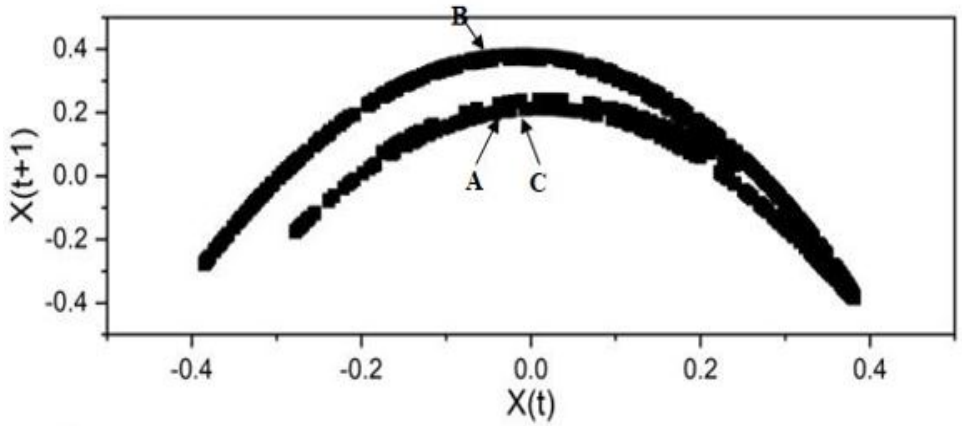



**Figure-5**: Phase space plot for Dimension (2) showing point A, B, and C

# Case Study

- Located in North-Eastern Switzerland in the middle of the Thur river basin
- Area - 3.31 km$^2$, Above MSL 682 to 950 m
- Land use Land cover – Pastureland (71.9%), Forest (25.6%), Orchards (1.2%), and settlements (1.3%)
- Data length – 488 months (1976 - 2015)
- Evapotranspiration (mm)

