A Discontinuous Galerkin Method for Sequences of Earthquakes and Aseismic Slip on Multiple Faults Using Unstructured Curvilinear Grids

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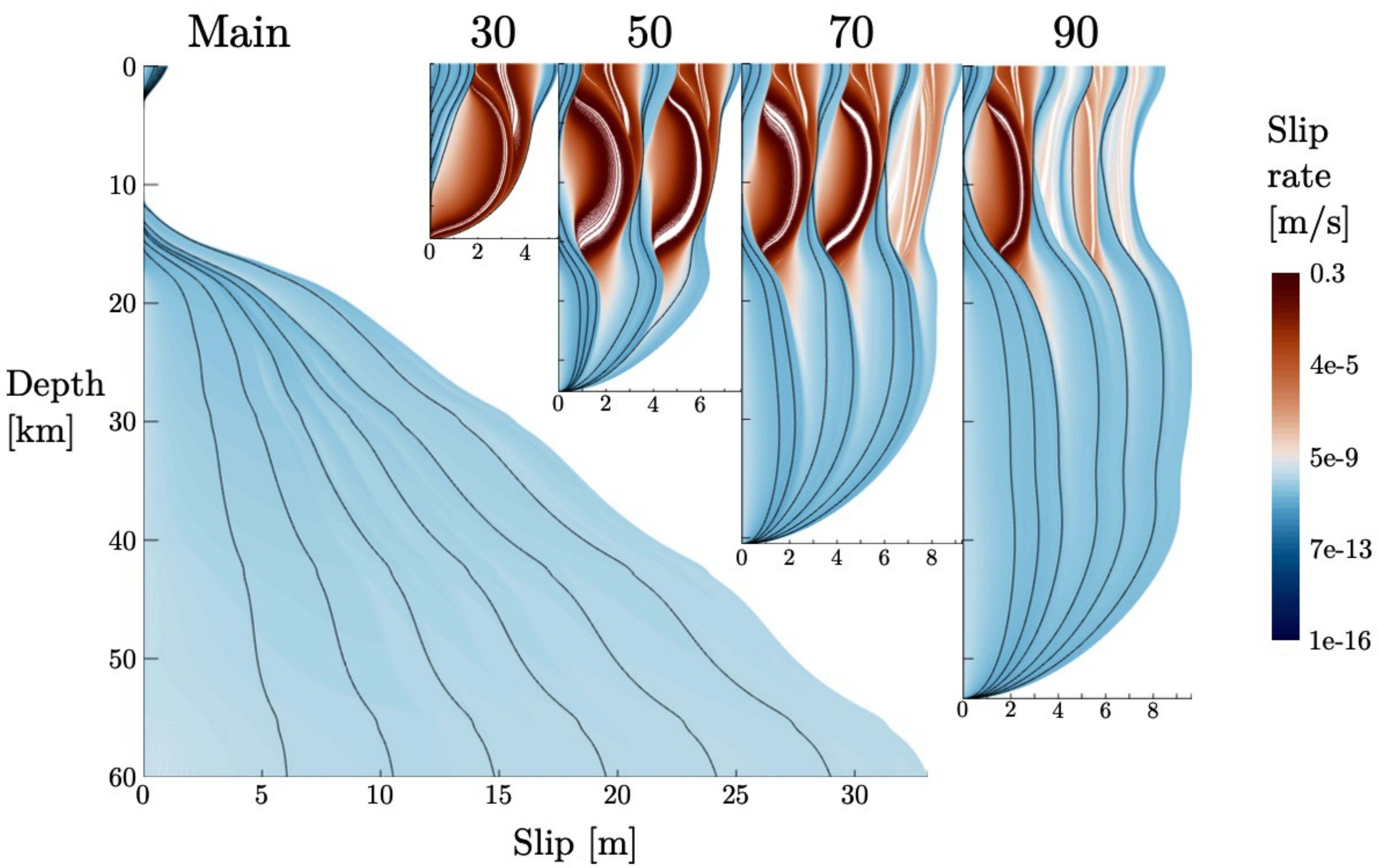
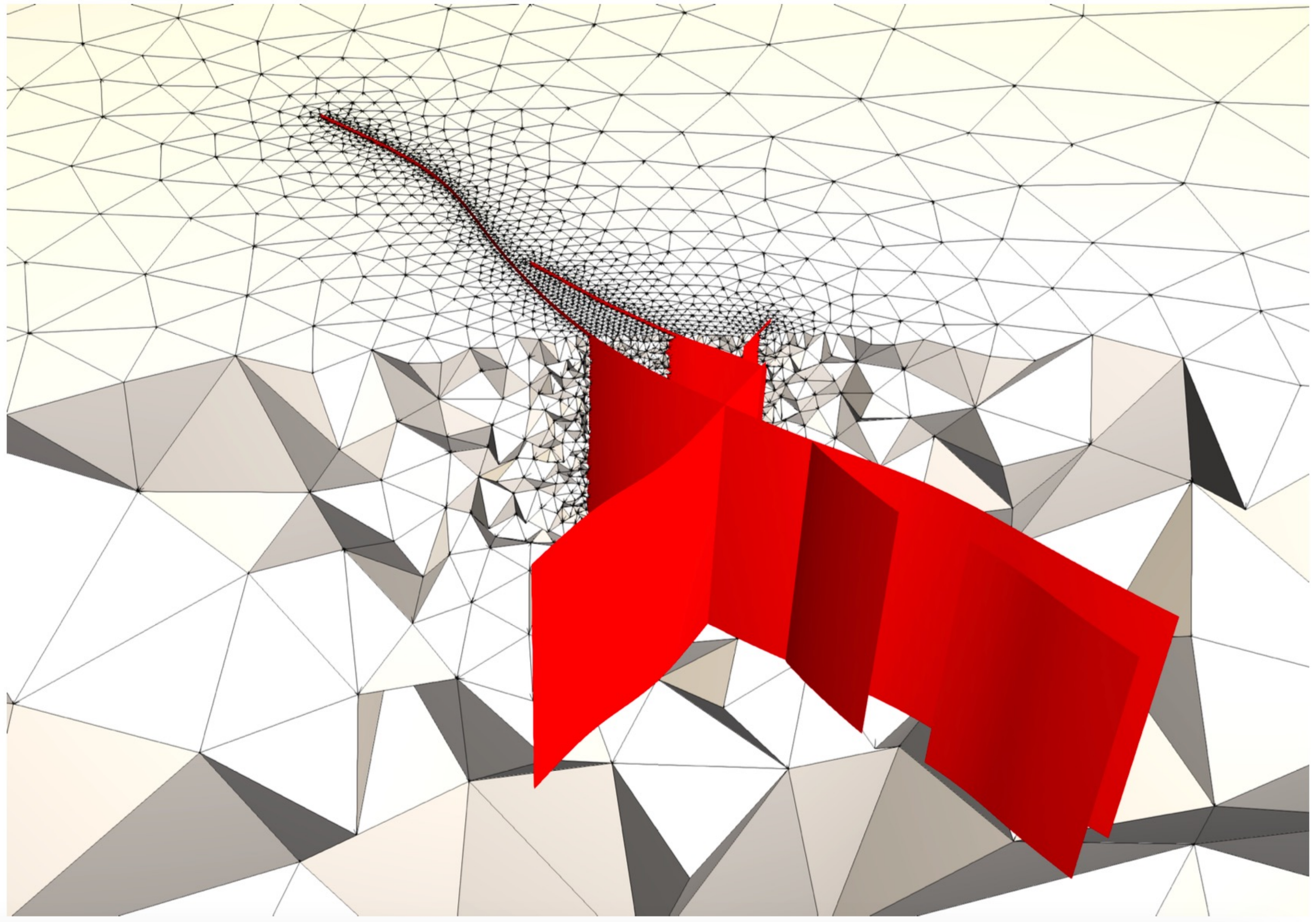


