# Fitting and extrapolation of transient behaviour in the presence of tipping points

Contribution to NP2.4 EGU 2022

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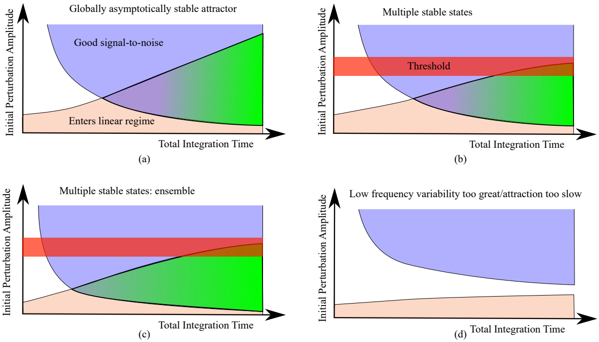
*"A key challenge is to study the limits of using the linear framework"*[3]

*Design→Selection→Fitting→Extrapolation*



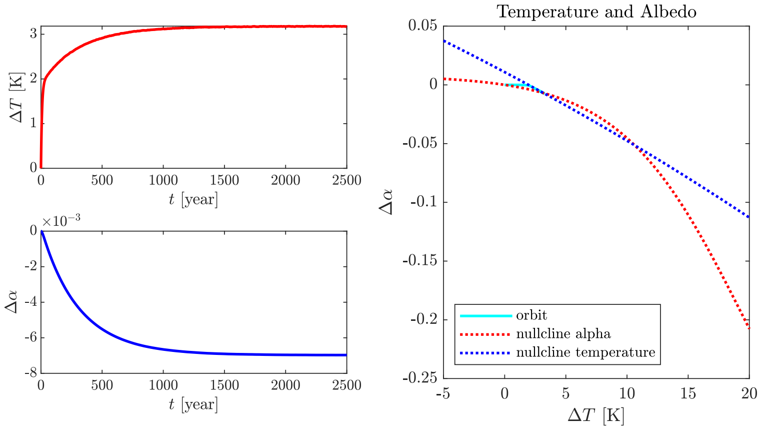

Trade-offs for estimating asymptotic properties of climate attractor for a nonlinear model. Green: region where accurate prediction of ECS is possible.

[3] Knutti & Rugenstein (2015)

We consider a Budyko-Sellers-Ghil type Energy Balance Model (EBM) with (slow) dynamic albedo and greenhouse $CO_2$ forcing $\mu$. We assume the GMST $T(t)$ evolves according to the model

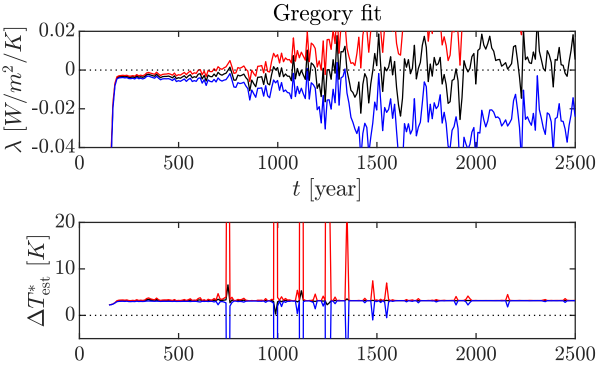
$$
\begin{aligned}
C\frac{dT}{dt} &= Q_0(1-\alpha) - \varepsilon_0(T)\sigma T^4 + \mu + \mu_{NV}(t), \\
\tau_\alpha \frac{d\alpha}{dt} &= [\alpha_0(T) - \alpha]
\end{aligned}
$$

with monotonic $\alpha_0(T)$ and $\varepsilon_0(T)$ limiting to constant values.

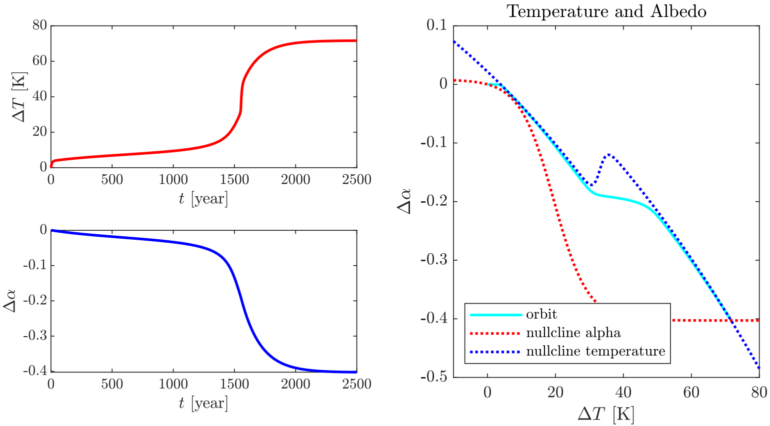



Warming for the EBM subjected to an abrupt2xCO2 forcing. The initial equilibrium is $T_0 = 255K$.