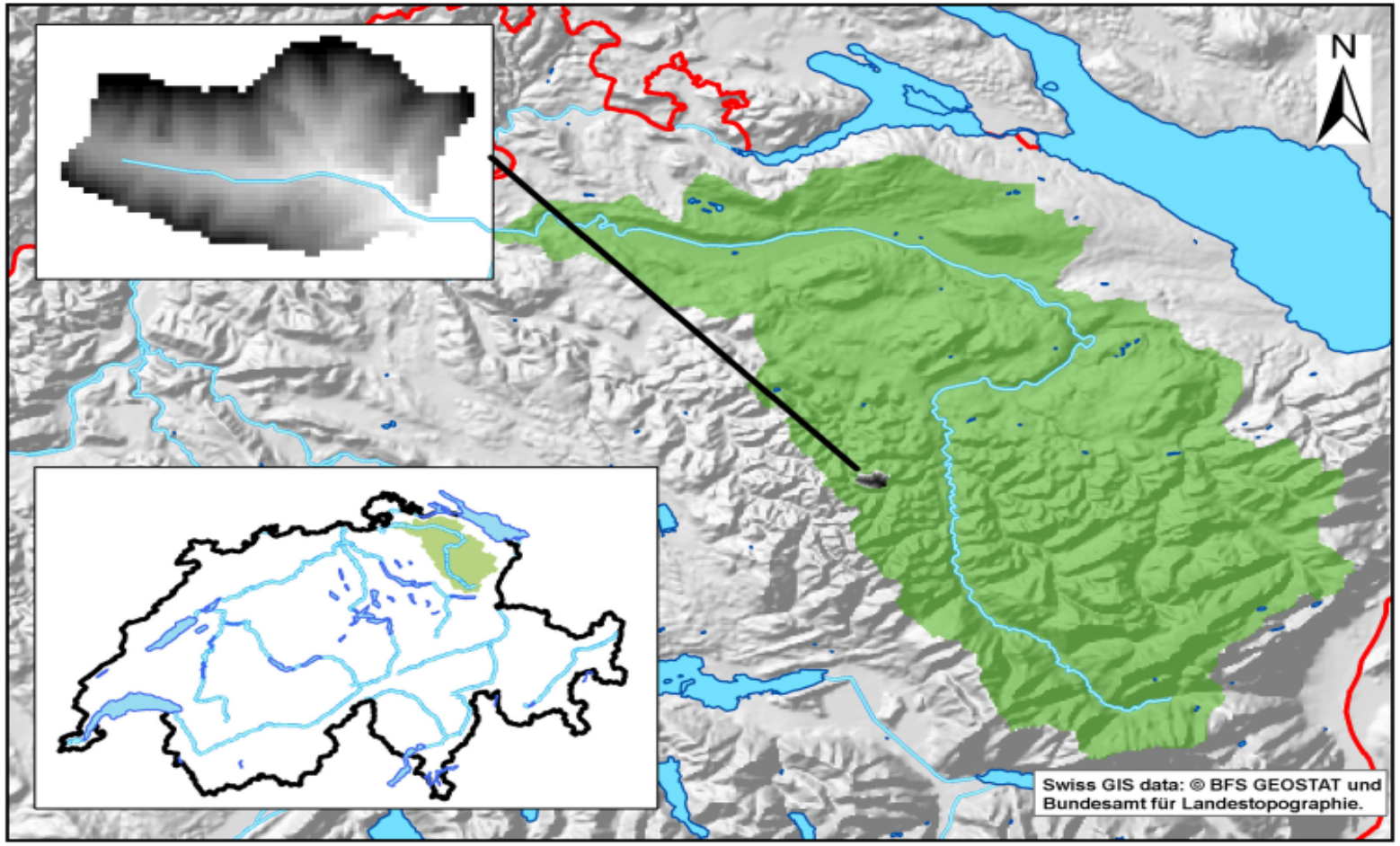
