

# Combined orbit and clock zero-difference solution at CODE: ambiguity resolution strategy

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# CONTENT

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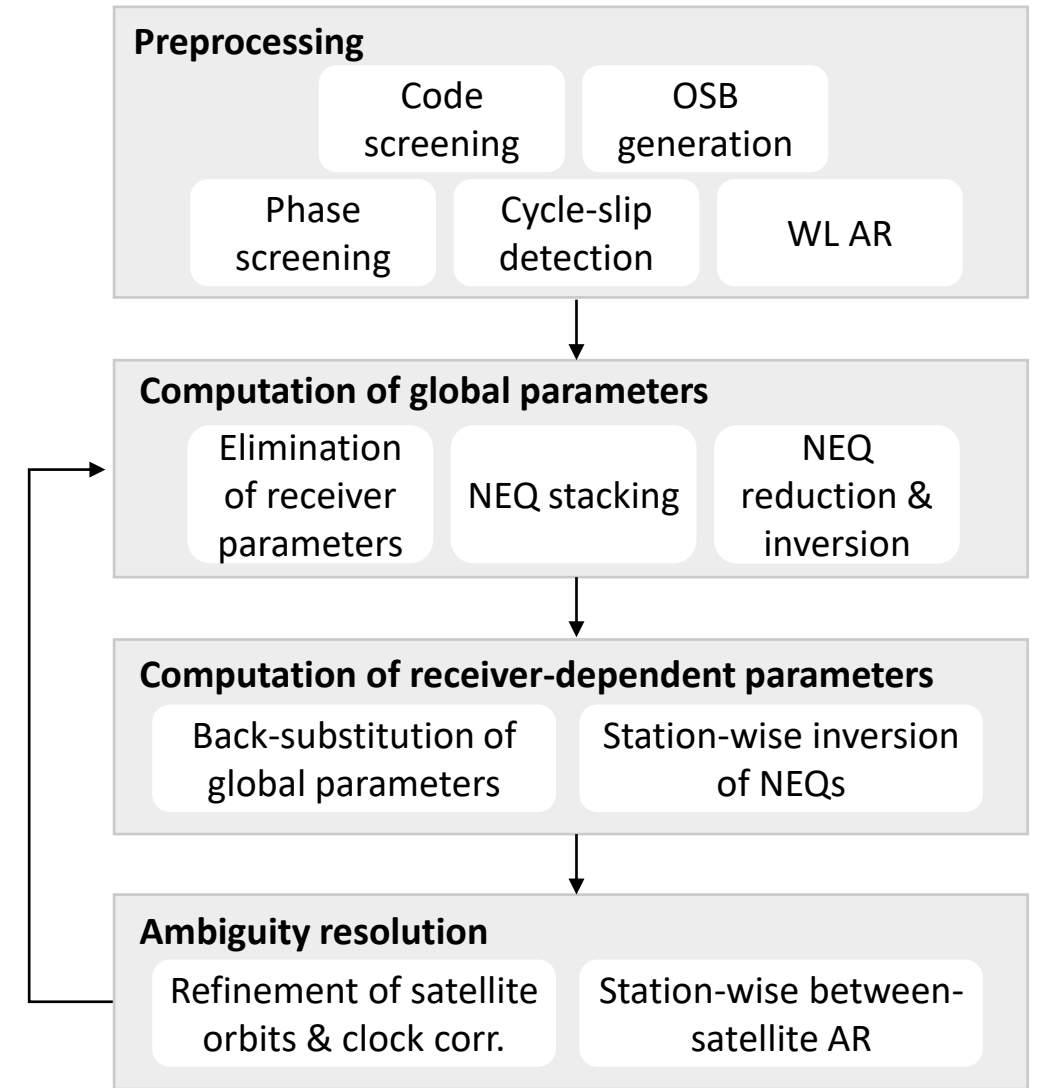
- **Design of the processing scheme**
- **Ambiguity resolution strategy**
- **Validation of the results**

Part I

# DESIGN OF PROCESSING SCHEME

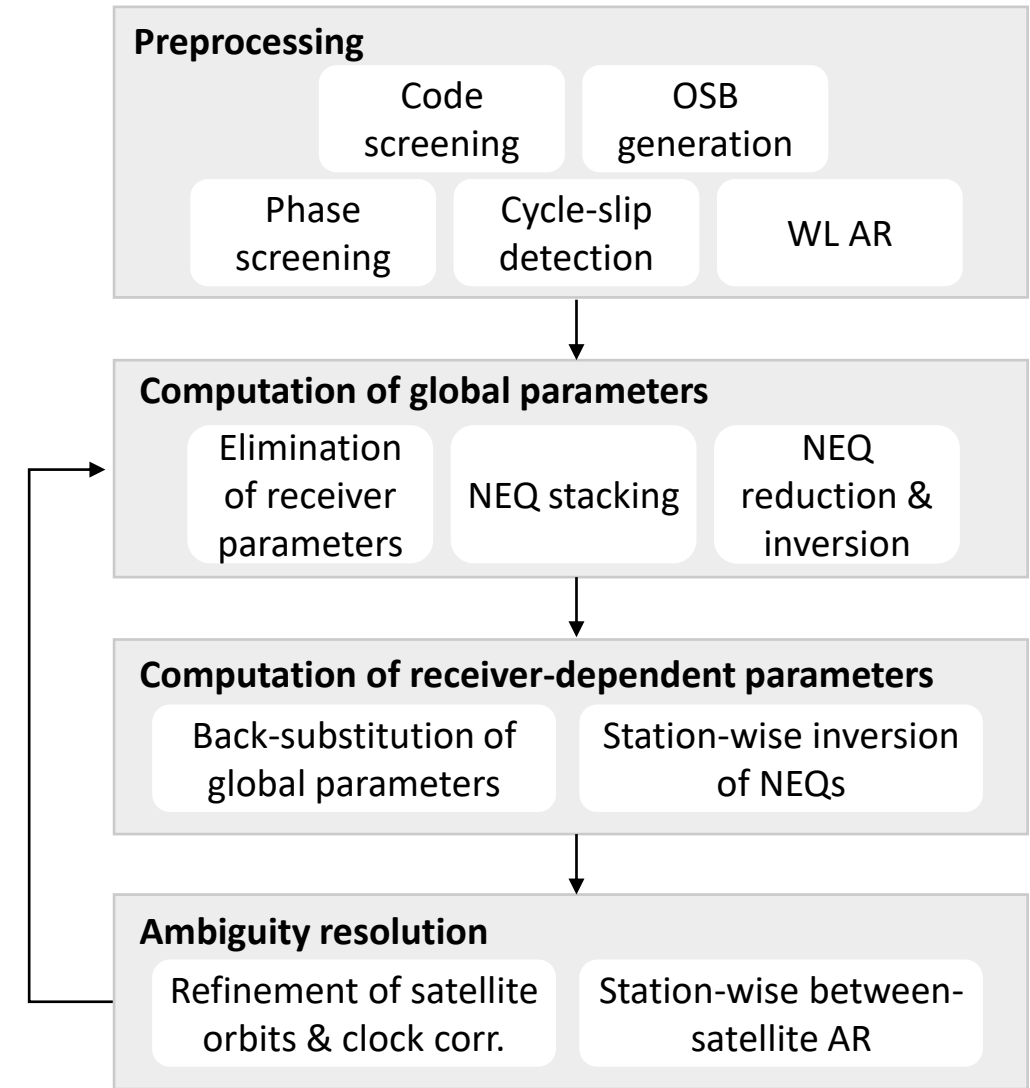
# PROCESSING SCHEME (I)

- ~300 stations and ~75 satellites (GPS, GLONASS and Galileo).
- ~3,000,000 observations (code and phase at 5 min. sampling) and ~100,000 parameters.
- Architecture based on pre-elimination of receiver-dependent parameters in a station-wise parallelization.



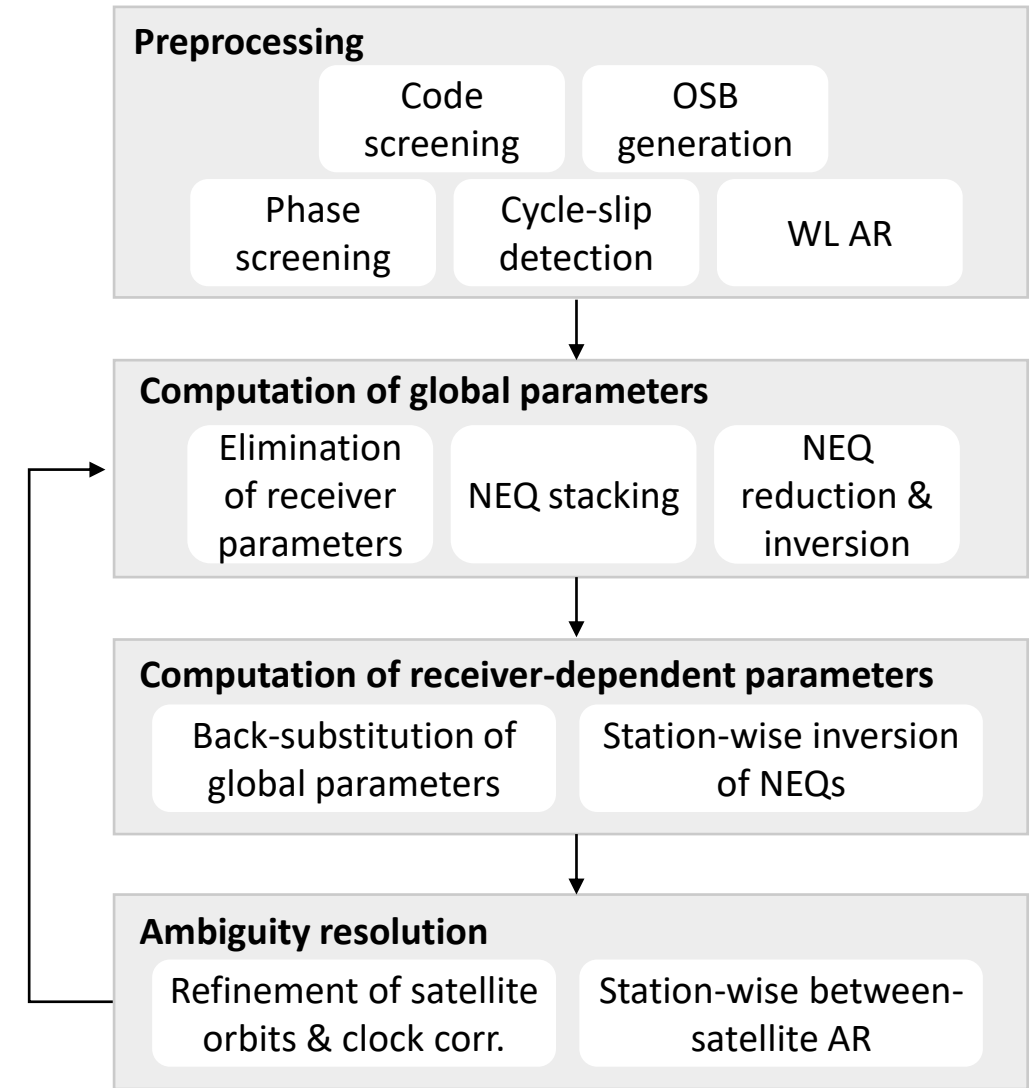
# PROCESSING SCHEME (II)

- **Preprocessing:** screening, calibration of code biases, cycle slip detection and wide-lane ambiguity resolution (based on real-valued ambiguity inspection; calibration of ‘fractional’ biases).