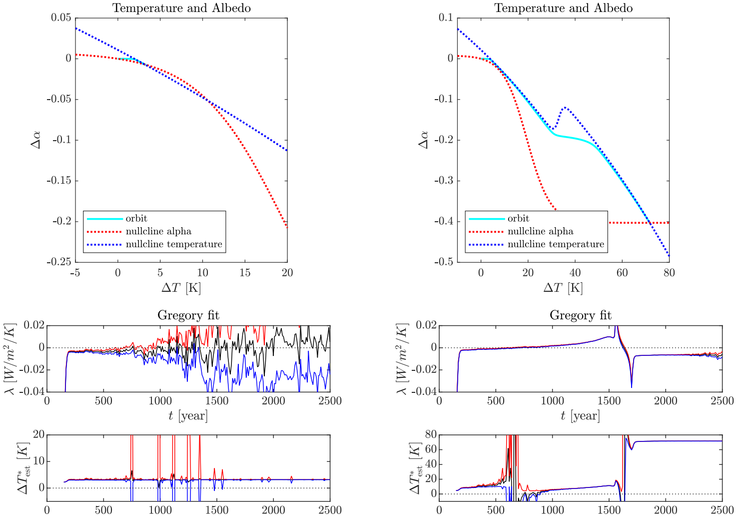
*Gregory method* (Gregory et al, GRL 2004): Linearly fit of transient temperature $\Delta T$ to top-of-atmosphere radiative imbalance $\Delta N$, for (a typically 150 year) abrupt $CO_2$-forcing experiment.



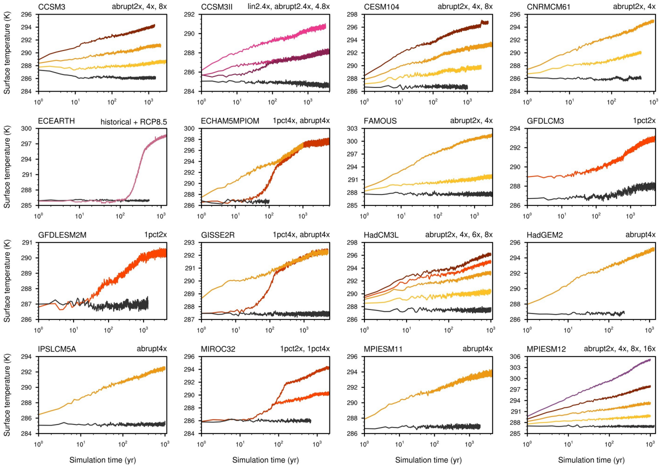

Warming for the EBM with abrupt2xCO2 forcing, Gregory fit using sliding time windows of 150 years, showing the estimated feedback parameter $\lambda$ and expected equilibrium warming $\Delta T^*_{\text{est}}$ over time, together with standard errors.




Warming for the EBM subjected to abrupt4xCO2 forcing. The initial equilibrium is $T_0 = 255K$. A late tipping event (ice-house to hot-house) happens as the dynamics drive the system over a fold point of the slow manifold.




Comparison of phase portraits and Gregory fits, using sliding time windows of 150 years for (left) 2xabruptCO2 and (right) 4xabruptCO2 forcing; note the late rapid tipping on the right.




Looking for late tipping in `http://www.longrunmip.org/` (Rugenstein et al 2019).

- Need to identify *trade-offs* made when designing protocols to estimate ECS.
- *Slow tipping points* mean that time frames can be arbitrarily long before asymptotic behaviour becomes visible.
- *Late tipping points* can appear in the presence of multiple-timescales with nonlinear feedbacks.
- Clear (and extreme) examples with *good fit* but *poor extrapolation* to an accurate estimate of the ECS.

Thank you for listening!

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*TiPES (Tipping Points in the Earth System)*