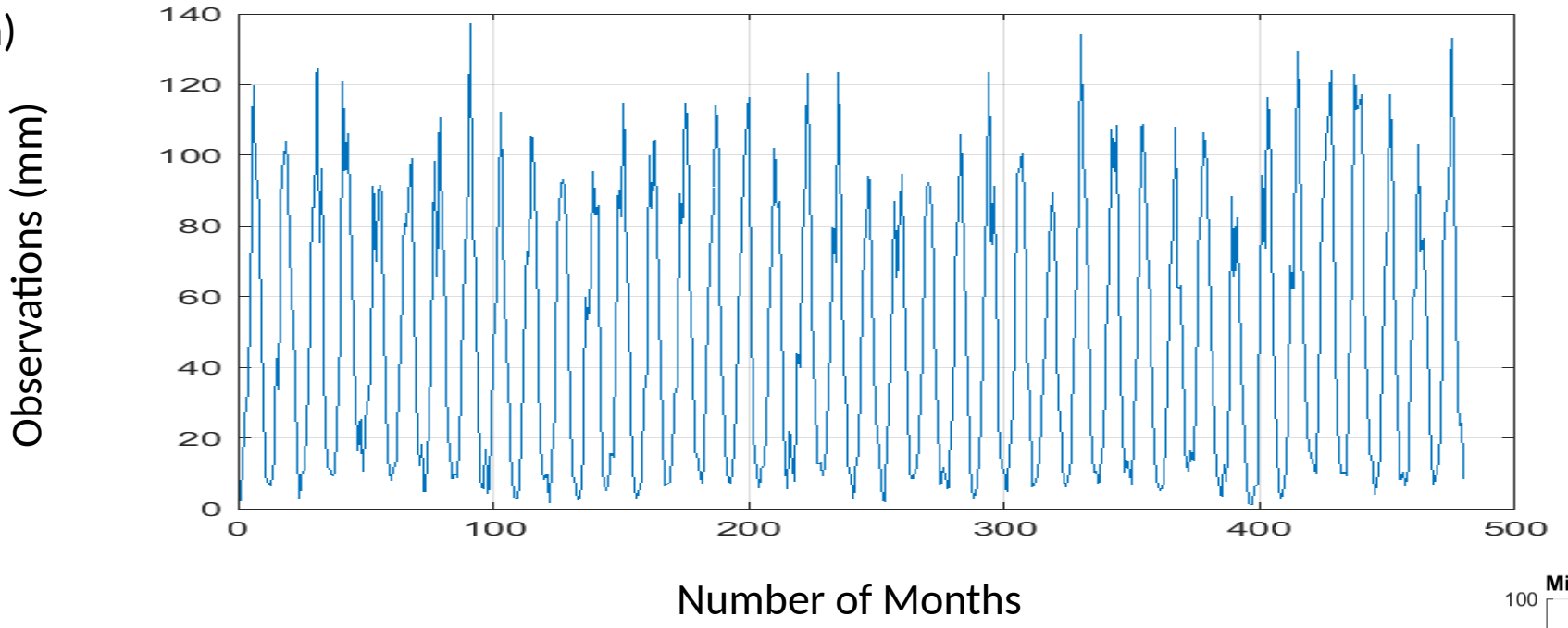