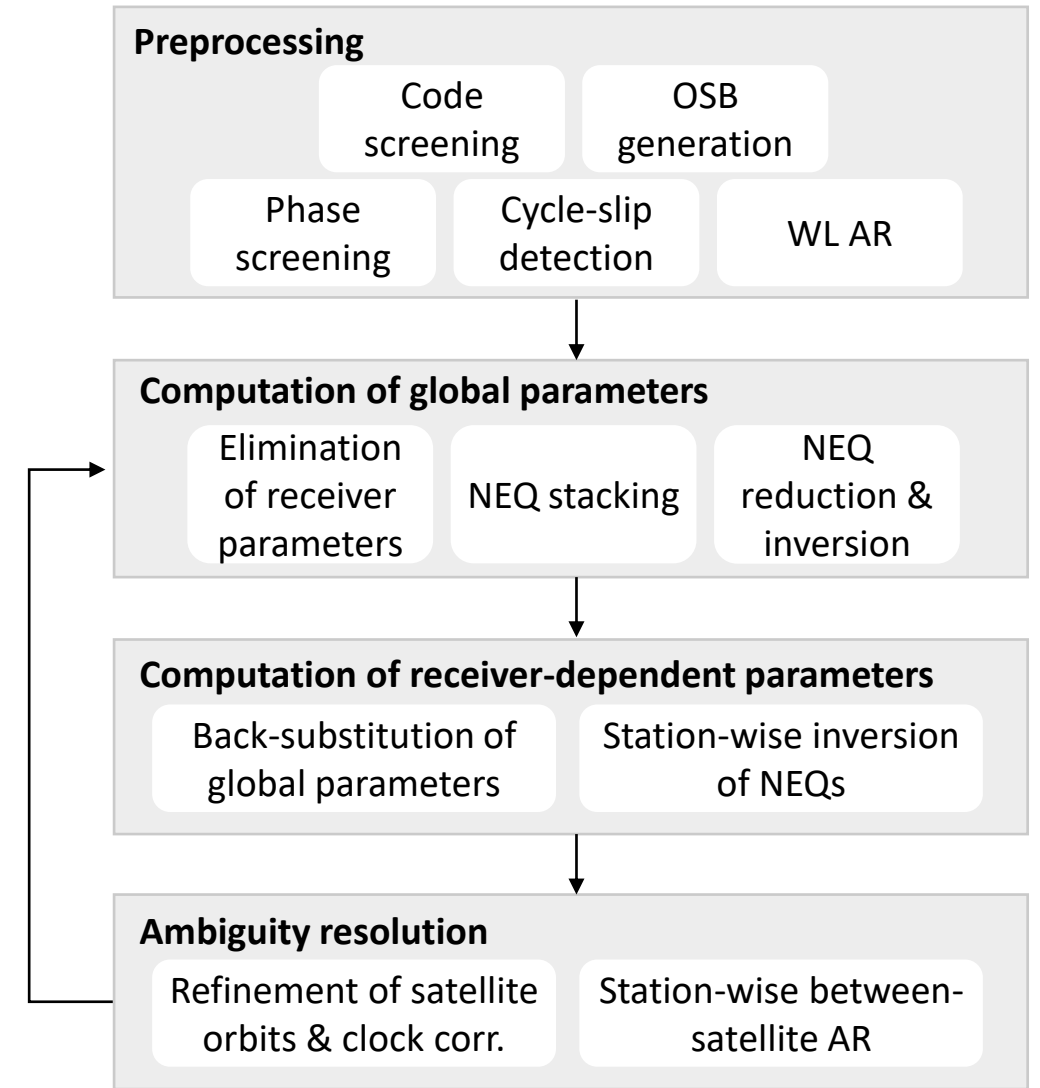
# PROCESSING SCHEME (III)

- **Computation of global parameters:** station-wise parallelization to pre-eliminate receiver parameters (i.e., receiver clock corrections, troposphere parameters and ambiguities). Then, the normal equations are stacked and inverted to compute global parameters (i.e., satellite orbits and clock corrections, ERP and station coordinates).



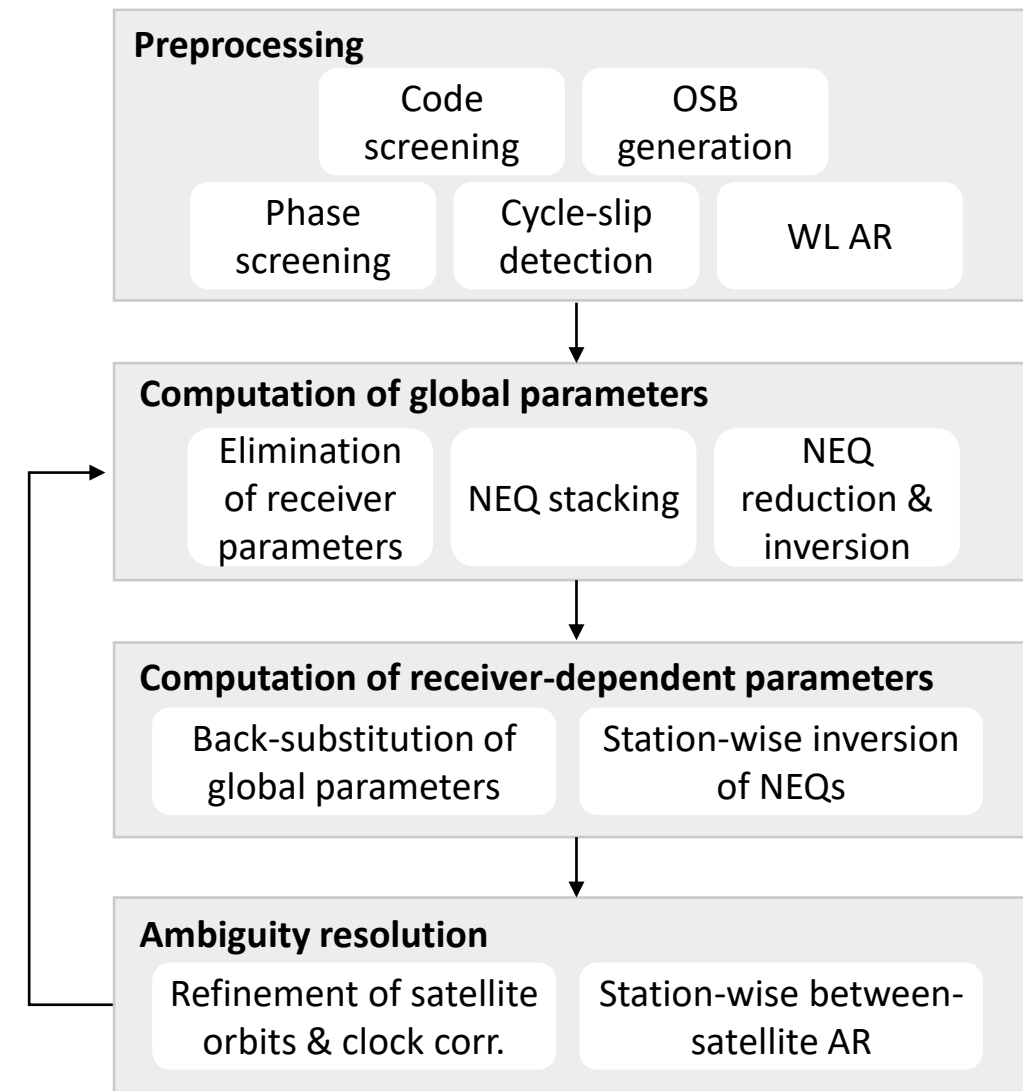
# PROCESSING SCHEME (IV)

- **Computation of receiver-dependent parameters:** once the global parameters are back-substituted, the receiver-dependent parameters are recovered station by station (parallel jobs).



# PROCESSING SCHEME (V)

- **Ambiguity resolution (for narrow-lane ambiguities):** the ambiguities are inspected by means of a mixed-integer model that allows to infer the coupling with the satellite products. Once this coupling is compensated for, the ambiguities are resolved station by station.



Part II

# AMBIGUITY RESOLUTION STRATEGY

# AMBIGUITY RESOLUTION: OVERVIEW (I)

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- Two-step ambiguity resolution strategy (based on **real-valued ambiguity inspection**):
  1. Resolving wide-lane ambiguities: inspection of real-valued wide-lane ambiguities to calibrate ‘fractional biases’.
  2. Resolving narrow-lane ambiguities: inspection of real-valued narrow-lane ambiguities to calibrate ‘coupling between ambiguities and satellite-dependent parameters’.

# AMBIGUITY RESOLUTION: OVERVIEW (II)

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The inspection of real-valued ambiguities to support ambiguity resolution has been explored by different authors, e.g.:

Ge M, Gendt G, Rothacher MA, Shi C, Liu J (2008) Resolution of GPS carrier-phase ambiguities in precise point positioning (PPP) with daily observations. J Geod, 82(7), 389-399.

<https://doi.org/10.1007/s00190-007-0187-4>

Laurichesse D, Mercier F, Berthias JP, Broca P, Cerri L (2009) Integer ambiguity resolution on undifferenced GPS phase measurements and its application to PPP and satellite precise orbit determination. Navigation, 56(2), 135-149. <https://doi.org/10.1002/j.2161-4296.2009.tb01750.x>

Schaer S, Villiger A, Arnold D, Dach R, Prange L, Jäggi A (2021) The CODE ambiguity-fixed clock and phase bias analysis products: generation, properties, and performance. J Geod, 95(7), 1-25.

<https://doi.org/10.1007/s00190-021-01521-9>

# AMBIGUITY RESOLUTION: WIDE-LANE (I)

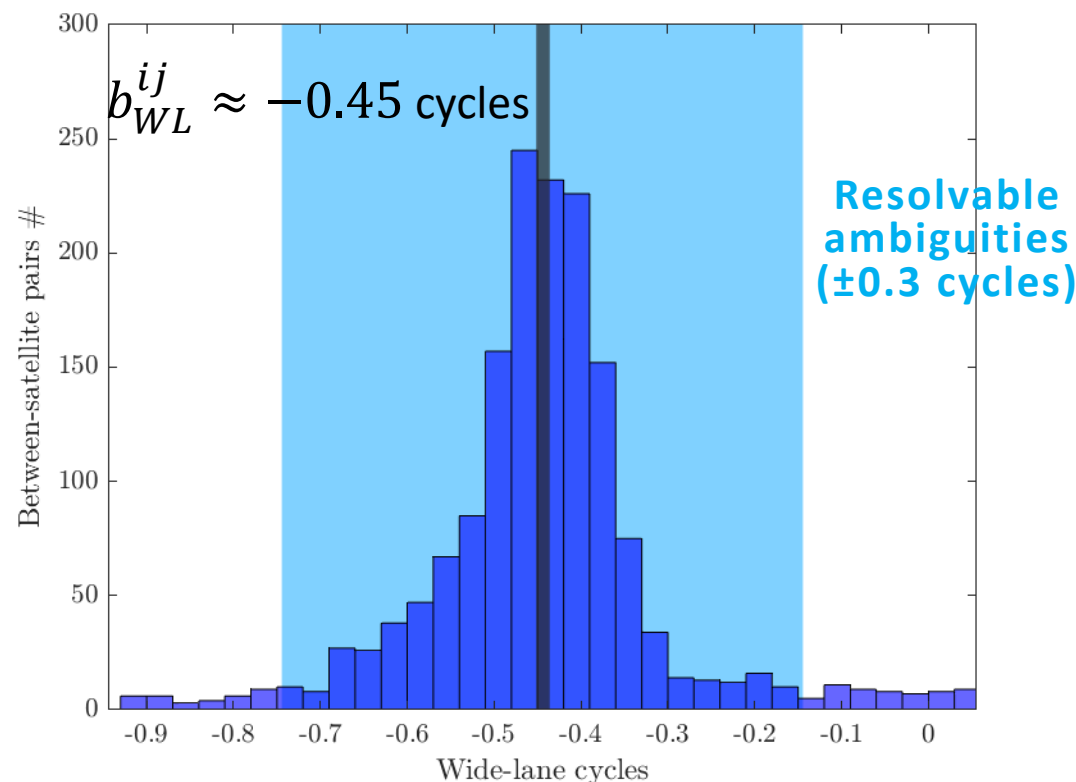
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$$MW_k^{ij} = \lambda_{WL} \underbrace{\left( N_{k,WL}^{ij} + b_{WL}^{ij} \right)}_{B_{k,WL}^{ij}}$$

- Inspecting the estimated parameters  $B_{k,WL}^{ij}$  for a network of stations, we should be able to calibrate the phase biases  $b_{WL}^{ij}$ , granting access to the integer ambiguities.