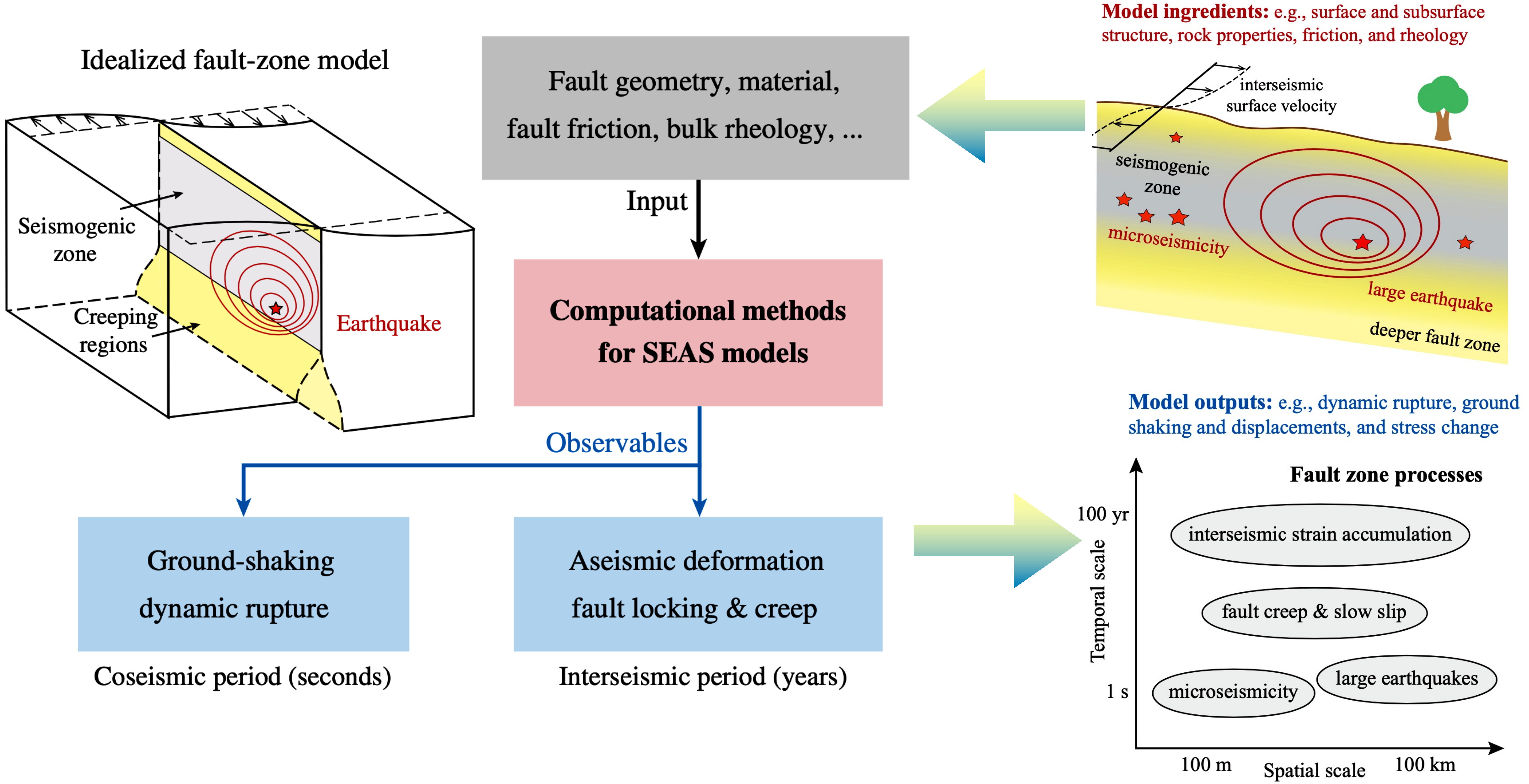


Seismic cycle modeling

- Modeling of inter-seismic phase, spontaneous earthquake nucleation as well as co-seismic and post-seismic slip to **connect long-term deformation and short-term seismicity**
- Very successful **community verification efforts**

Need for open-source community methods and utilization of HPC !

Erickson et al., 2020, SRL, **Jiang** et al., 2022, JGR, Erickson et al., subm., EarthArxiv



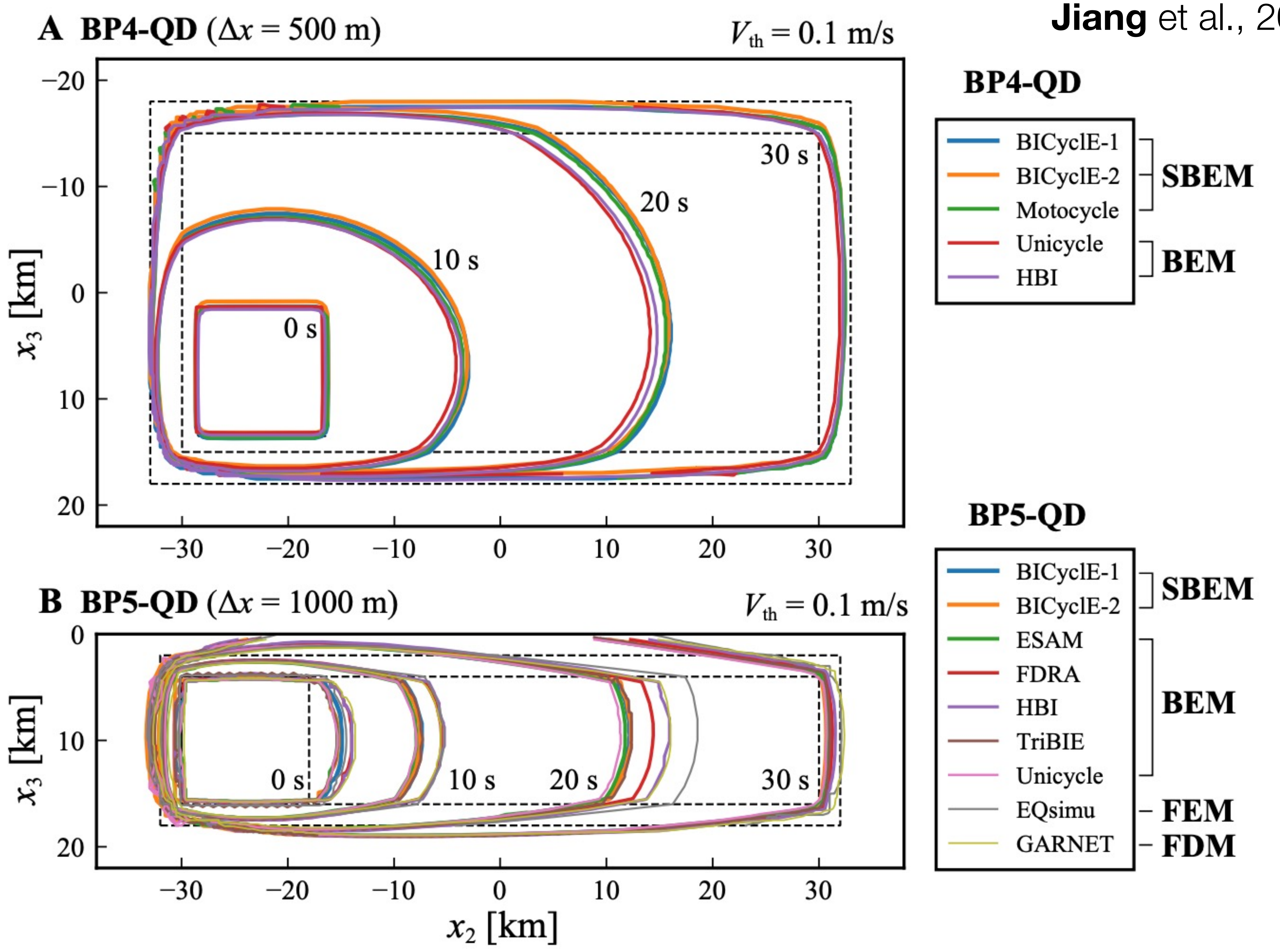
Seismic cycle modeling

- **Boundary Integral/Element Method (BIM/BEM/SBEM):** Boundary discretization, lower-dimensional, fast, but requires a fundamental solution. Mostly also relies on using analytical Green's function.

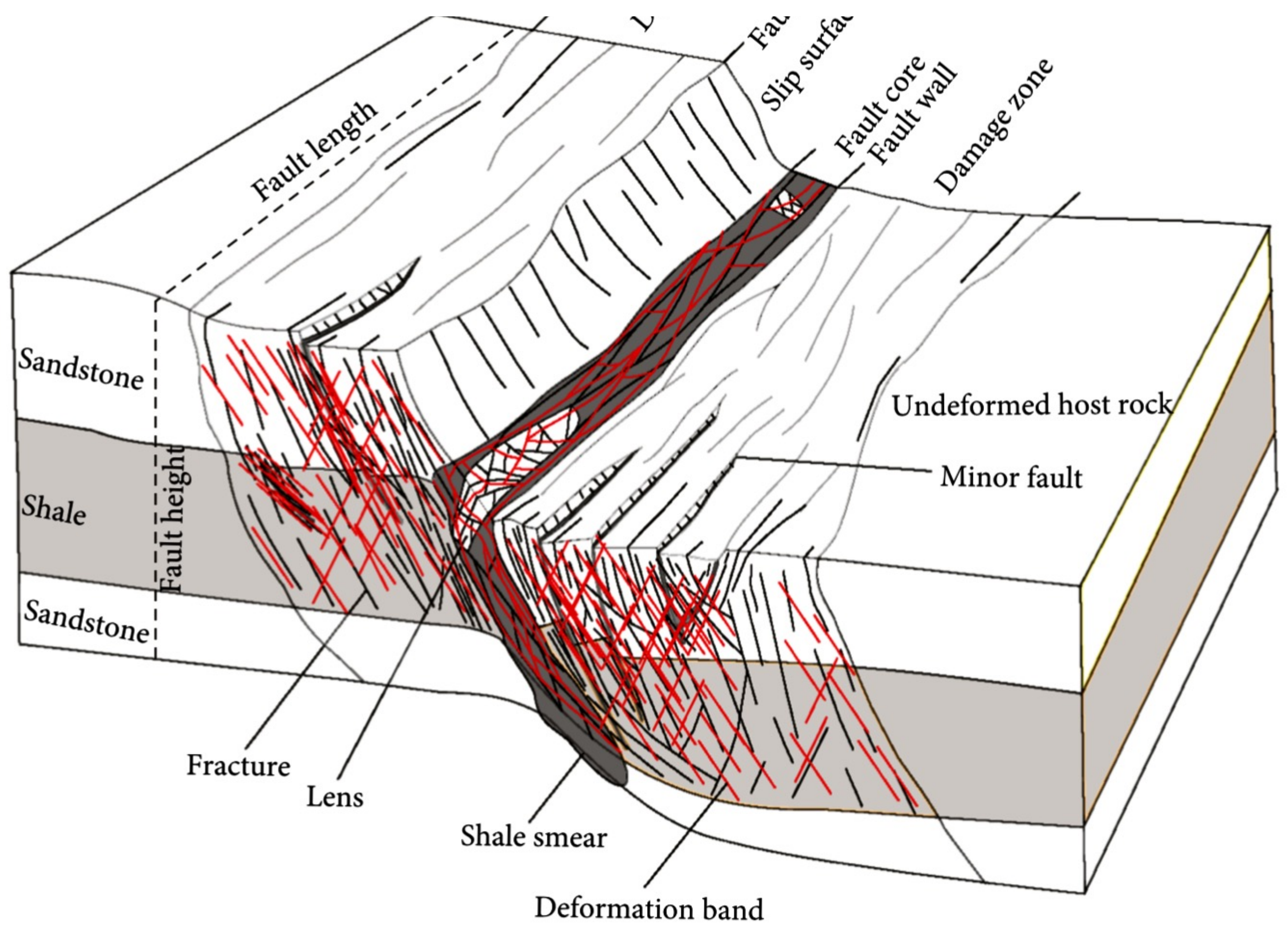
Typically restricted to homogeneous and linear off-fault materials