**Figure-6**: Location of Rietholzbach catchment in Switzerland: Thur river basin (green) and Rietholzbach catchment (black). Swiss GIS elements reproduced with the authorization of swisstopo (JA100118) (Seneviratne et al. 2012)

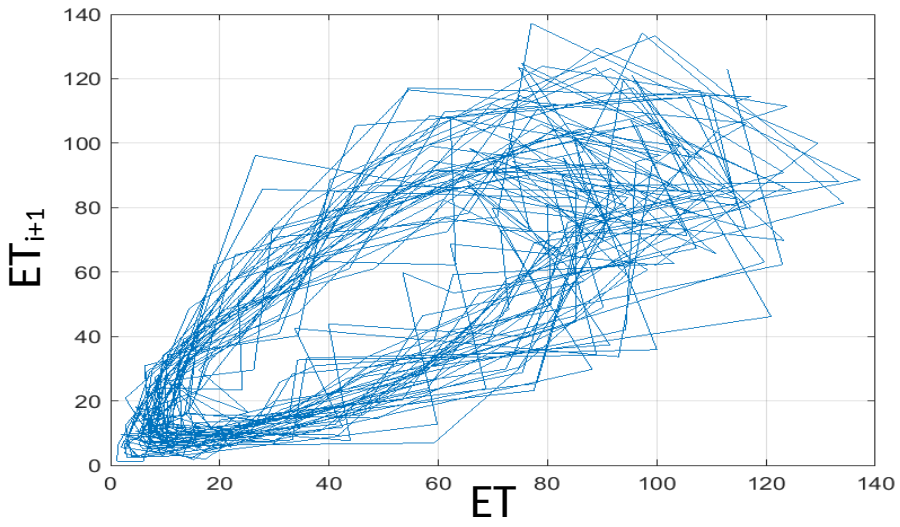
# Evapotranspiration

a)




Length - 488 months
Units - mm
Max - 137.2
Min - 1.2
Avg. - 47.03
Lag - 1
Dim - 4

b)




c)