# AMBIGUITY RESOLUTION: WIDE-LANE (II)

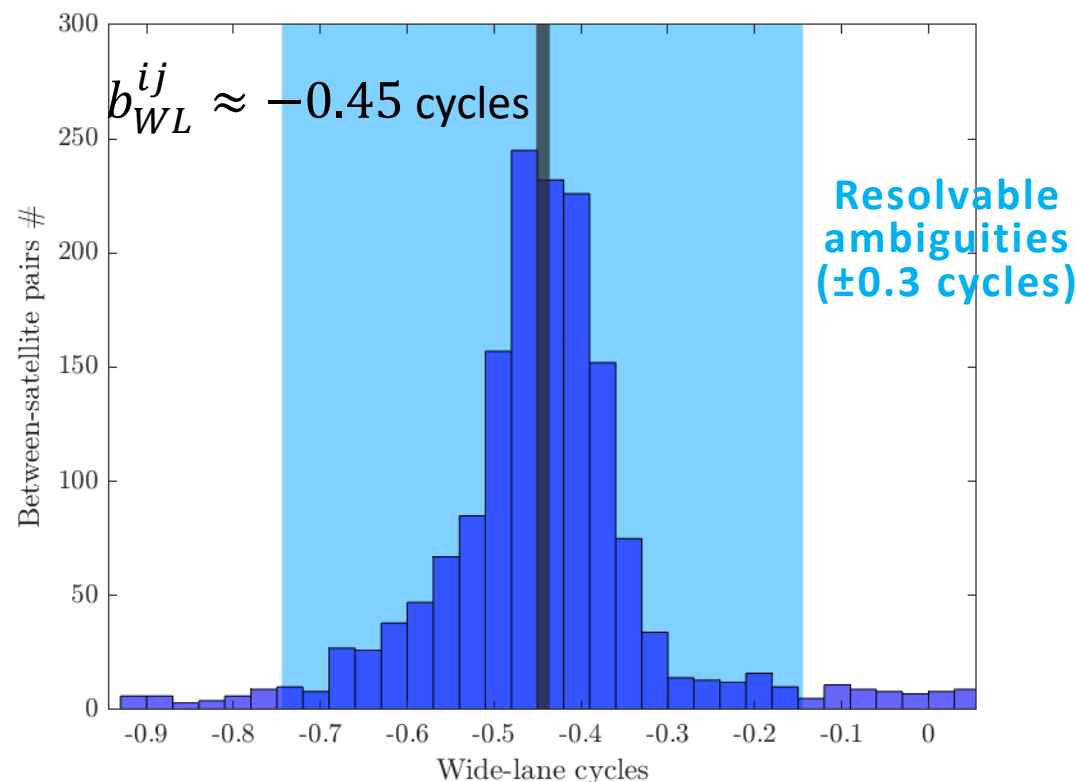
Example of  $B_{k,WL}^{ij}$  inspection and  $b_{WL}^{ij}$  calibration on February 19<sup>th</sup>, 2021 for the between-satellite pair G21/G22



The histogram shows the fractional part of the  $B_{k,WL}^{ij}$  ambiguities for a fixed between-satellite pair and a network of ~300 stations. Deviations from a theoretical gaussian distribution may originate from deficiencies in the calibration of code biases for the various signal types.

# AMBIGUITY RESOLUTION: WIDE-LANE (III)

Example of  $B_{k,WL}^{ij}$  inspection and  $b_{WL}^{ij}$  calibration on February 19<sup>th</sup>, 2021 for the between-satellite pair G21/G22



Once the phase biases are calibrated, the integer nature of the between-satellite WL ambiguities is unveiled, allowing to resolve them.

Satellite-dependent biases are calibrated with a set of redundant, properly aligned, between-satellite pairs.

# AMBIGUITY RESOLUTION: NARROW-LANE (I)

$$L_k^{ij} = \rho_k^{ij} - \tau^{ij} + T_k^{ij} + \lambda_{NL} \left( N_{k,NL}^{ij} + b_{NL}^{ij} + \frac{\lambda_2}{\lambda_{WL}} N_{k,WL}^{ij} \right)$$

- The narrow-lane ambiguities ( $N_{k,NL}^{ij}$ ) does not only absorb phase biases, but, due to the correlations, they are also contaminated by other error sources (e.g., related to the satellite orbits).

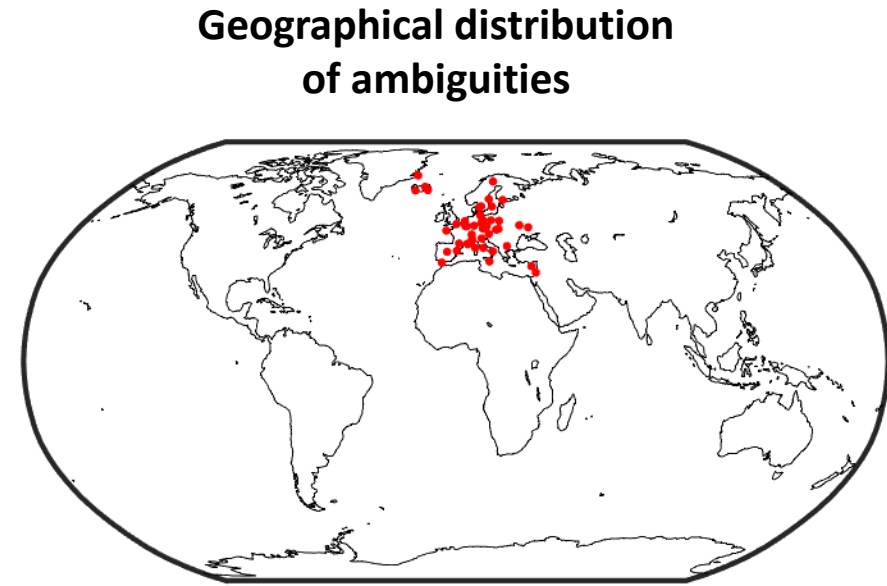
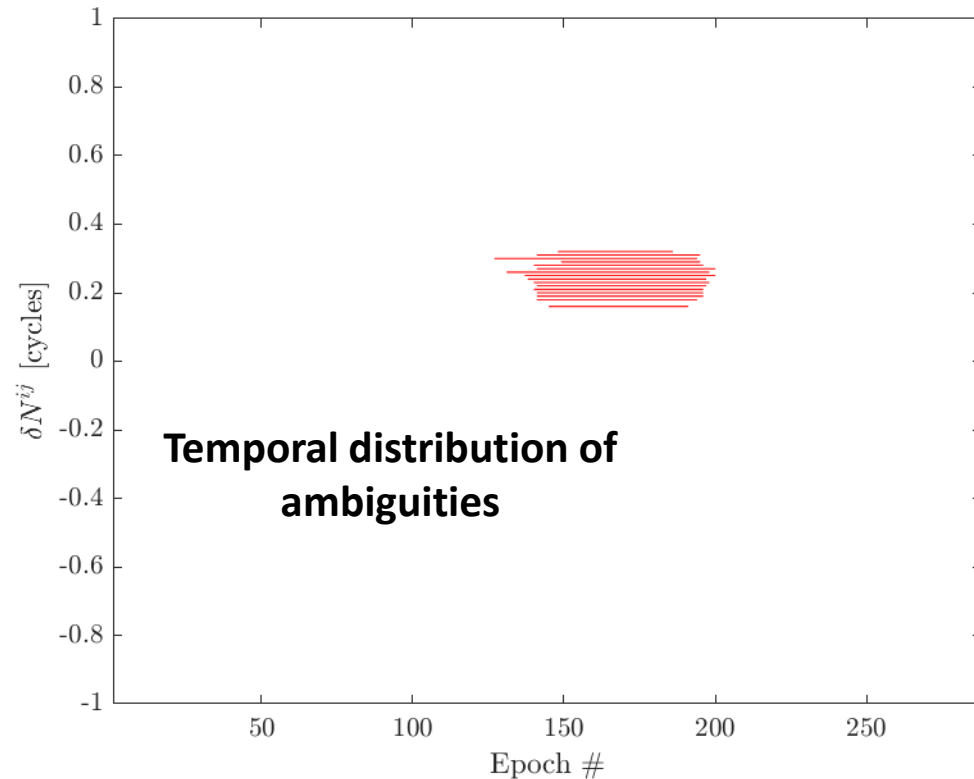
# AMBIGUITY RESOLUTION: NARROW-LANE (II)

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- It does not make sense to talk about ‘fractional part’ of the NL ambiguities. However, it does make sense to talk about **local** fractional part. Those ambiguities which...
  - ... are close in time (spanning similar time intervals), and
  - ... are close in space (belonging to stations that are close within the surface of the Earth)share a similar fractional part.
- The NL ambiguities are systematically grouped into **ambiguity clusters** according to the aforementioned physical conditions

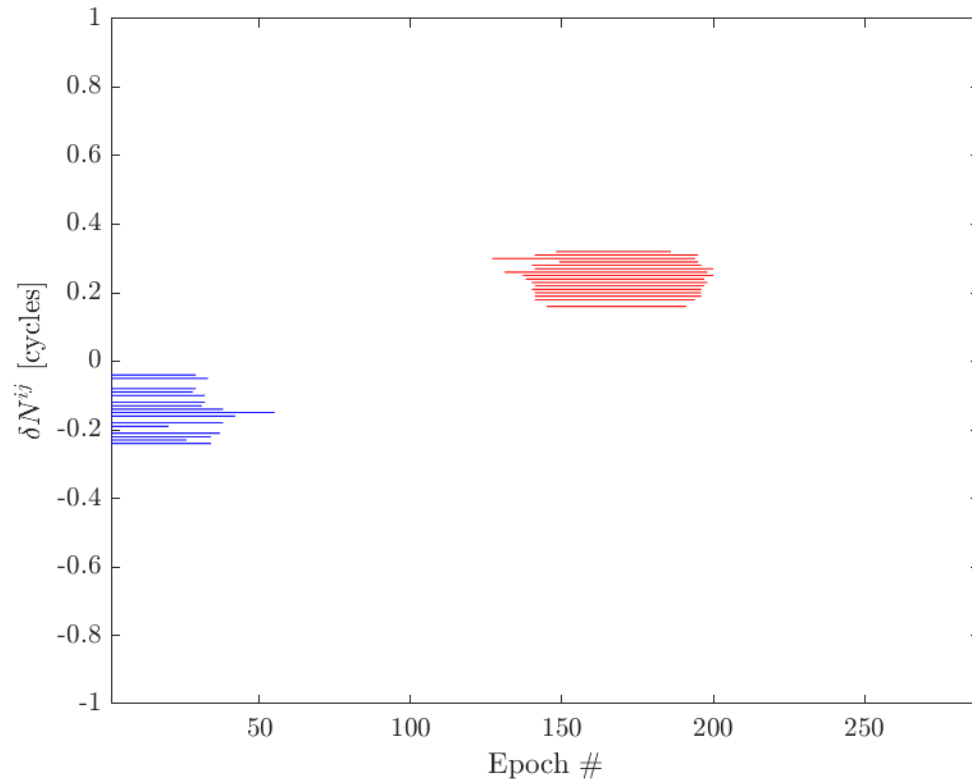
# GENERATION OF AMBIGUITY CLUSTERS (ITE #01)