- **Finite Difference / Volume / Element Methods (FDM/FVM/FEM):** Volume discretization, higher-dimensional, more demanding

More universally applicable: e.g., including heterogeneous/non-linear off-fault materials



Jiang et al., 2022, JGR

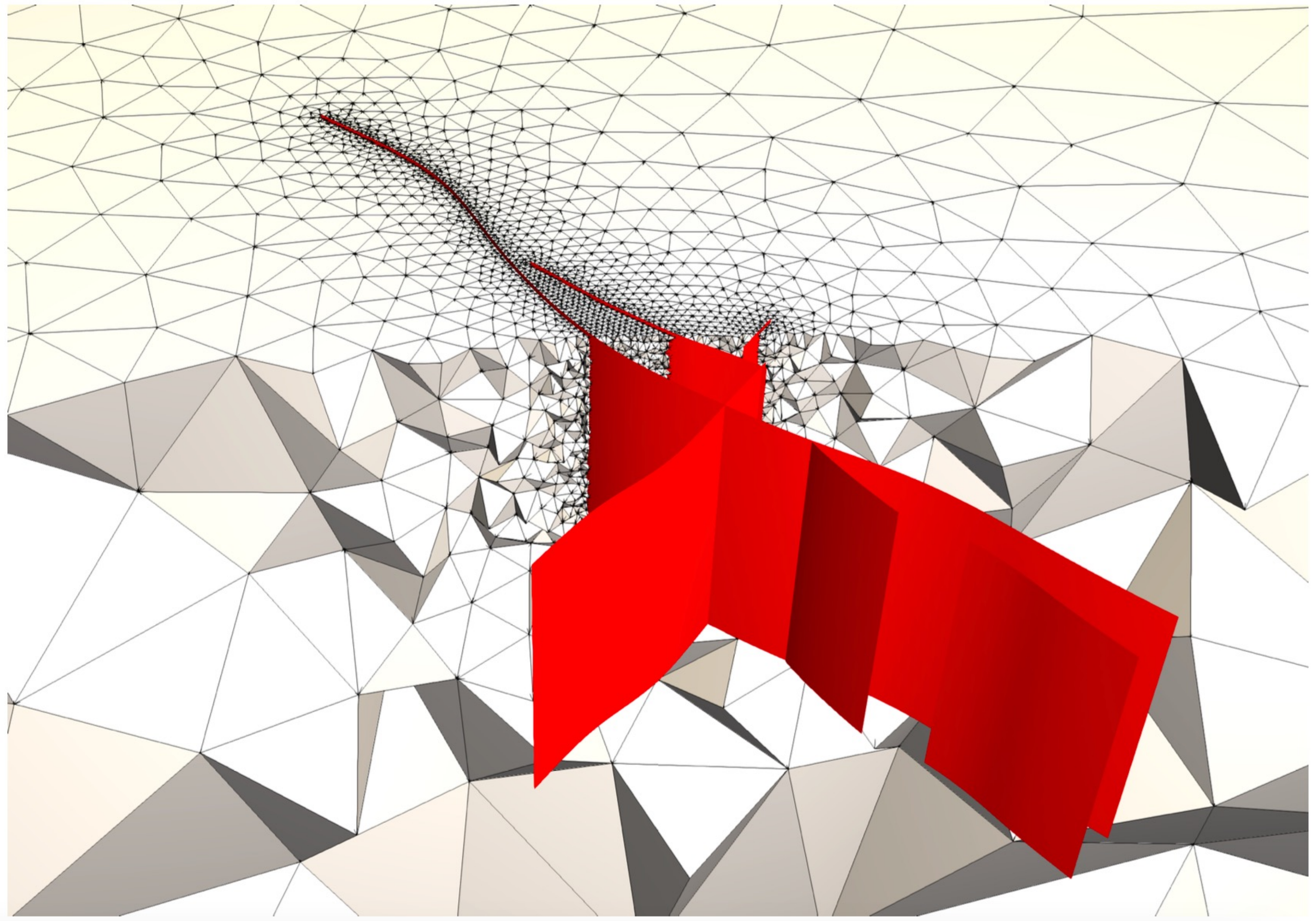


Tobari et al, 2019

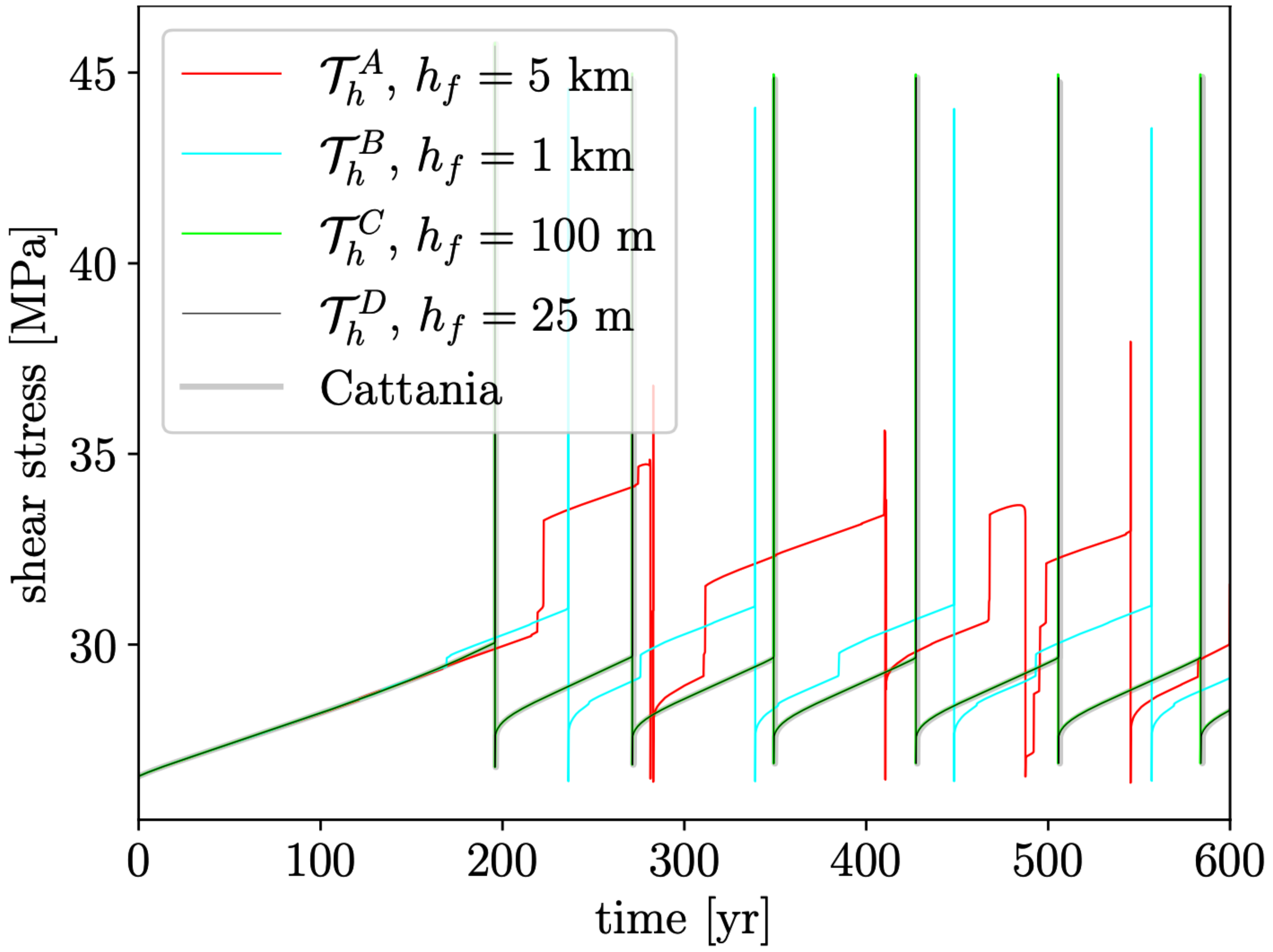
tandem - the Symmetric Interior Penalty Galerkin (SIPG) Discontinuous Galerkin (DG) method

We introduce ***tandem***, an **open-source** PETSc SIPG-DG solver for elliptic SEAS problems, that allows for

- **geometric flexibility:** enables **unstructured, curvilinear, triangular/tetrahedral** grids (future seamless linking to hyperbolic dynamic rupture solvers such as SeisSol)

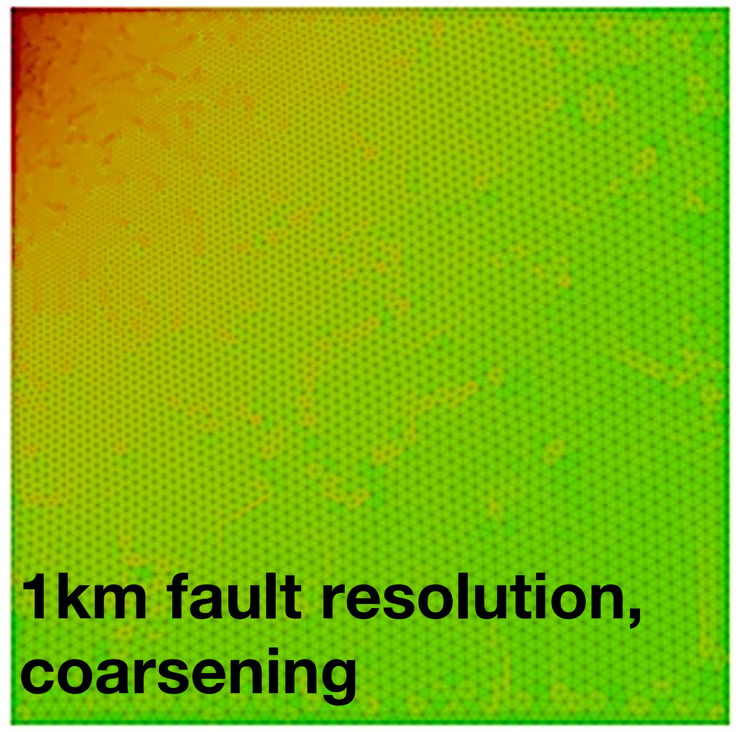


Close up and cut-away of the unstructured tetrahedral mesh used for a tandem static Ridgecrest example. The local refinement near the fault results in element edge lengths of 250 m. Away from the fault, the element edge length is coarsened up to a value of 20 km at the domain boundary. The mesh contained a total of 421,154 tetrahedra.