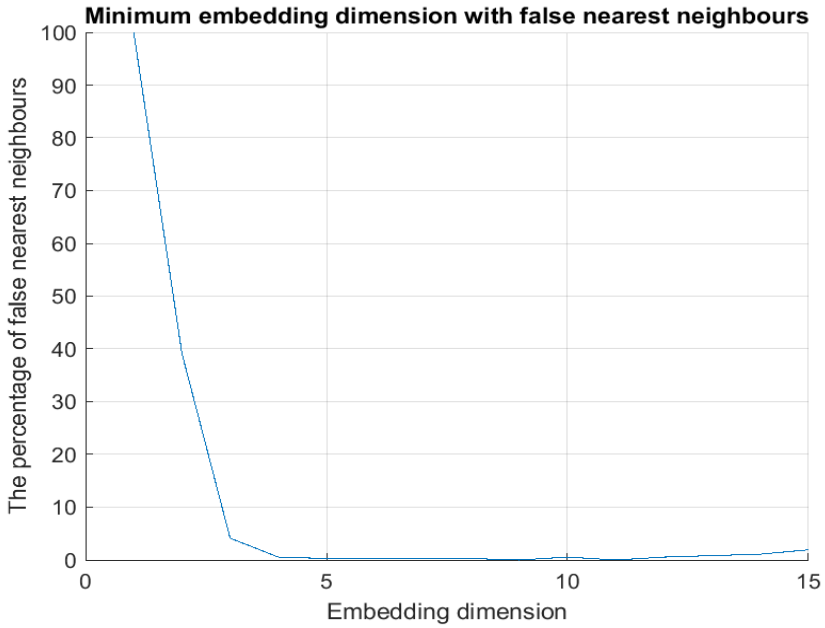
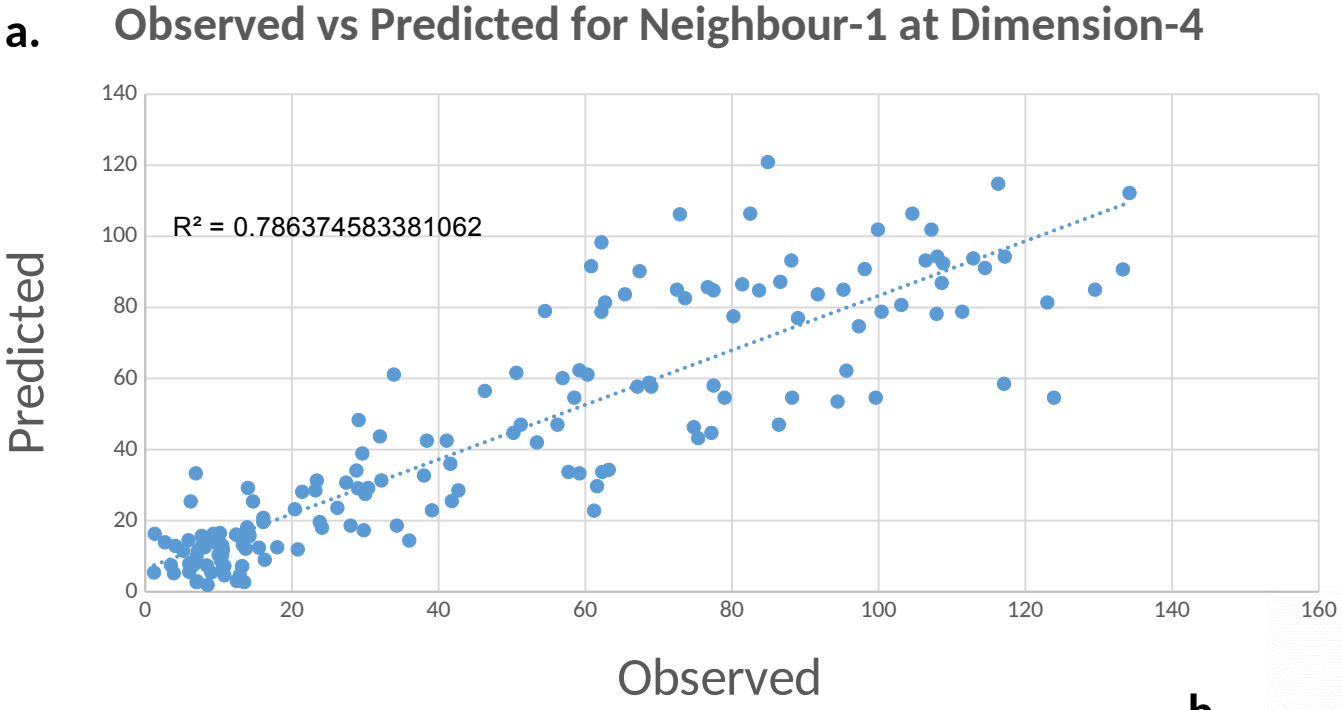



**Figure-7:** a) Time series of evapotranspiration b) Phase Space reconstruction, and c) False Nearest Neighbour plot

a.




b.