- Between-satellite pair E11/E25 on February 19<sup>th</sup>, 2021

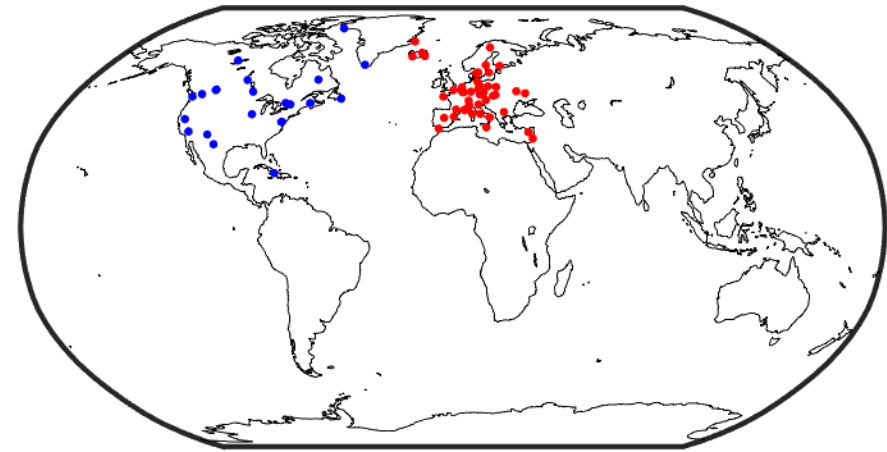


# GENERATION OF AMBIGUITY CLUSTERS (ITE #02)

- Between-satellite pair E11/E25 on February 19<sup>th</sup>, 2021

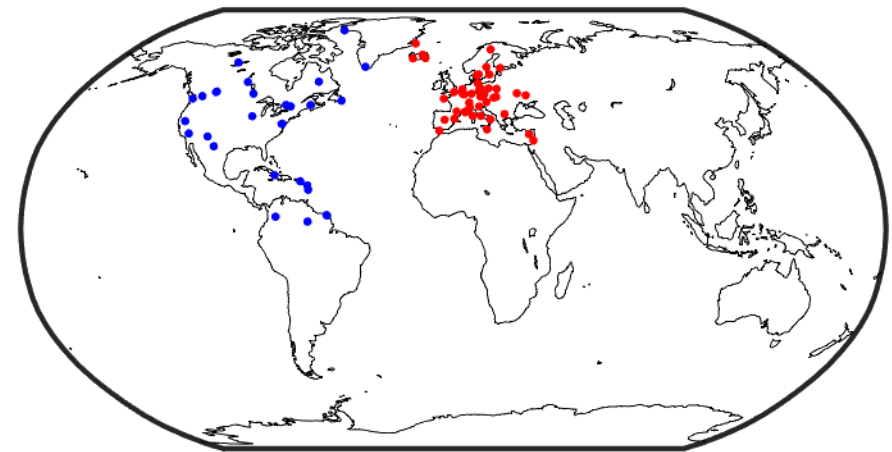
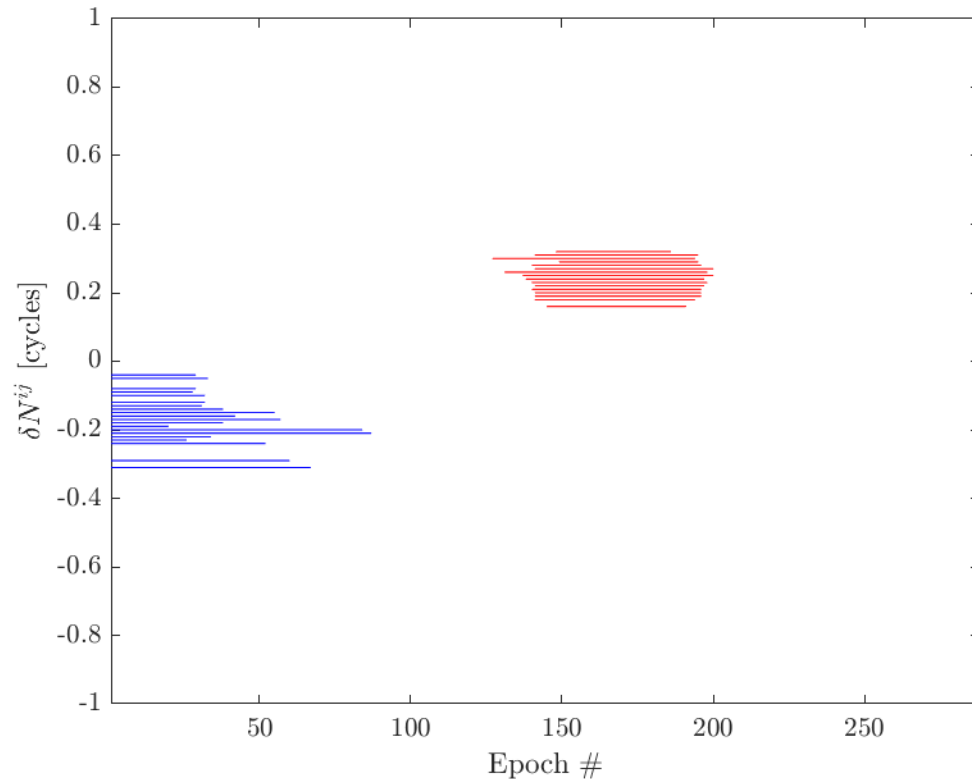


Different ambiguity clusters  
denoted by different colours



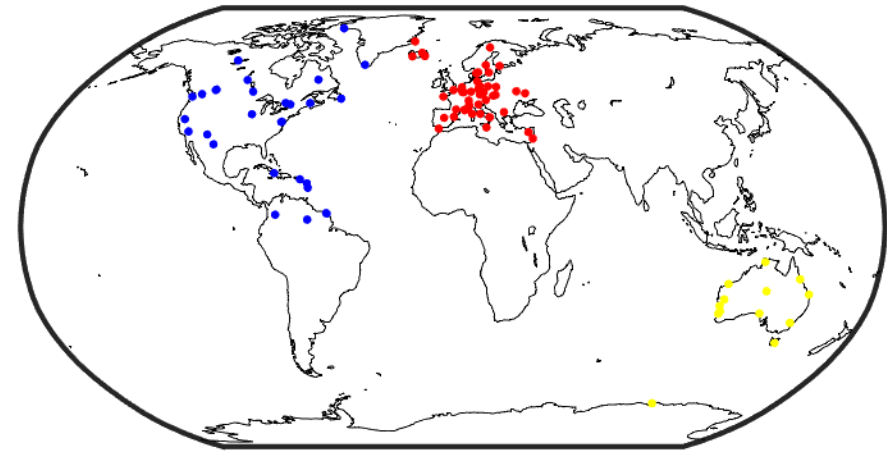
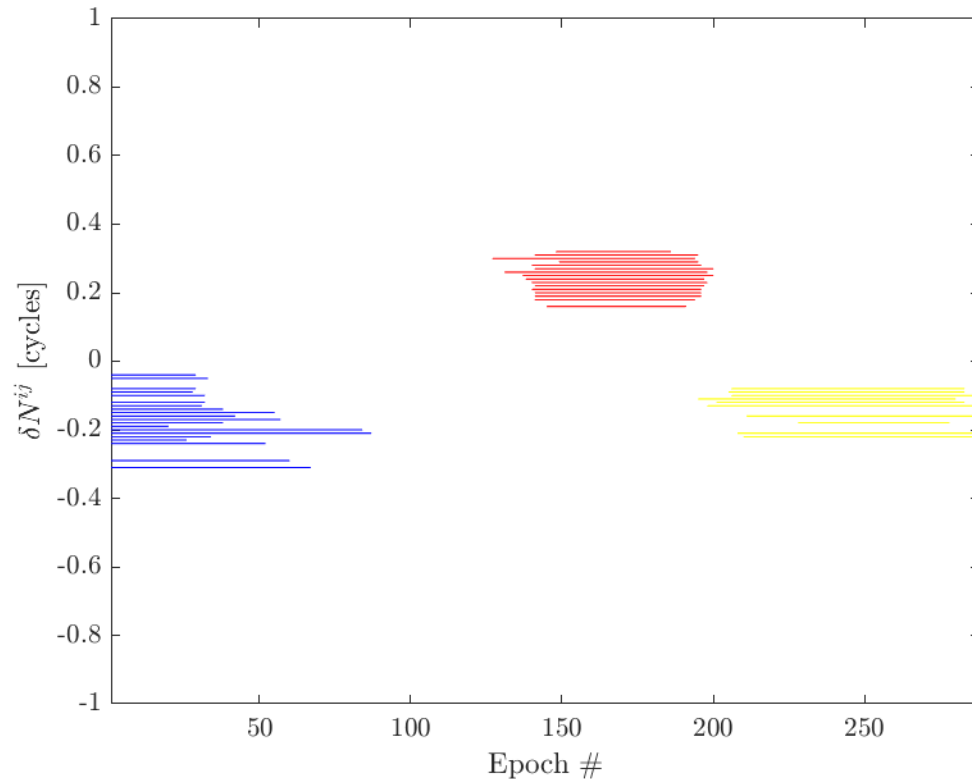
# GENERATION OF AMBIGUITY CLUSTERS (ITE #03)

- Between-satellite pair E11/E25 on February 19<sup>th</sup>, 2021



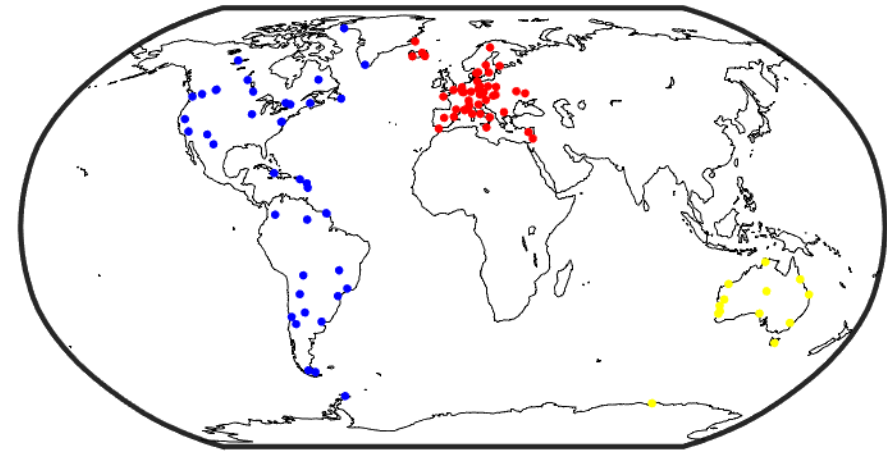
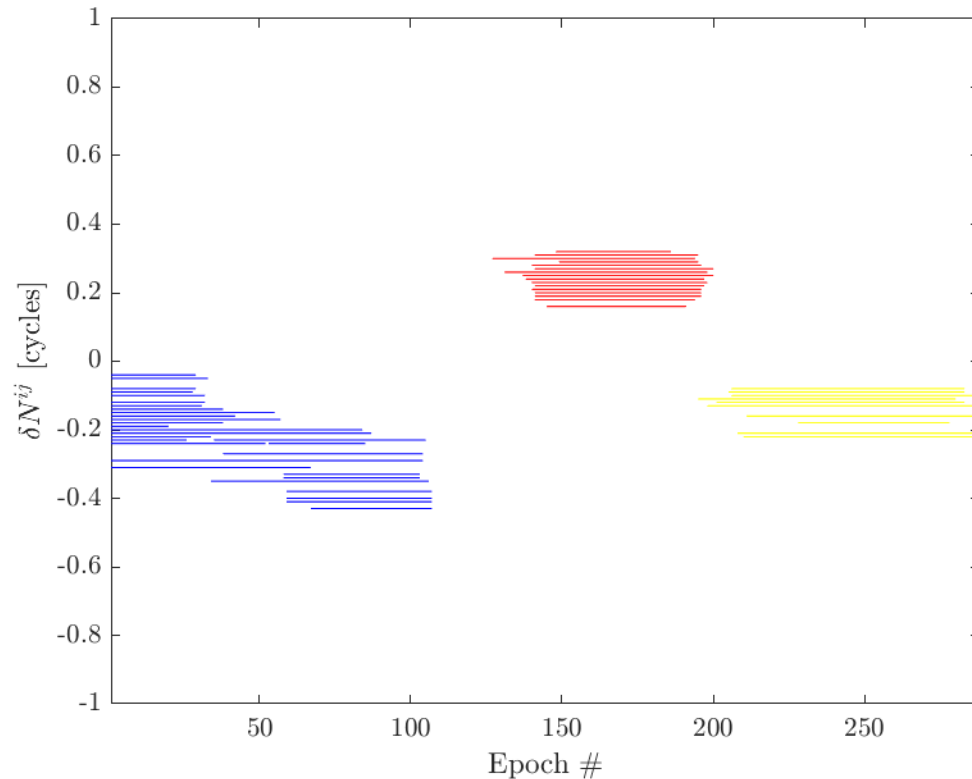
# GENERATION OF AMBIGUITY CLUSTERS (ITE #04)

- Between-satellite pair E11/E25 on February 19<sup>th</sup>, 2021



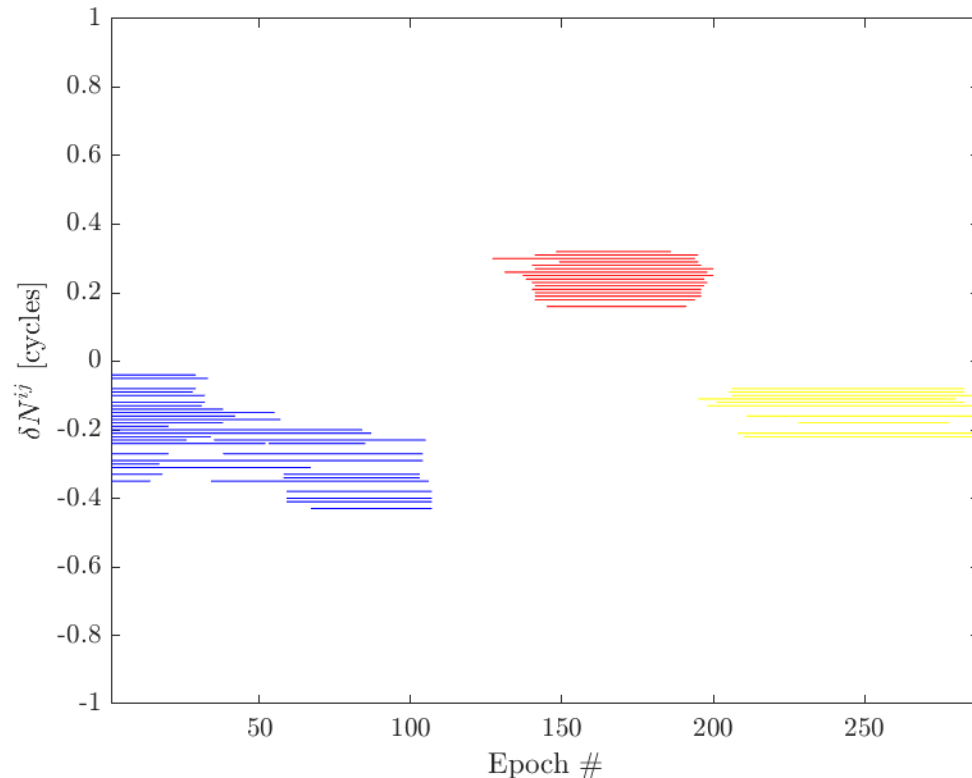
# GENERATION OF AMBIGUITY CLUSTERS (ITE #05)