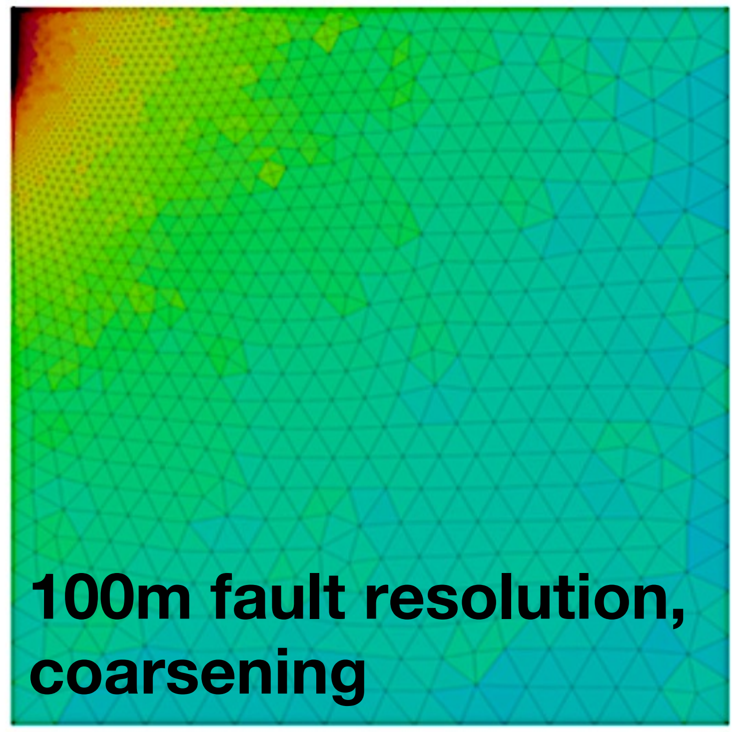


Benchmark comparison, all 4 tandem meshes have 150k elements but very different refinement.

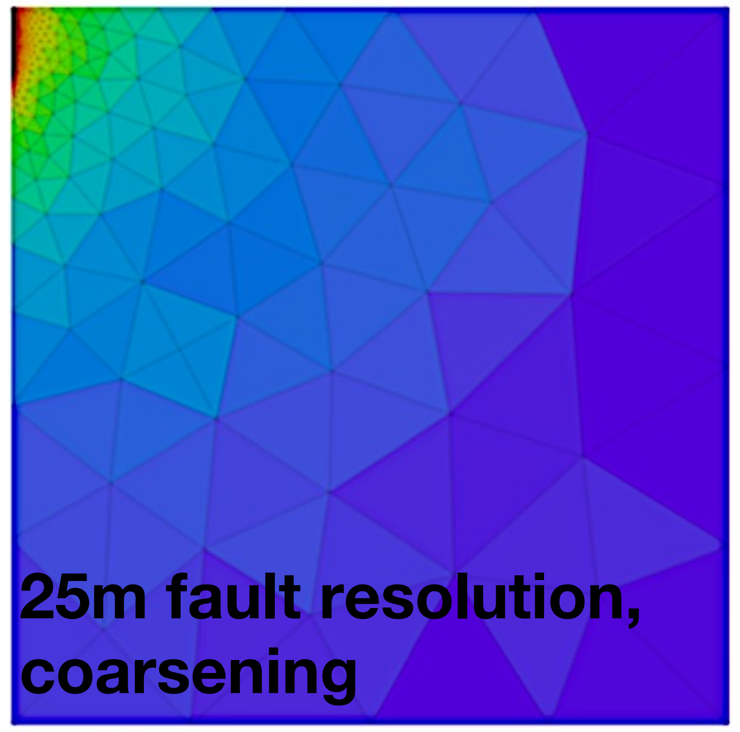
(a) $\mathcal{T}_h^A : h_f = 5$ km, $h_b = 5$ km, $N_e = 14982$



(b) $\mathcal{T}_h^B : h_f = 1$ km, $h_b = 8.5$ km, $N_e = 15296$



(c) $\mathcal{T}_h^C : h_f = 100$ m, $h_b = 25$ km, $N_e = 15289$



(d) $\mathcal{T}_h^D : h_f = 25$ m, $h_b = 130$ km, $N_e = 14314$

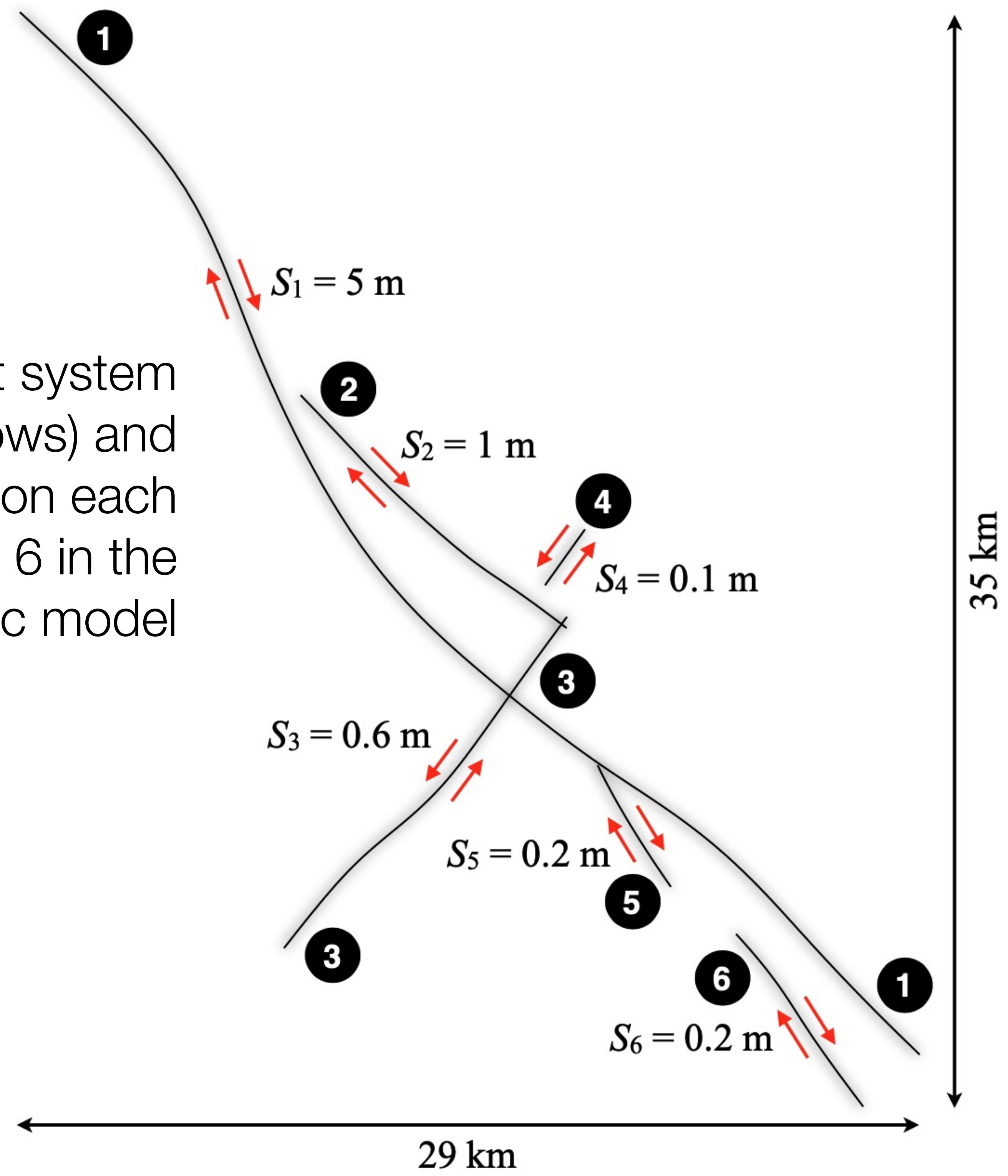
H-refinement experiment using 4 meshes **all containing 150k elements** to demonstrate the usefulness of unstructured meshes and aggressive coarsening.