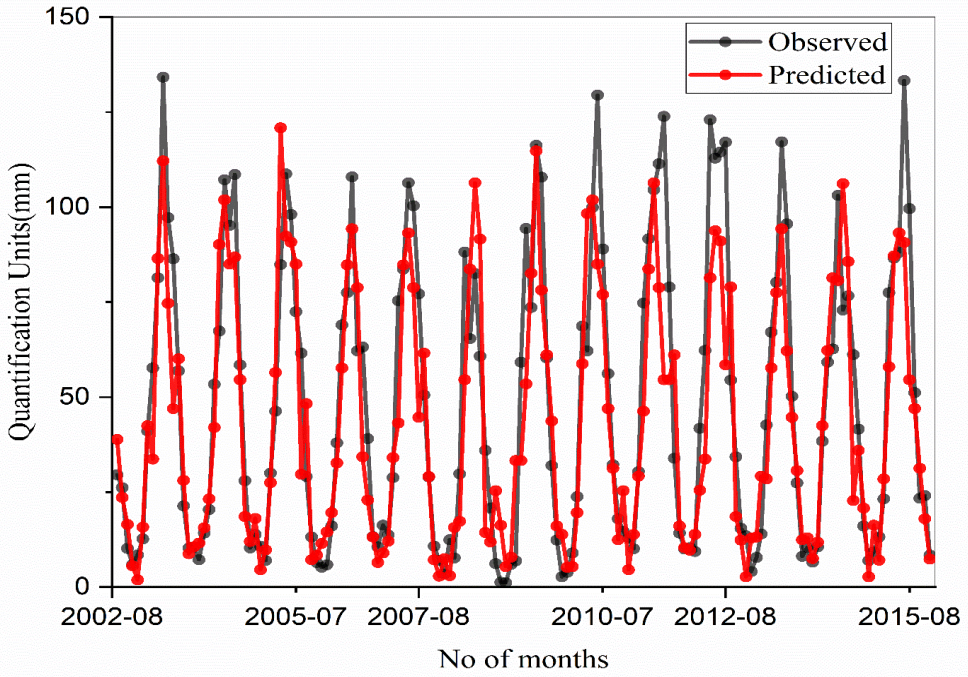
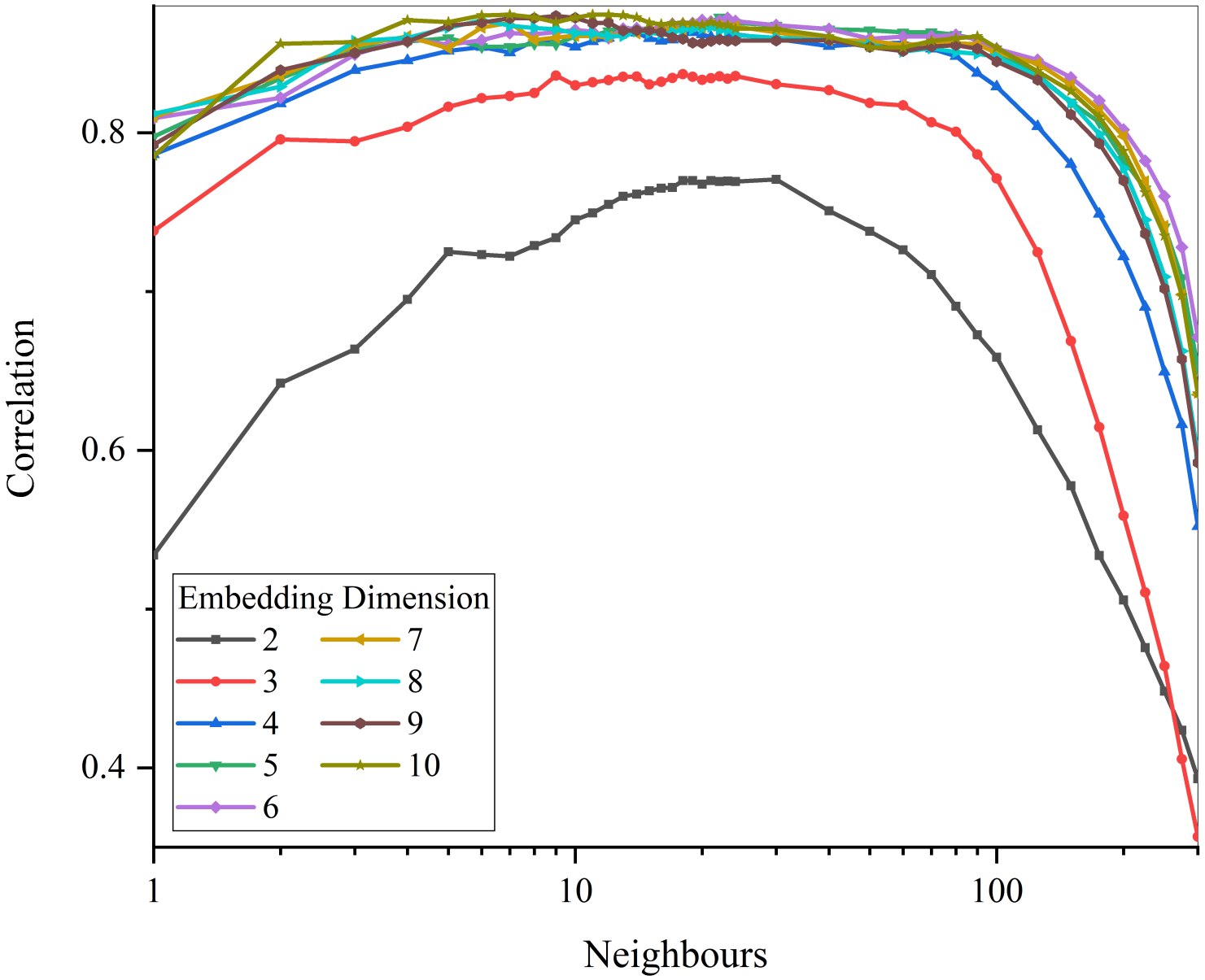