- Between-satellite pair E11/E25 on February 19<sup>th</sup>, 2021



# GENERATION OF AMBIGUITY CLUSTERS (ITE #06)

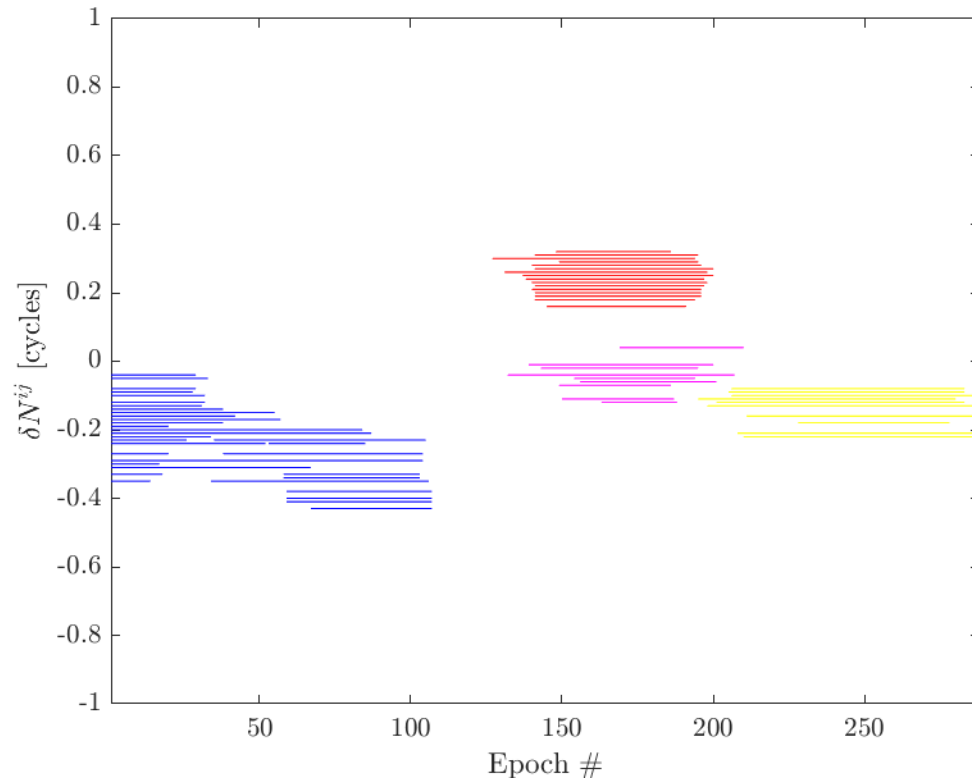
- Between-satellite pair E11/E25 on February 19<sup>th</sup>, 2021



Adding new ambiguities will increase the size of the clusters, which will be eventually combined if one ambiguity could be part of two clusters simultaneously

# GENERATION OF AMBIGUITY CLUSTERS (ITE #07)

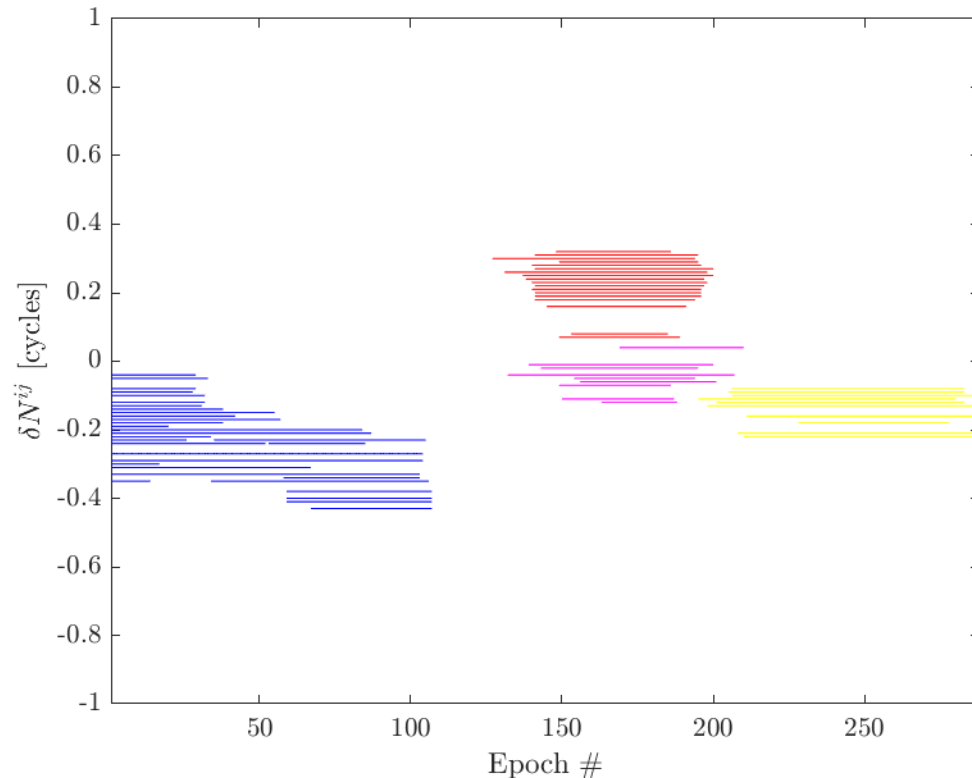
- Between-satellite pair E11/E25 on February 19<sup>th</sup>, 2021



Adding new ambiguities will increase the size of the clusters, which will be eventually combined if one ambiguity could be part of two clusters simultaneously

# GENERATION OF AMBIGUITY CLUSTERS (ITE #08)

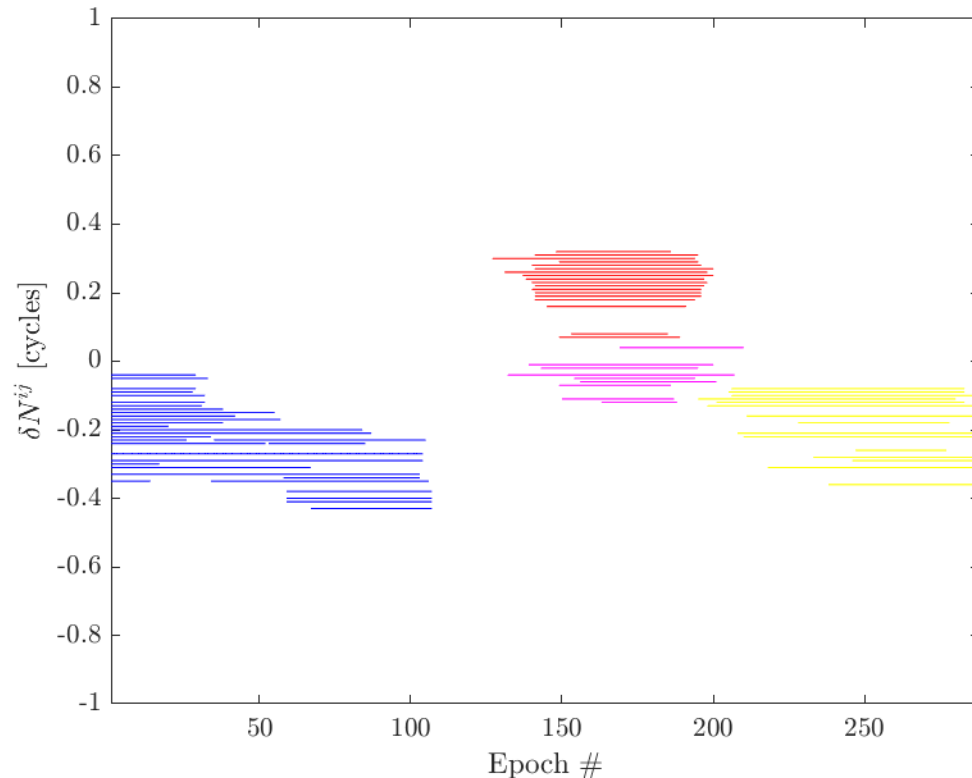
- Between-satellite pair E11/E25 on February 19<sup>th</sup>, 2021



Adding new ambiguities will increase the size of the clusters, which will be eventually combined if one ambiguity could be part of two clusters simultaneously

# GENERATION OF AMBIGUITY CLUSTERS (ITE #9)

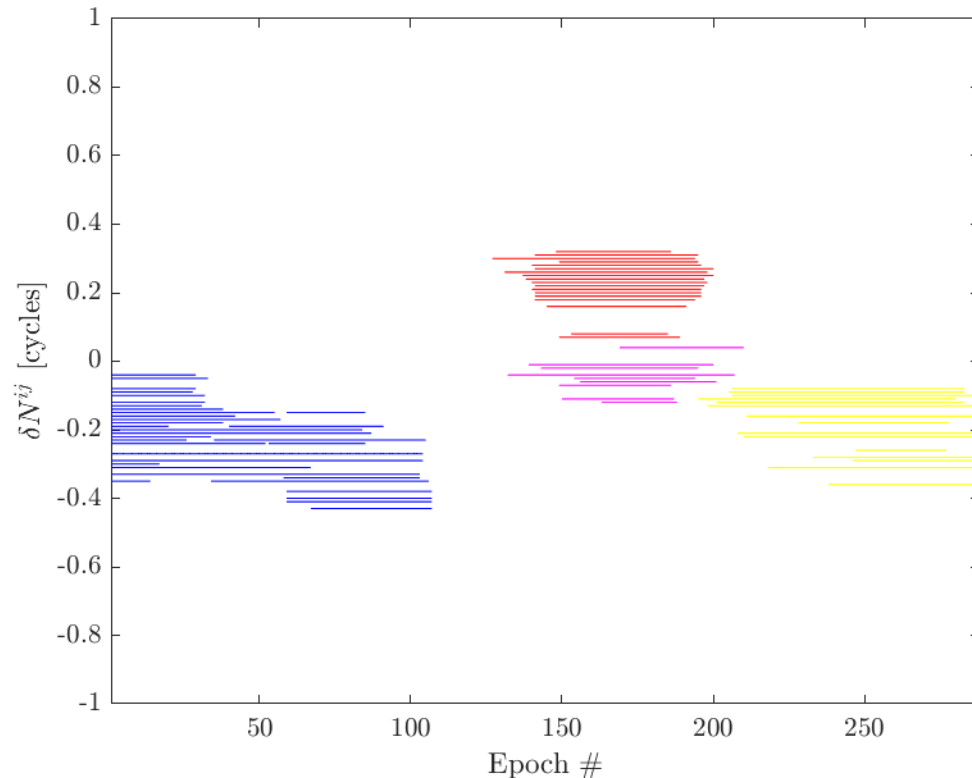
- Between-satellite pair E11/E25 on February 19<sup>th</sup>, 2021



Adding new ambiguities will increase the size of the clusters, which will be eventually combined if one ambiguity could be part of two clusters simultaneously