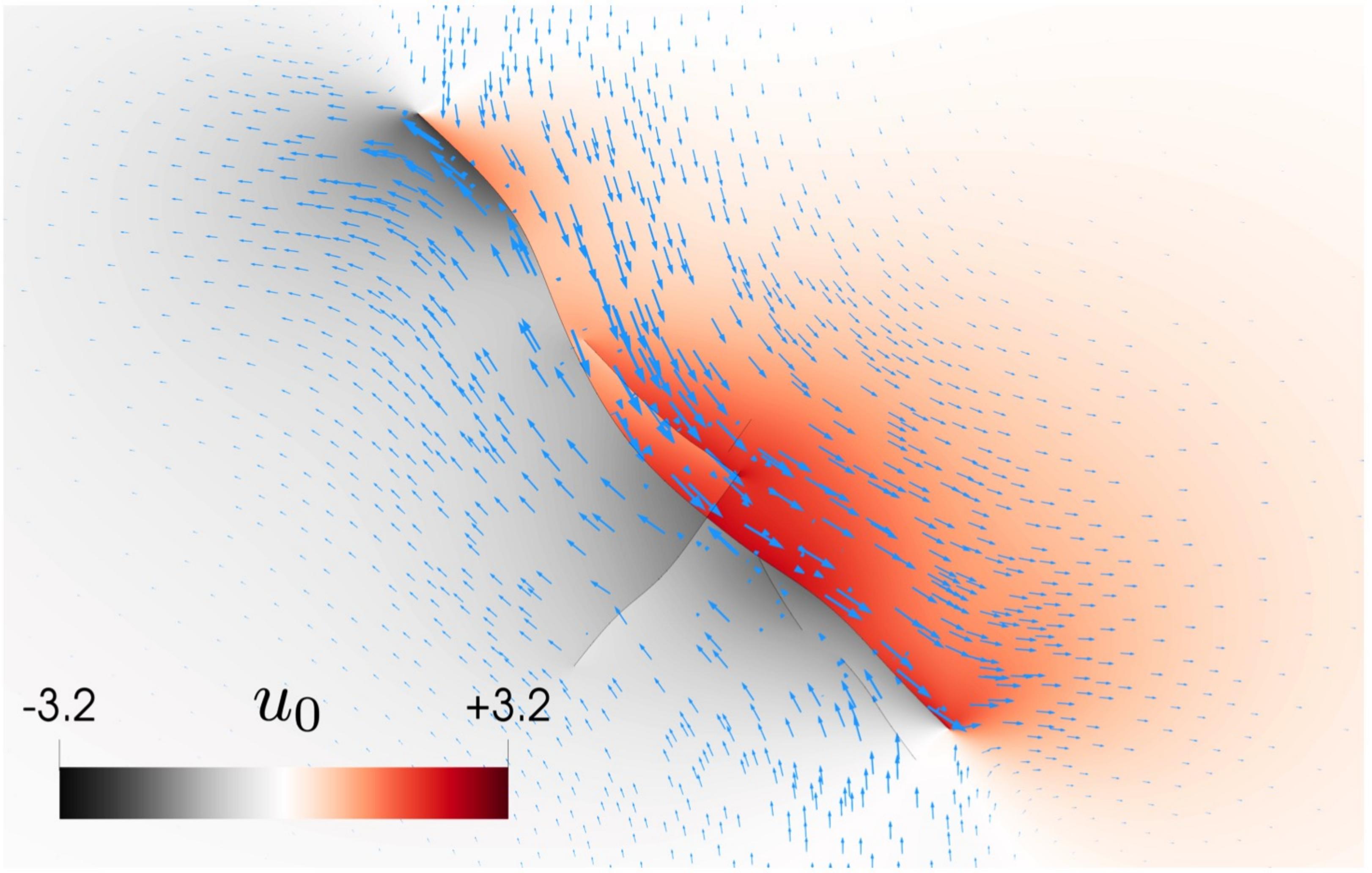
tandem - the Symmetric Interior Penalty Galerkin (SIPG) Discontinuous Galerkin (DG) method

We introduce ***tandem***, an **open-source** PETSc SIPG-DG solver for elliptic SEAS problems, that allows for

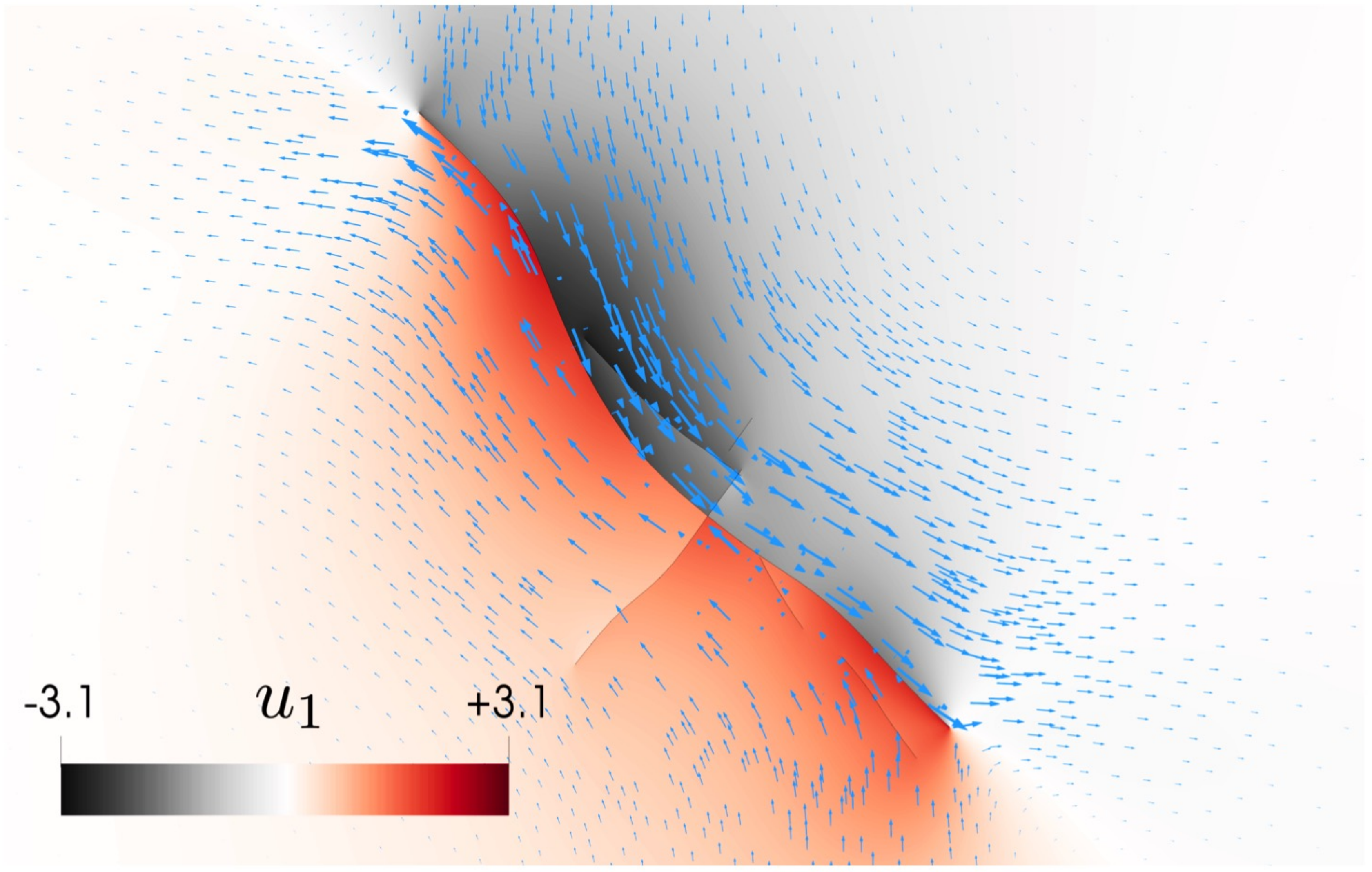
- **natural handling of discontinuities:** using a using a double-valued numerical flux there is no need to split nodes or to introduce Lagrange multipliers at the fault

Schematic of the Ridgecrest fault system geometry, the sense of slip (red arrows) and values for the imposed strike-slip (Sk) on each fault segment k = 1, . . . , 6 in the instantaneous, kinematic elasto-static model

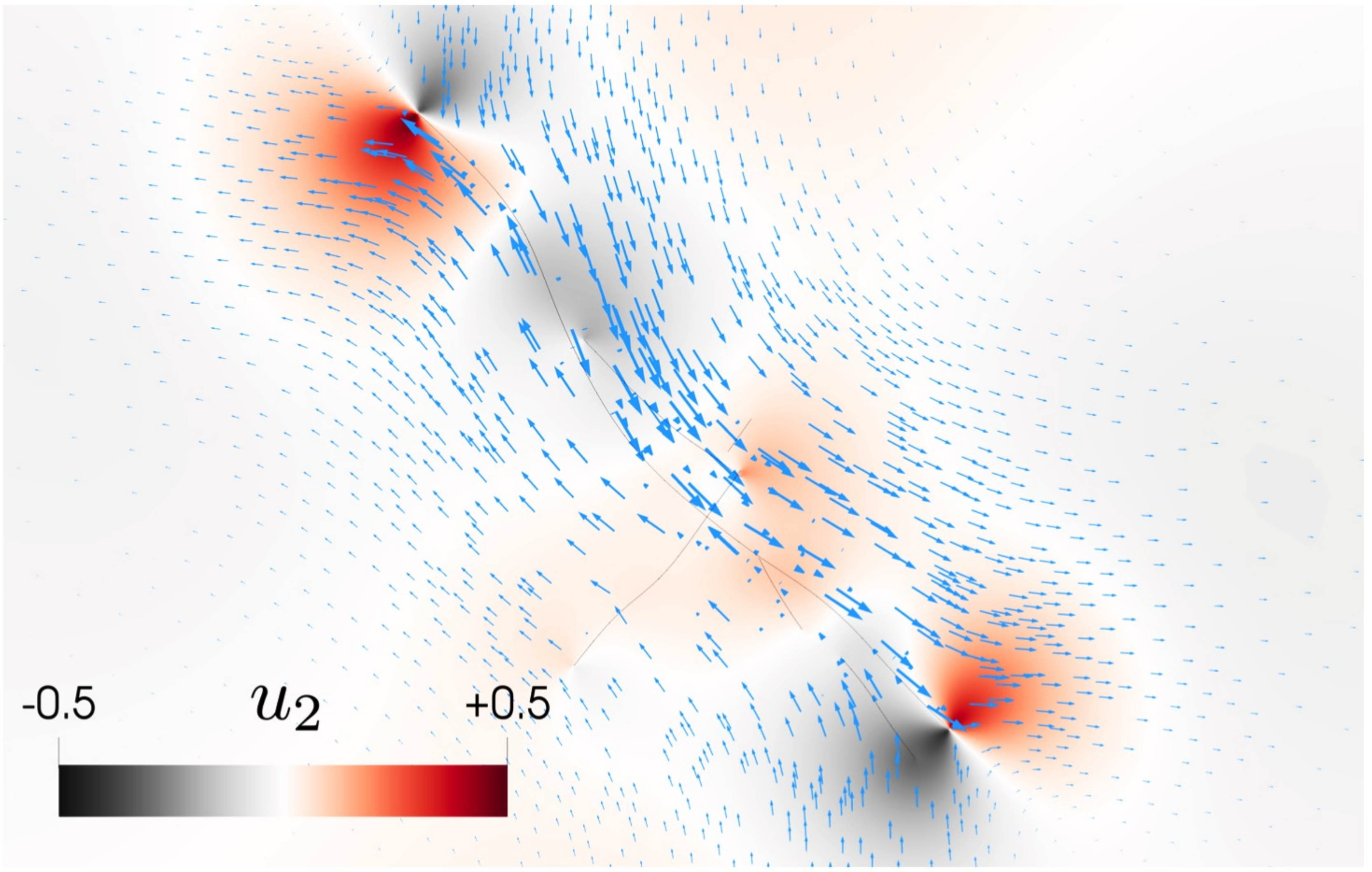




(a) x displacement (u_0)