**Figure: 8** (a) Scatterplot and (b) Time series plot for ET data (Neighbour-1; Dimension-4)

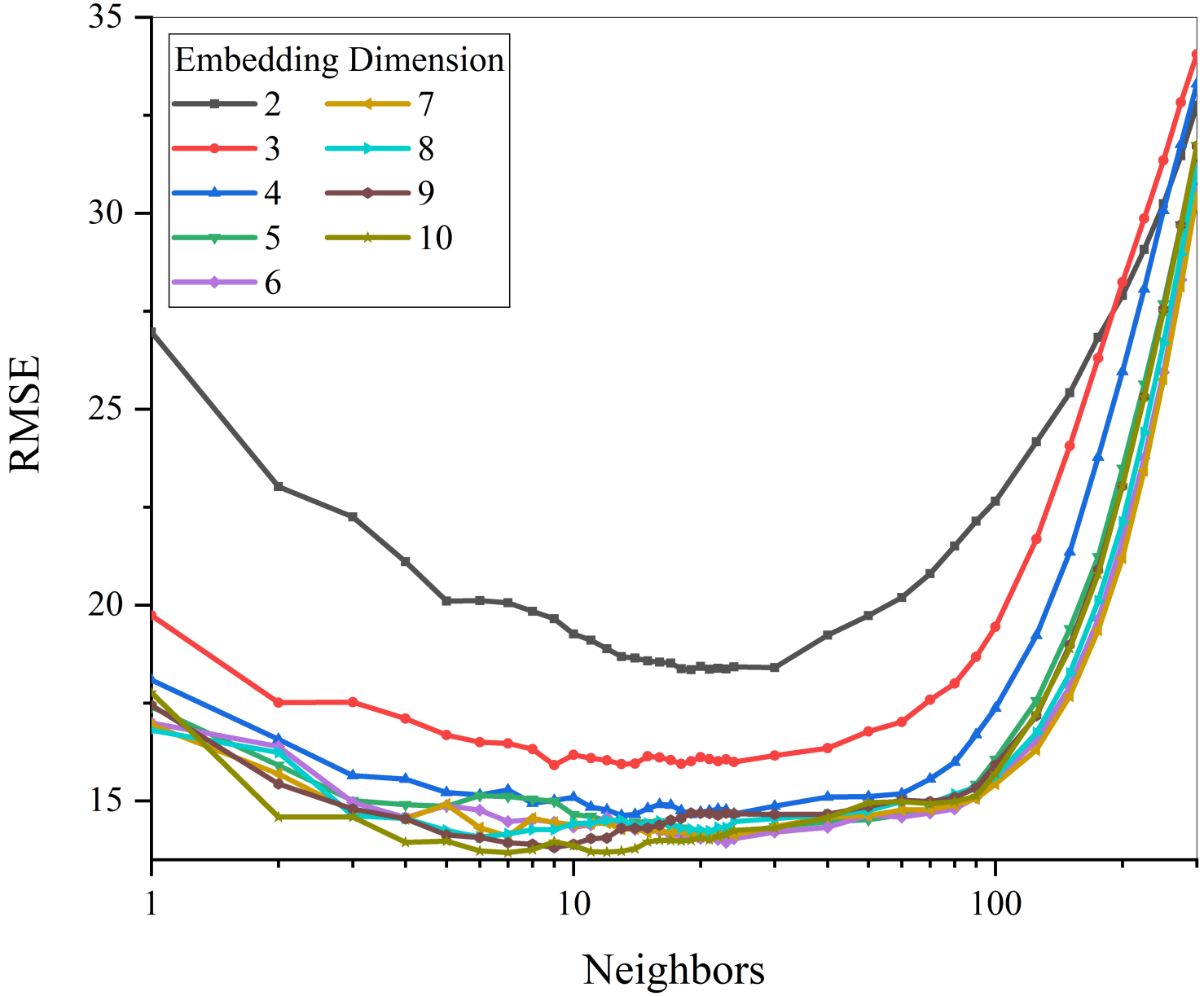
**Figure-9:** Prediction accuracy vs Number of Neighbours for different embedding dimensions ($R^2$)




## Evapotranspiration Data

- Length of Predicted data – 160 months (Direct Prediction)
- Optimum dimension – 4 (FNN)
- Less improved after dimension-4
- Influence of neighbourhood is more for lower dimensions
- $R^2$ value for optimum dimension is around 0.8 for a single neighbour

**Figure-10:** Prediction accuracy vs Number of Neighbours for different embedding dimensions (RMSE)




## Evapotranspiration Data

- Inverted pattern for both $R^2$ and RMSE
- RMSE is comparatively low at optimum dimension
- At 10 th neighbourhood, lowest RMSE

# Conclusions

- Evapotranspiration shows **chaotic** behaviour
- It requires **4** number of independent variables to predict ET
- A **simple** model will suffice to model evapotranspiration
- The data shows **less to no noise** in the measured monthly evapotranspiration