# GENERATION OF AMBIGUITY CLUSTERS (ITE #10)

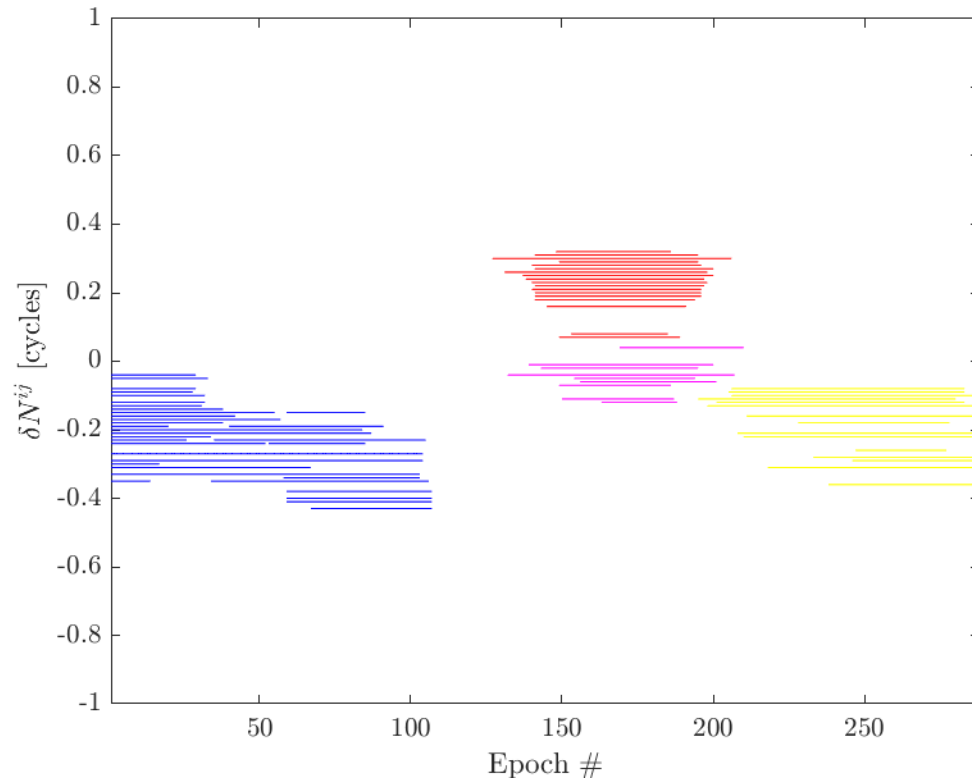
- Between-satellite pair E11/E25 on February 19<sup>th</sup>, 2021



Adding new ambiguities will increase the size of the clusters, which will be eventually combined if one ambiguity could be part of two clusters simultaneously

# GENERATION OF AMBIGUITY CLUSTERS (ITE #11)

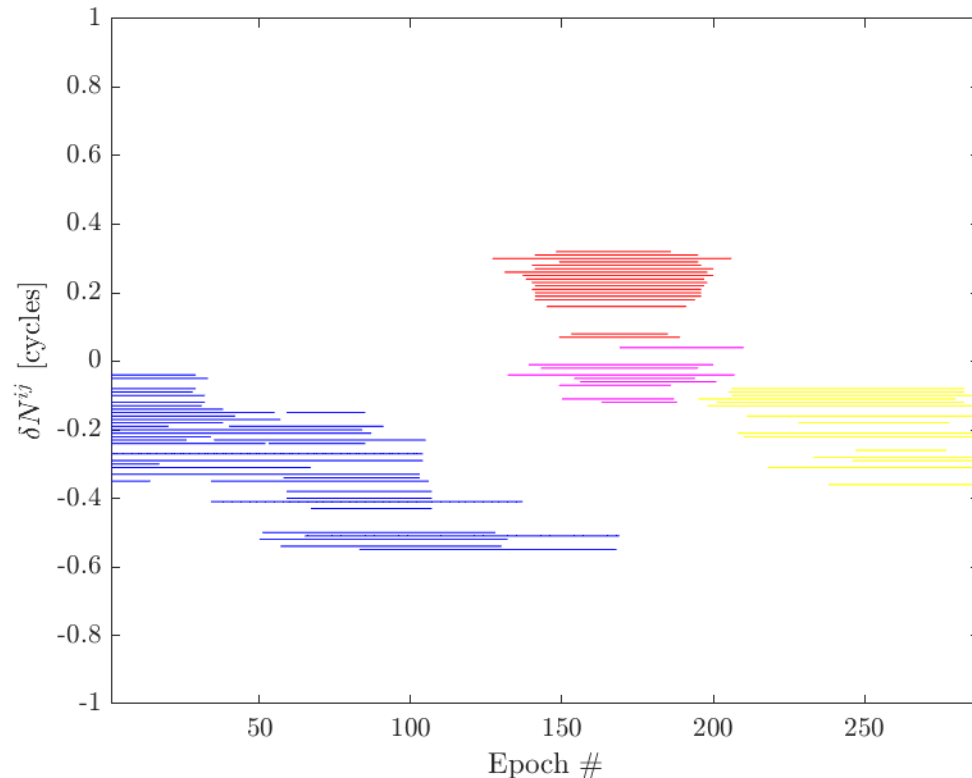
- Between-satellite pair E11/E25 on February 19<sup>th</sup>, 2021



Adding new ambiguities will increase the size of the clusters, which will be eventually combined if one ambiguity could be part of two clusters simultaneously

# GENERATION OF AMBIGUITY CLUSTERS (ITE #12)

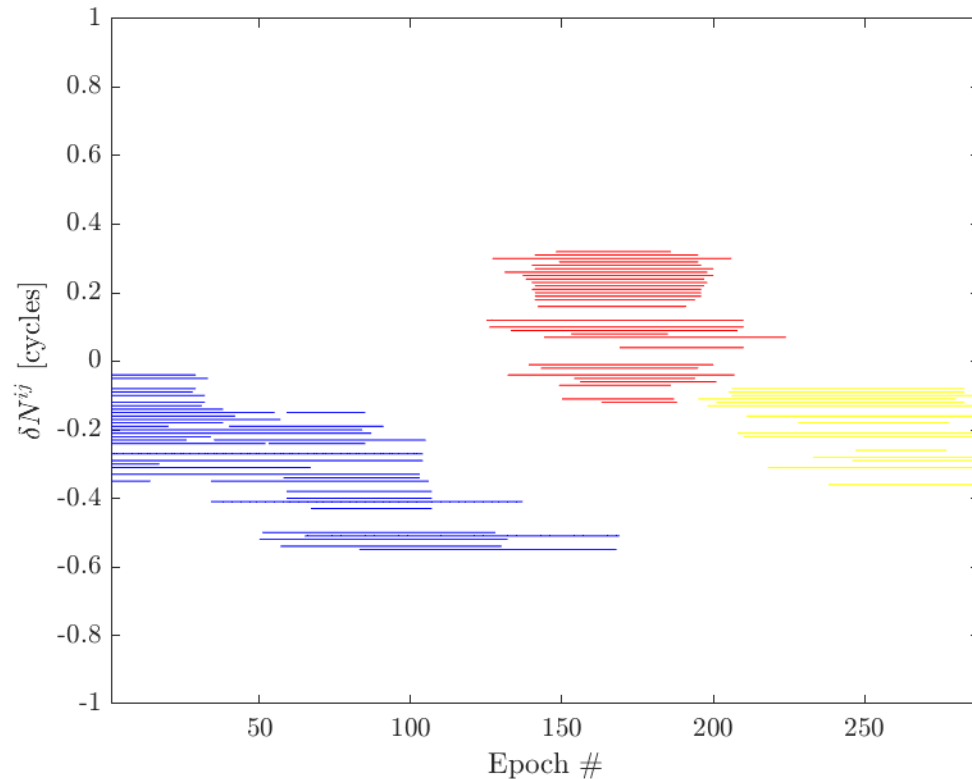
- Between-satellite pair E11/E25 on February 19<sup>th</sup>, 2021



Adding new ambiguities will increase the size of the clusters, which will be eventually combined if one ambiguity could be part of two clusters simultaneously

# GENERATION OF AMBIGUITY CLUSTERS (ITE #13)

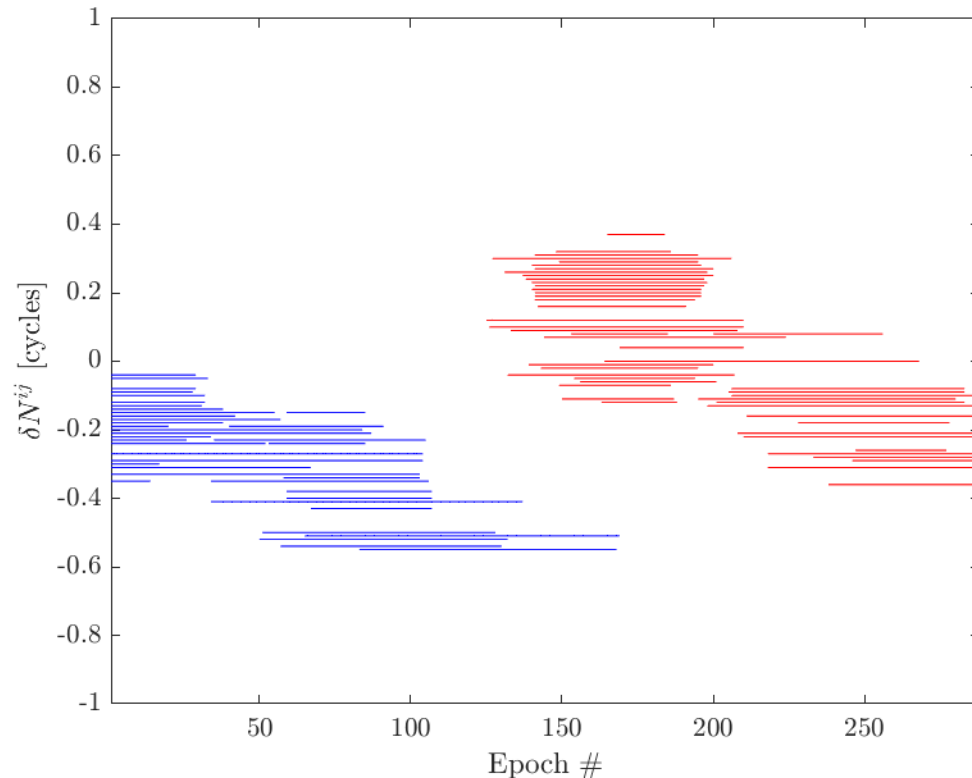
- Between-satellite pair E11/E25 on February 19<sup>th</sup>, 2021



Adding new ambiguities will increase the size of the clusters, which will be eventually combined if one ambiguity could be part of two clusters simultaneously

# GENERATION OF AMBIGUITY CLUSTERS (ITE #14)

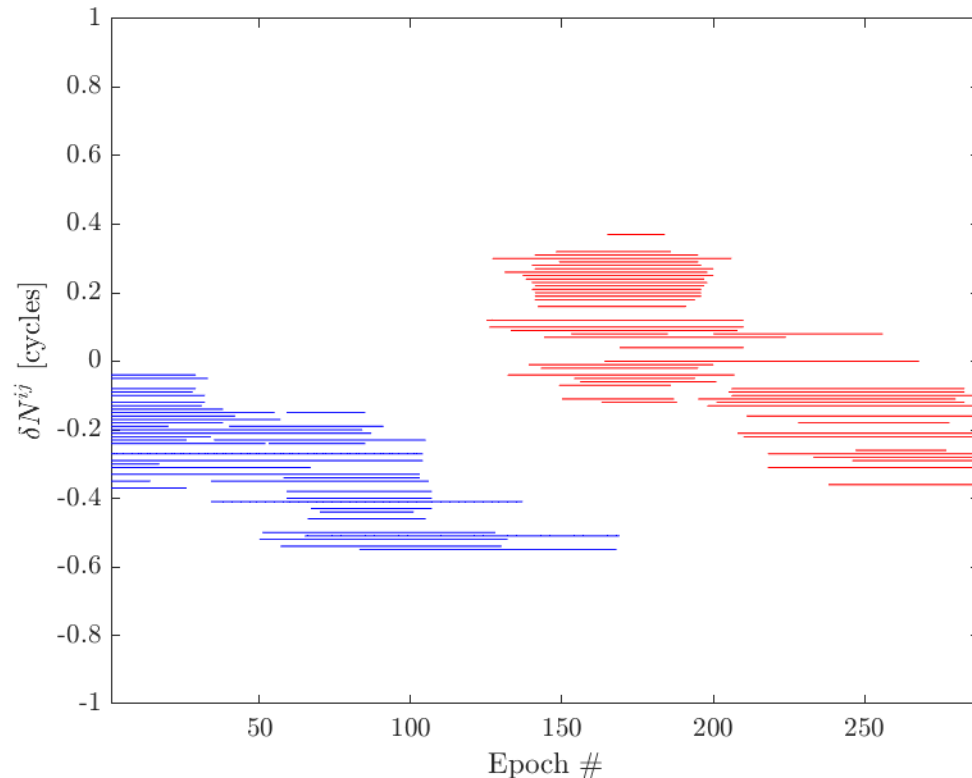
- Between-satellite pair E11/E25 on February 19<sup>th</sup>, 2021



Adding new ambiguities will increase the size of the clusters, which will be eventually combined if one ambiguity could be part of two clusters simultaneously