(b) y displacement (u_1)



(c) z displacement (u_2)

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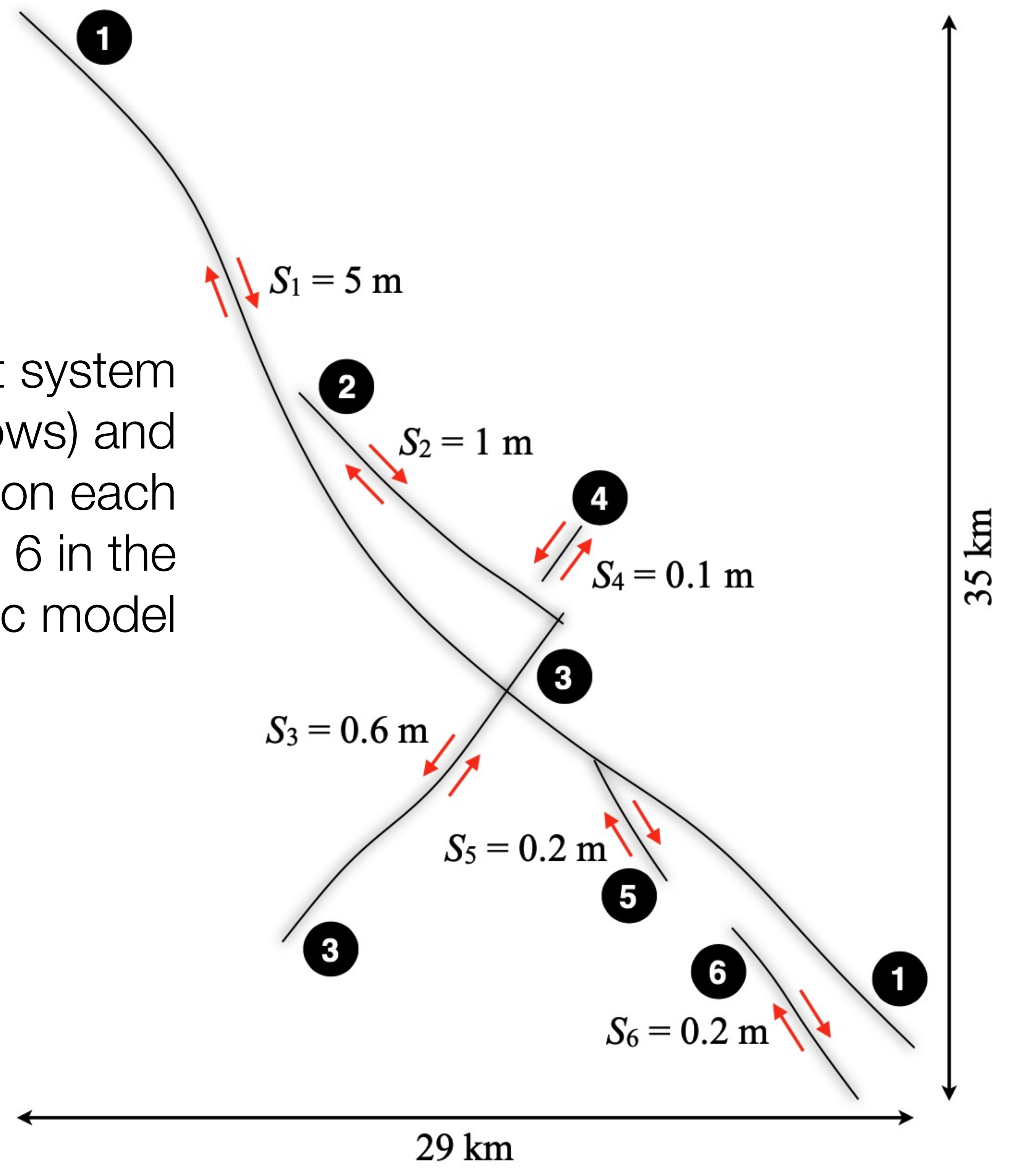
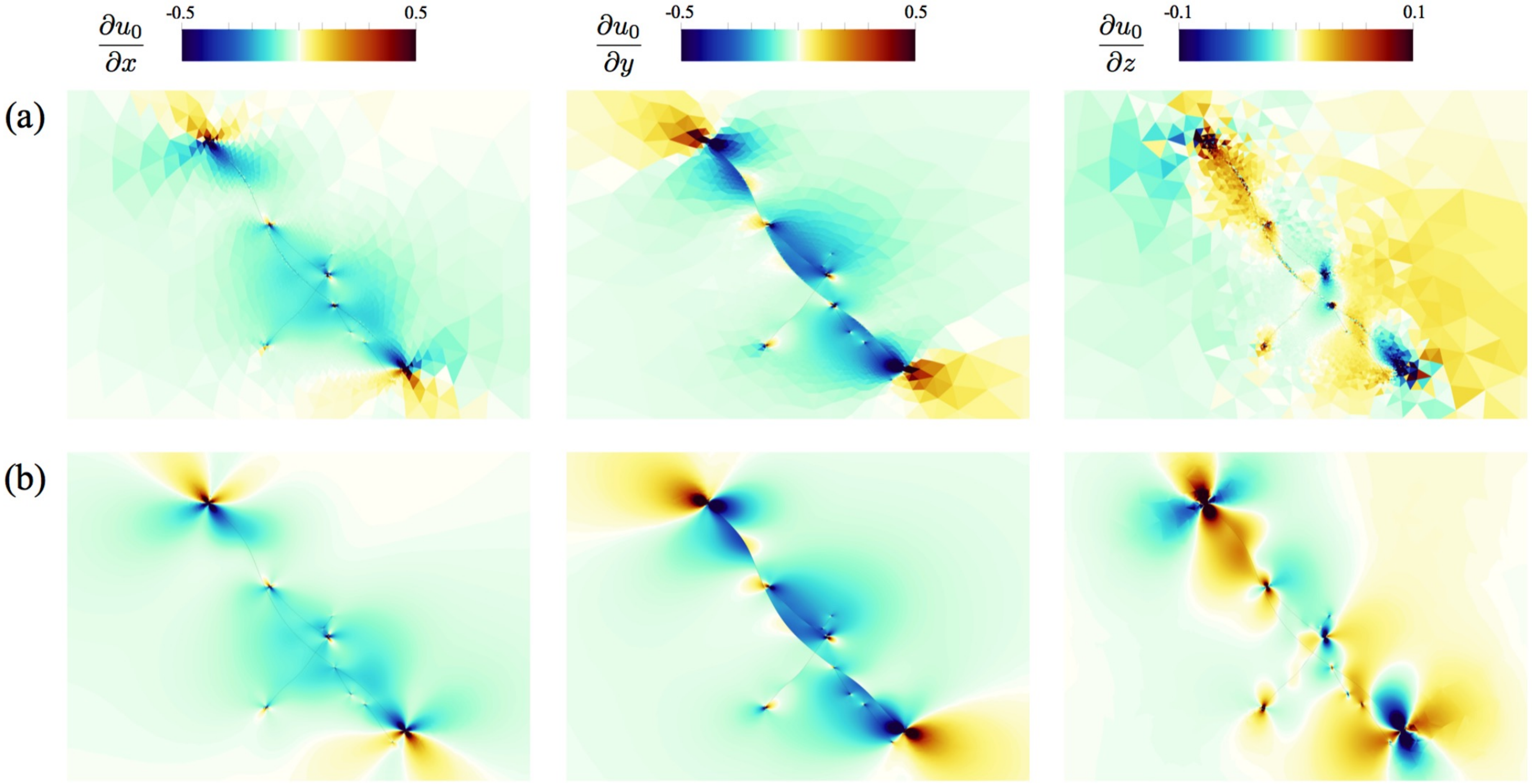




Demonstration of the advantage of using a curvilinear mesh also for static solves.

top: gradient of displacements in an **affine** mesh

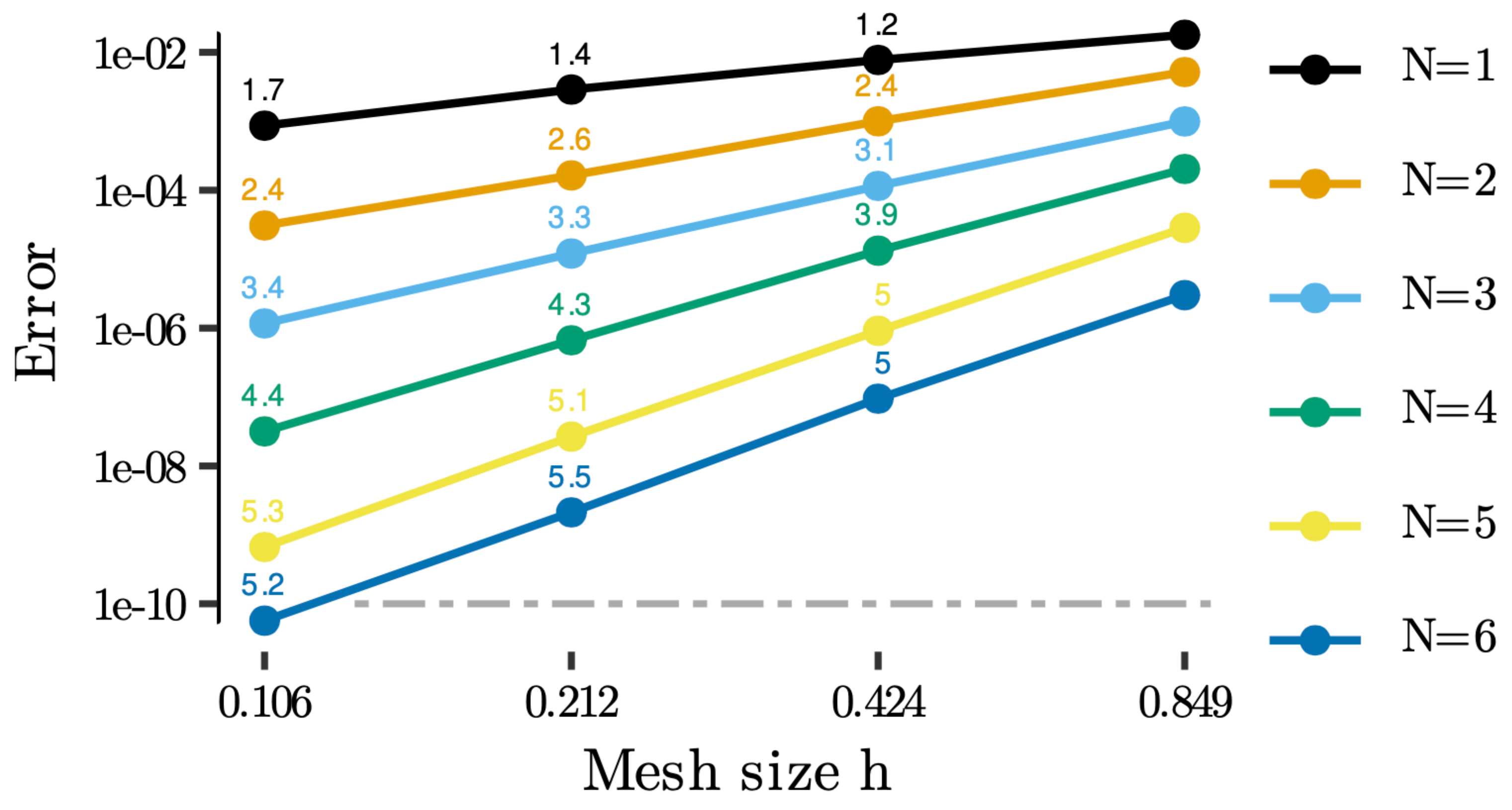
vs.

Bottom: Much **improved** in a non-affine (**curvilinear**) mesh with fault geometry approximated by polynomial of degree N=2

tandem - the Symmetric Interior Penalty Galerkin (SIPG) Discontinuous Galerkin (DG) method

We introduce ***tandem***, an **open-source** PETSc SIPG-DG solver for elliptic SEAS problems, and

- we show that SIPG achieves **arbitrary high-order accuracy for analytic, manufactured, and SEAS applications**



Convergence analysis under h- and p-refinement matching the theoretical high-order expectation. For a **3D SEAS manufactured solution** (top), a 3D spherical hole analytical problem verifying **curvilinear grids** (bottom right) and **heterogeneous materials** varying within elements (bottom left).

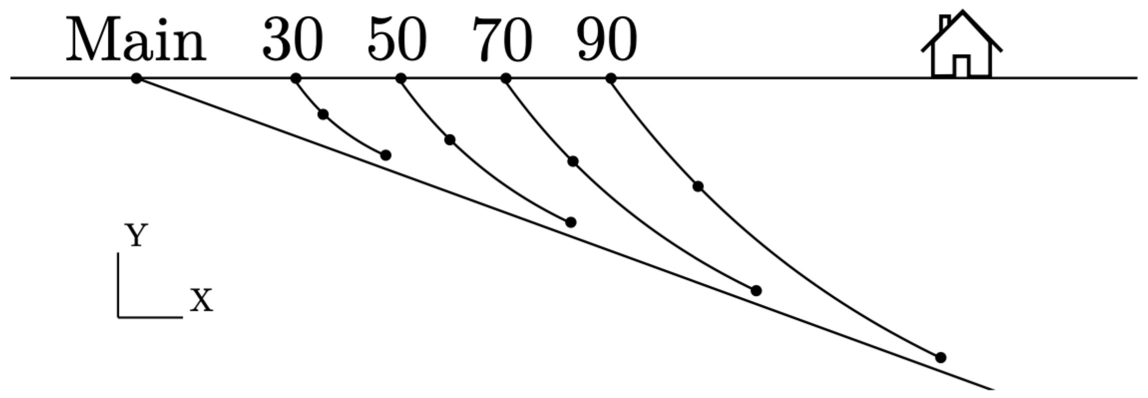
tandem - the Symmetric Interior Penalty Galerkin (SIPG) Discontinuous Galerkin (DG) method

We introduce ***tandem***, an **open-source** PETSc SIPG-DG solver for elliptic SEAS problems, that is

- well suited to complex large scale simulations and **scalable up to the largest-available supercomputers** (tested on ~**5000 MPI ranks** on SuperMUC-NG)

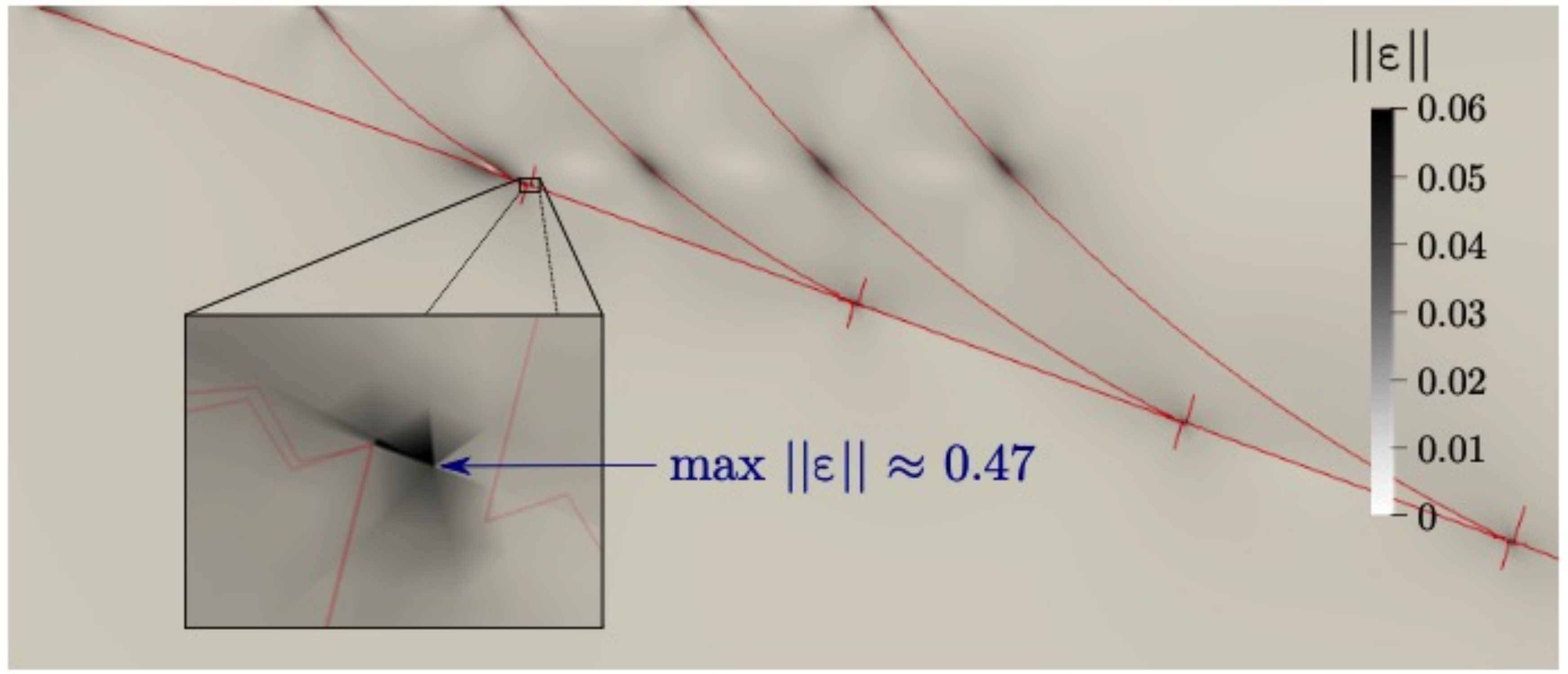
(i) Support for 2D and 3D spatial discretisations using unstructured meshes comprised of triangles (2D) and tetrahedra (3D);
(ii) Support for high-order (curvilinear) representation of exterior boundaries and interfaces (faults);
(iii) Sub-element (high-order accurate) representation of material properties and sub-element (high-order accurate) representation of slip, slip-rate in SEAS problems;
(iv) Efficient kernels for the assembly of discontinuous Galerkin operators and residual evaluation;
(v) A fully parallel implementation, including mesh loading, solution stage, output and visualisation;
(vi) Access to state-of-the art solvers, preconditioners and time integrators by using the Portable, Extensible Toolkit for Scientific Computation (PETSc);
(vii) Support to optionally compute the discrete Green's function (i.e., the affine function which maps slip to traction) via efficient scalable (algorithmic and parallel) solvers.