# GENERATION OF AMBIGUITY CLUSTERS (ITE #16)

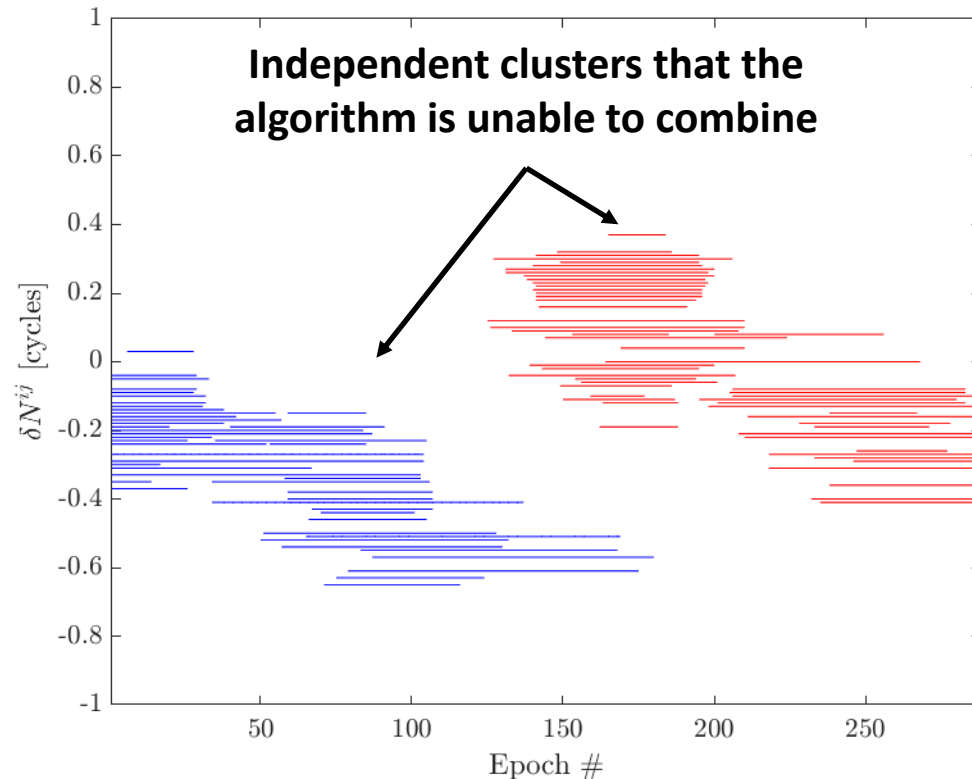
- Between-satellite pair E11/E25 on February 19<sup>th</sup>, 2021



Adding new ambiguities will increase the size of the clusters, which will be eventually combined if one ambiguity could be part of two clusters simultaneously

# GENERATION OF AMBIGUITY CLUSTERS (ITE #n)

- Between-satellite pair E11/E25 on February 19<sup>th</sup>, 2021



If the blue cluster is shifted one cycle up, then we infer how the ambiguities are contaminated

# MIXED-INTEGER MODEL FOR NL AMBIGUITIES (I)

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- Real-valued NL ambiguities inspected by means of a model:

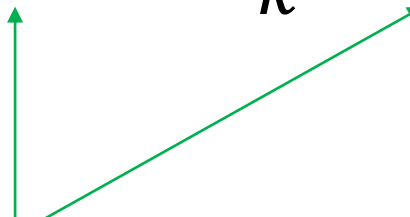
$$\delta N_{k_{NL}}^{ij} = - \left( \mathbf{v}_k^i \cdot \delta \mathbf{x}^i - \mathbf{v}_k^j \cdot \delta \mathbf{x}^j \right) + \delta \tau^{ij} + B_{cluster}$$



**The ambiguities are treated as observations for a least-squares adjustment**

# MIXED-INTEGER MODEL FOR NL AMBIGUITIES (II)

- Real-valued NL ambiguities inspected by means of a model:

$$\delta N_{k_{NL}}^{ij} = - \left( \mathbf{v}_k^i \cdot \delta \mathbf{x}^i - \mathbf{v}_k^j \cdot \delta \mathbf{x}^j \right) + \delta \tau^{ij} + B_{cluster}$$


Orbit-like parameters represented by a sum of polynomials. They describe a kinematic solution (i.e., they represent only the geometry without any physical model)

# MIXED-INTEGER MODEL FOR NL AMBIGUITIES (III)

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- Real-valued NL ambiguities inspected by means of a model:

$$\delta N_{k_{NL}}^{ij} = - \left( \mathbf{v}_k^i \cdot \delta \mathbf{x}^i - \mathbf{v}_k^j \cdot \delta \mathbf{x}^j \right) + \delta \tau^{ij} + B_{cluster}$$

**Clock-like parameters represented by a sum  
of polynomials**

# MIXED-INTEGER MODEL FOR NL AMBIGUITIES (IV)

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- Real-valued NL ambiguities inspected by means of a model:

$$\delta N_{k_{NL}}^{ij} = - \left( \mathbf{v}_k^i \cdot \delta \mathbf{x}^i - \mathbf{v}_k^j \cdot \delta \mathbf{x}^j \right) + \delta \tau^{ij} + B_{cluster}$$

**Integer cluster biases intended to align independent ambiguity clusters**

# MIXED-INTEGER MODEL FOR NL AMBIGUITIES (IV)

---

- Real-valued NL ambiguities inspected by means of a model:

$$\delta N_{k_{NL}}^{ij} = - \left( \mathbf{v}_k^i \cdot \delta \mathbf{x}^i - \mathbf{v}_k^j \cdot \delta \mathbf{x}^j \right) + \delta \tau^{ij} + B_{cluster}$$

**Important note:** these parameters do not correspond to any physical reality. They are only a comfortable way to unveil the integer nature of the NL ambiguities.

# MIXED-INTEGER MODEL FOR NL AMBIGUITIES (VI)

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- Real-valued NL ambiguities inspected by means of a model:

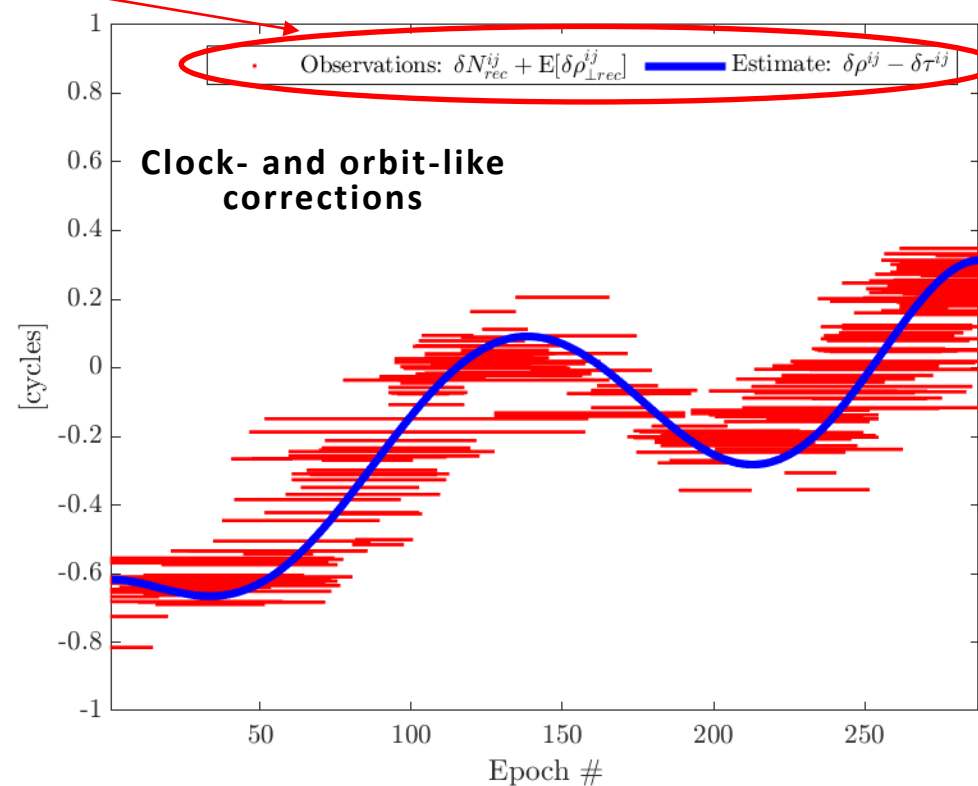
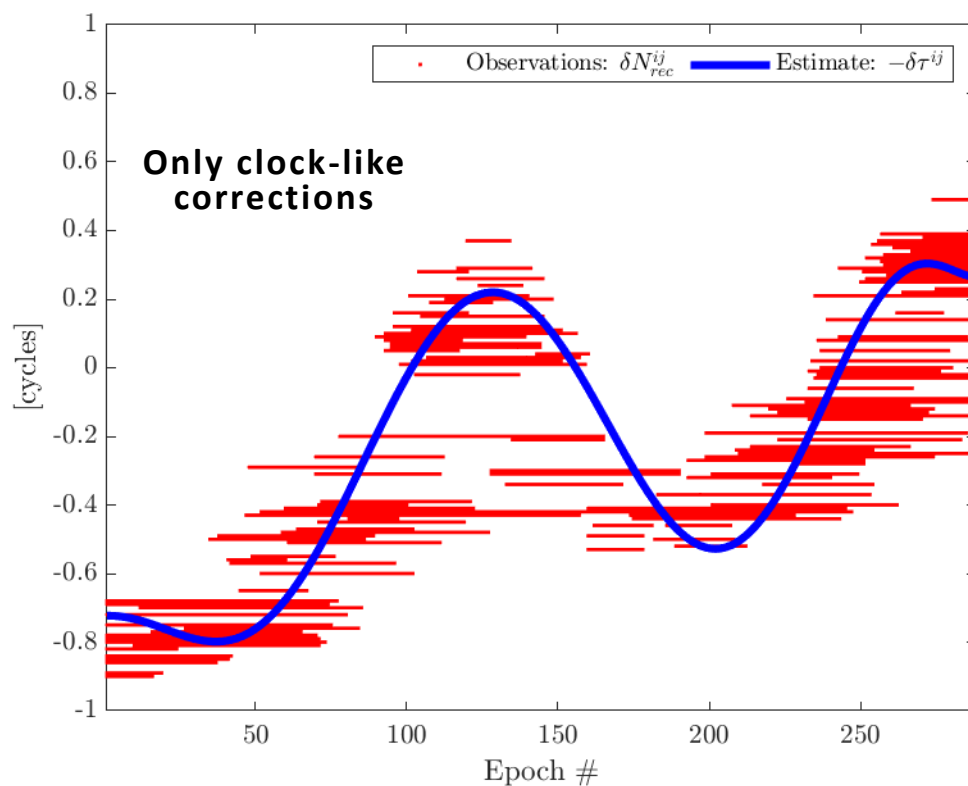
$$\delta N_{k_{NL}}^{ij} = - \left( \mathbf{v}_k^i \cdot \delta \mathbf{x}^i - \mathbf{v}_k^j \cdot \delta \mathbf{x}^j \right) + \delta \tau^{ij} + B_{cluster}$$

- For the sake of simplicity, no station-dependent parameters are considered.
- Mixed-integer model with ~200 integer parameters. It can be solved using, e.g., LAMBDA.
- Once the ambiguities are reduced with this model, they can be resolved as integers station by station.

# IMPACT OF ORBIT-LIKE PARAMETERS (I)

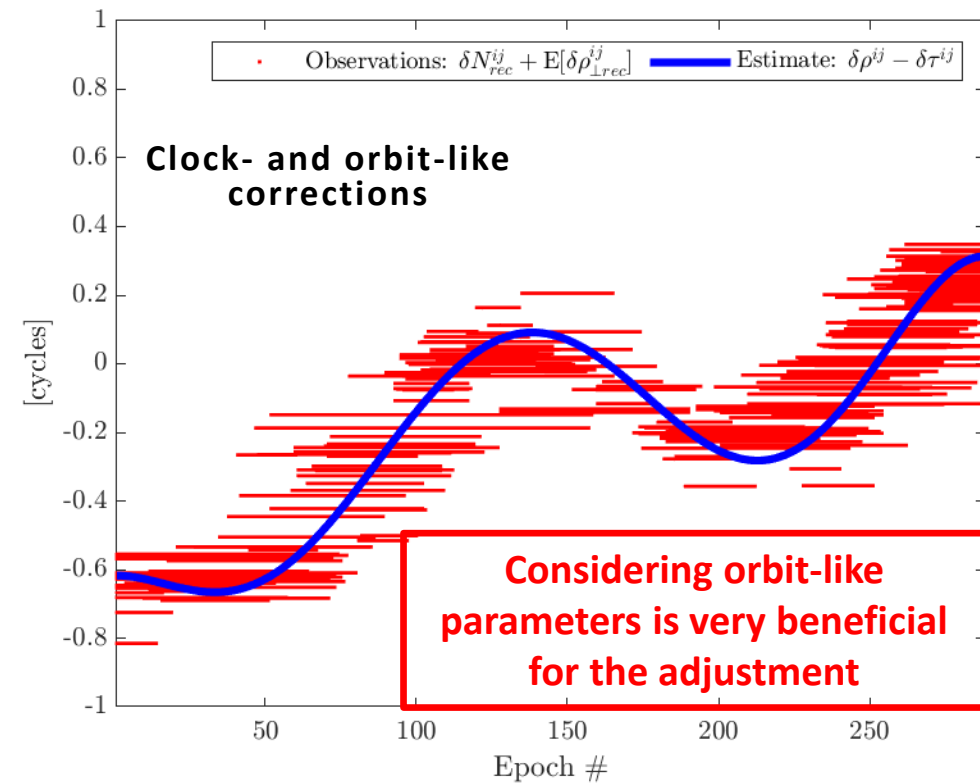
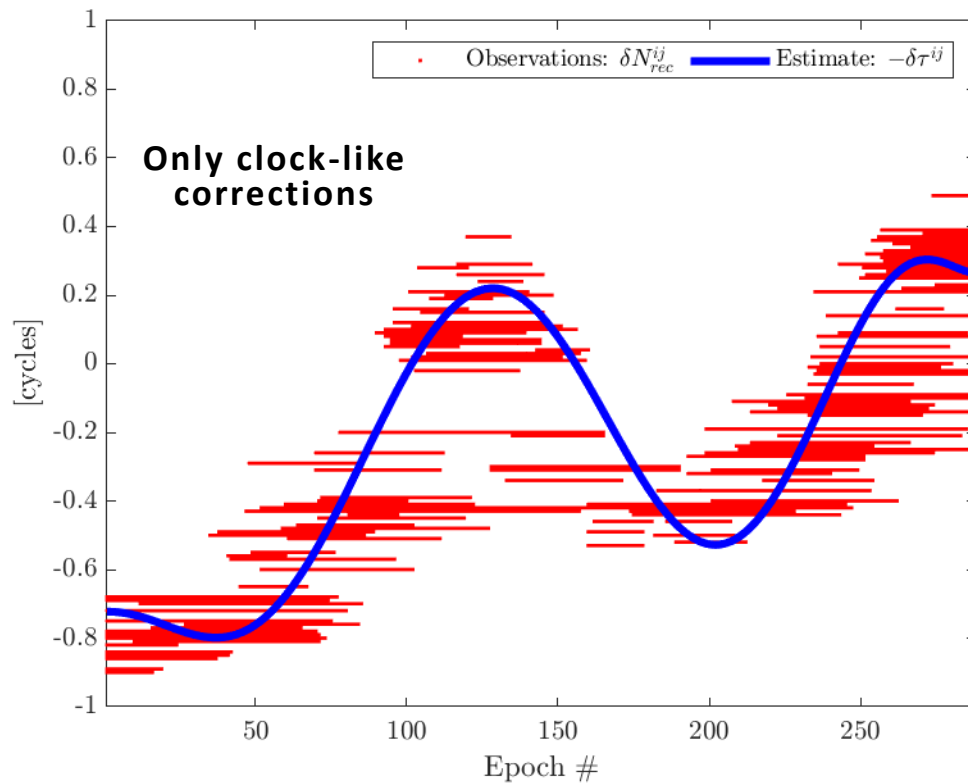
- Fit for between-satellite pair E11/E12 on March 3<sup>rd</sup>, 2021.

For a fair comparison, the observations are corrected with the transversal component of the orbit-like parameters and the estimate includes clock-like corrections + radial component of orbit-like corrections



# IMPACT OF ORBIT-LIKE PARAMETERS (II)

- Fit for between-satellite pair E11/E12 on March 3<sup>rd</sup>, 2021

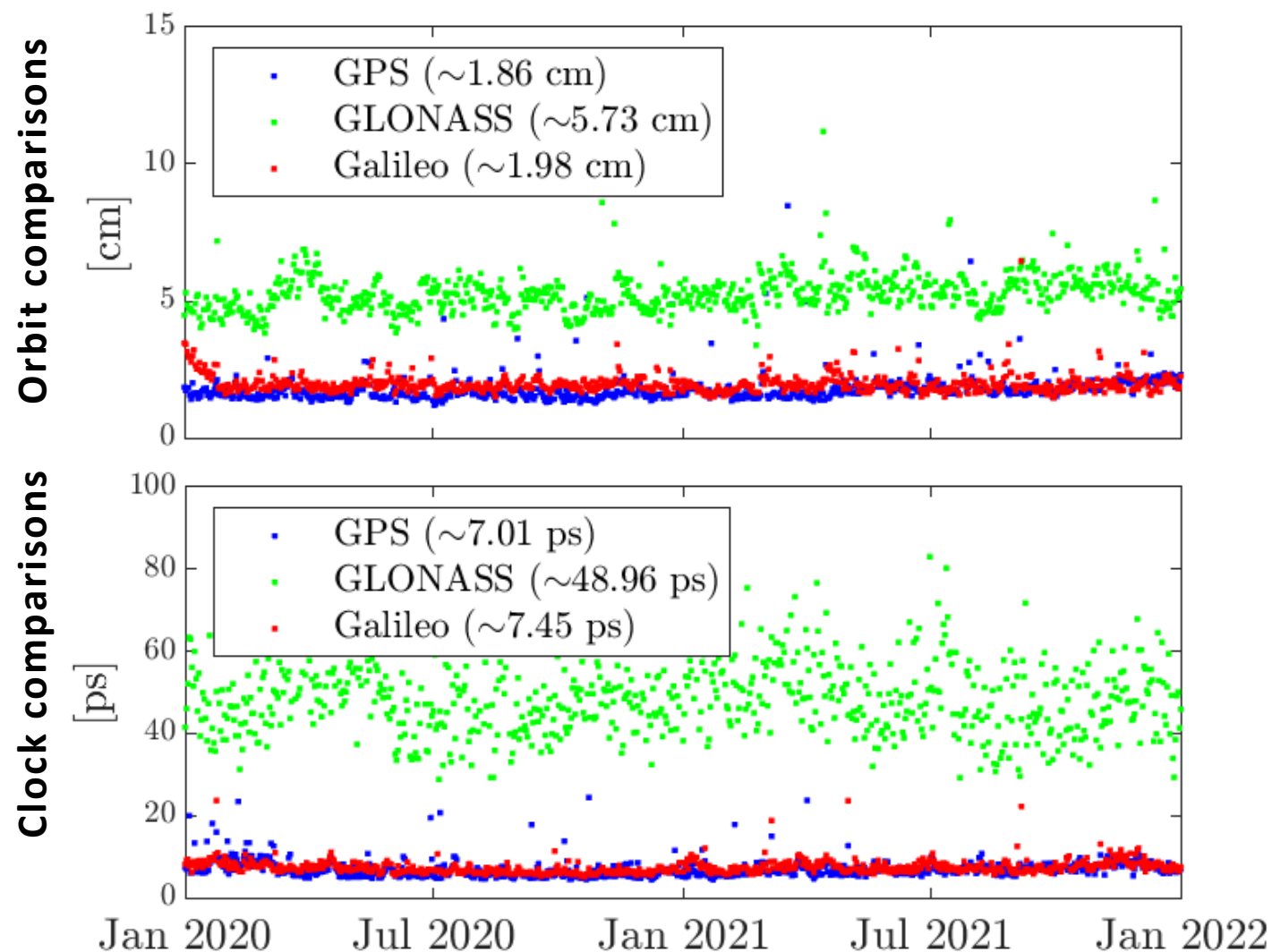


## Part III

# VALIDATION OF THE RESULTS

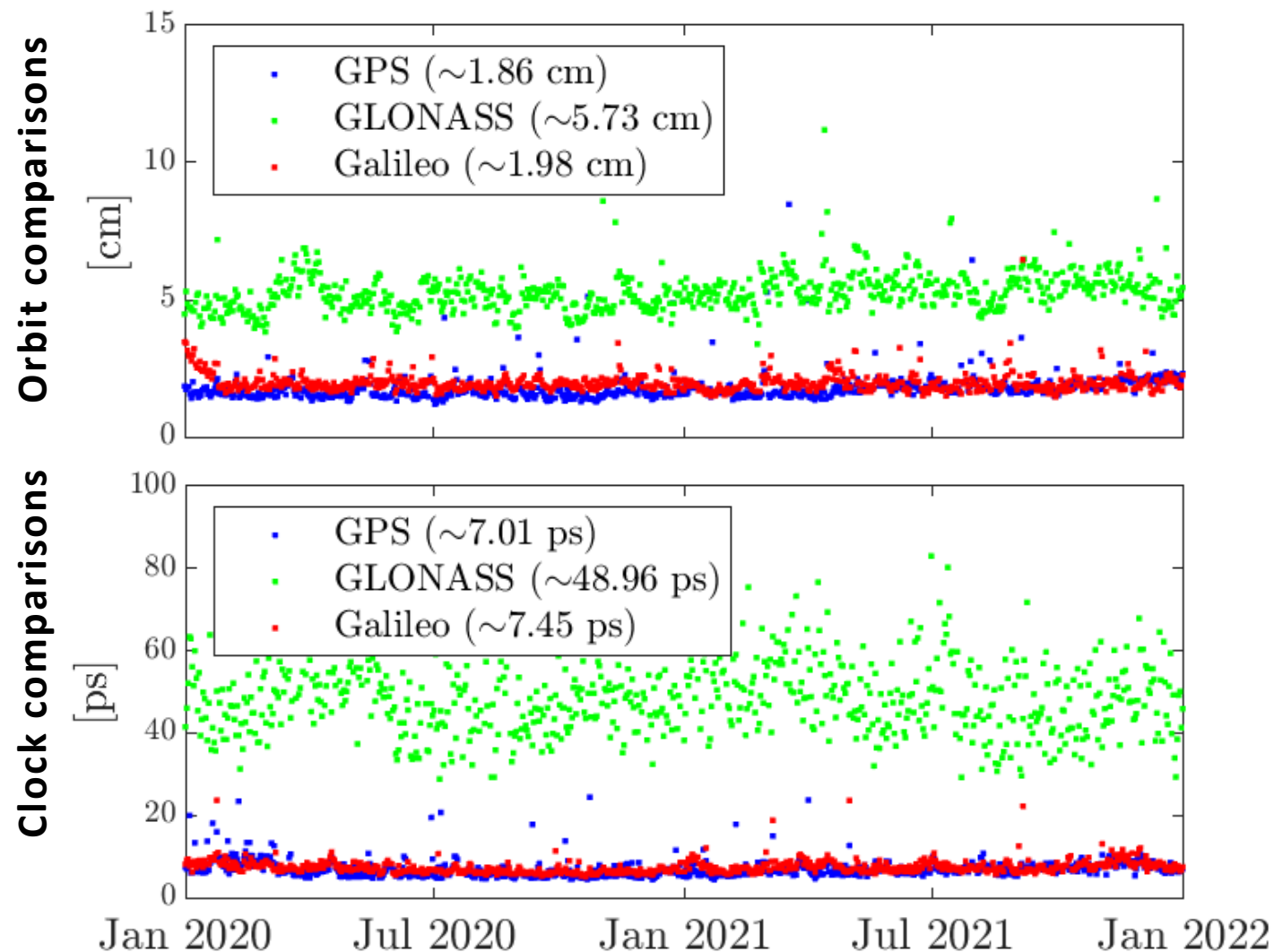
# VALIDATION OF RESULTS (I)

- Two years (2020/2021) of data (undifferenced solutions) compared against CODE MGEX.
- The orbit comparisons represent the daily 3D-RMS differences.
- The clock comparisons represent the daily standard deviation of the Signal In Space Range Error (SISRE).



# VALIDATION OF RESULTS (II)

- GLONASS does not benefit from ambiguity resolution → poorer, yet competitive, performance.
- Great agreement for the GPS and Galileo products in both orbits and clock corrections.



# SUMMARY

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- An undifferenced processing scheme based on a **station-wise architecture** has been designed.
  - The existing ambiguity resolution strategy based on **real-valued ambiguity inspection** is complemented by a mixed-integer model that decorrelates ambiguities from satellite products. The use of **ambiguity clusters** plays a fundamental role in reducing the computational complexity of the problem.
- The validation against CODE MGEX evidences that the undifferenced results show **similar performance as the legacy double-differenced-based solutions**.