A multi-fault scenario on a shallowly dipping normal fault with four curved splay faults

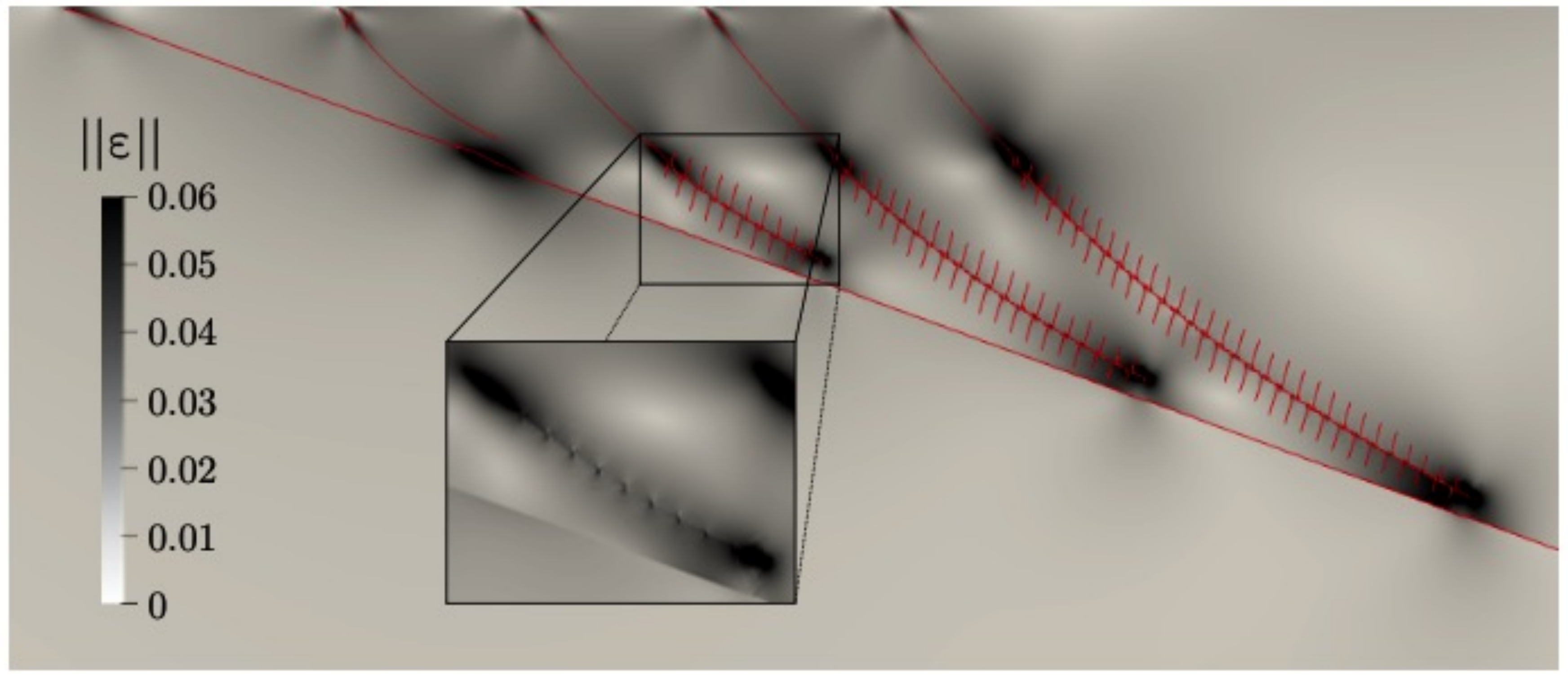


Parameter	Value	Parameter	Value
$a_{\max}$	0.025	a_0	0.010
L	0.05 m	b	0.015
f_0	0.6	ν	0.25
ρ_0	2670 $\mathrm{kg/m^3}$	c_s	3.464 $\mathrm{km\,s^{-1}}$
V_p	$-1 \times 10^{-9}\,\mathrm{m\,s^{-1}}$	V_0	$1 \times 10^{-6}\,\mathrm{m\,s^{-1}}$
σ_n^0	50 MPa	T^0	26.5461 MPa
η	$\rho_0 c_s/2\,\mathrm{Pa\,s\,m^{-1}}$	t_{end}	1500 yr

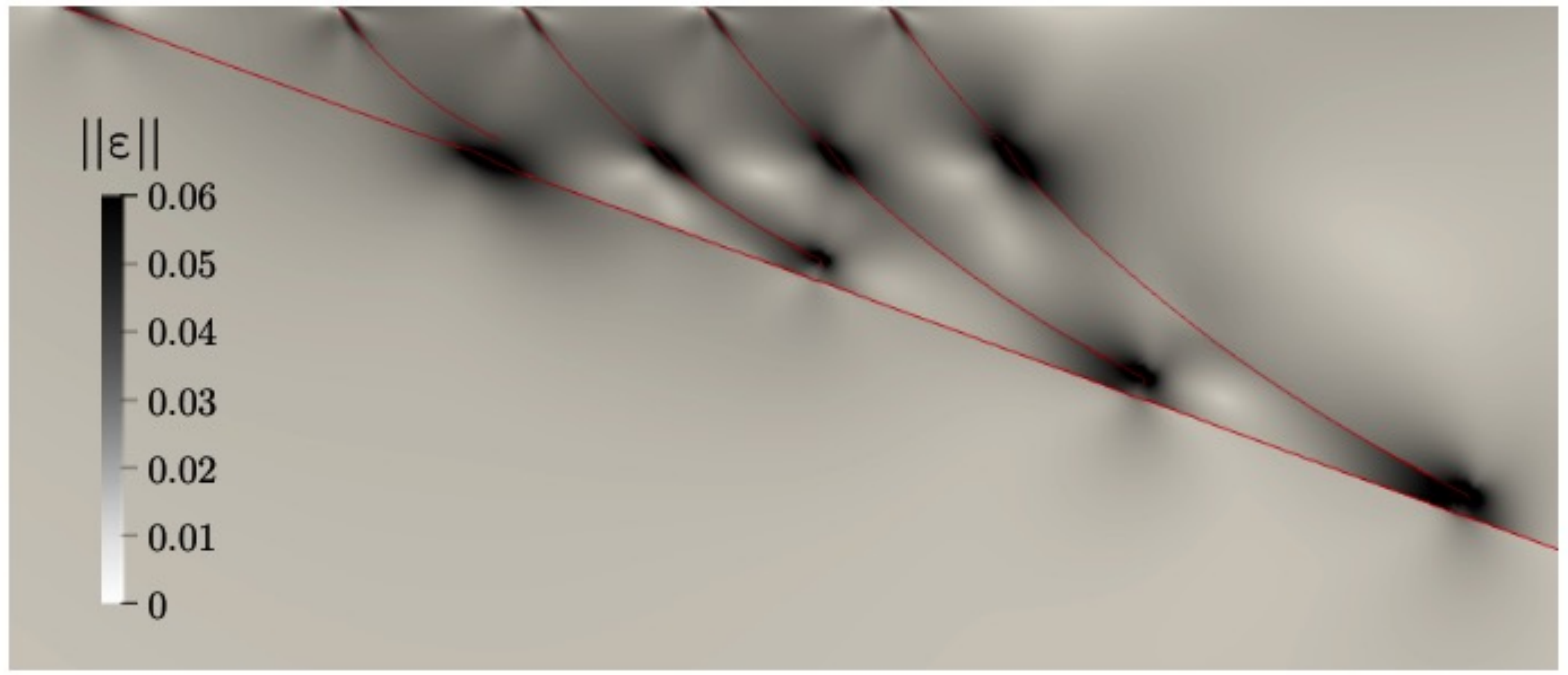
- **demonstration** example of **detachment splay-low-angle-normal fault systems** (see also rupture dynamics talk, Friday pm)
- **domain size** 2000 x 1000 km, homogeneous material, simple frictional parameters a/b profiles
- **1400 years** (no spin-up) of slip and slip rate evolution on the main low-angle normal fault and all four curved splay faults in response to steady tectonic loading
- **run-time** (120 ranks): 0.5 km on-fault resolution, Green's functions 1.1 d, time integration 0.9 d (96.4k time-steps, 45.5k time-step rejections)



(a) Splays intersecting the main fault cause large strain at the intersection point.

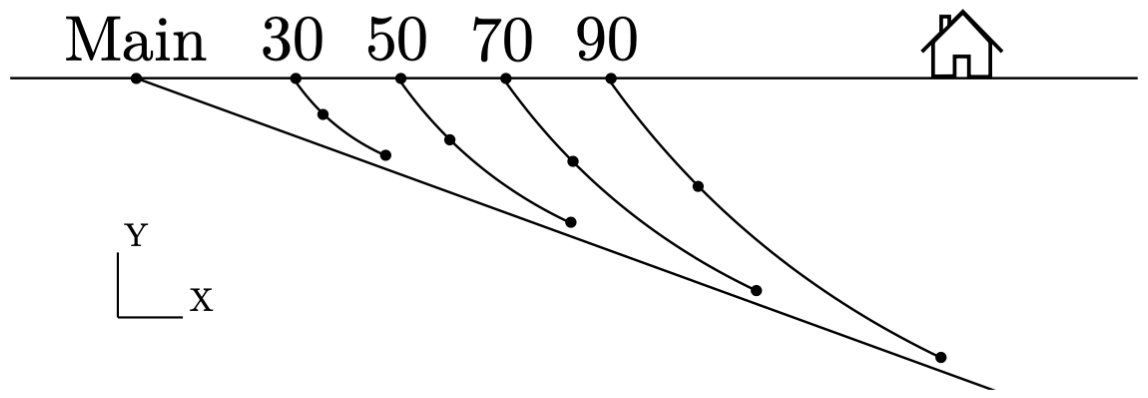


(b) Artifacts in strain and traction are visible on an affine mesh (non-intersecting) due to the linear approximation of the boundary.



(c) No visible artifacts with non-intersecting faults and curvilinear mesh.

A multi-fault scenario on a shallowly dipping normal fault with four curved splay faults



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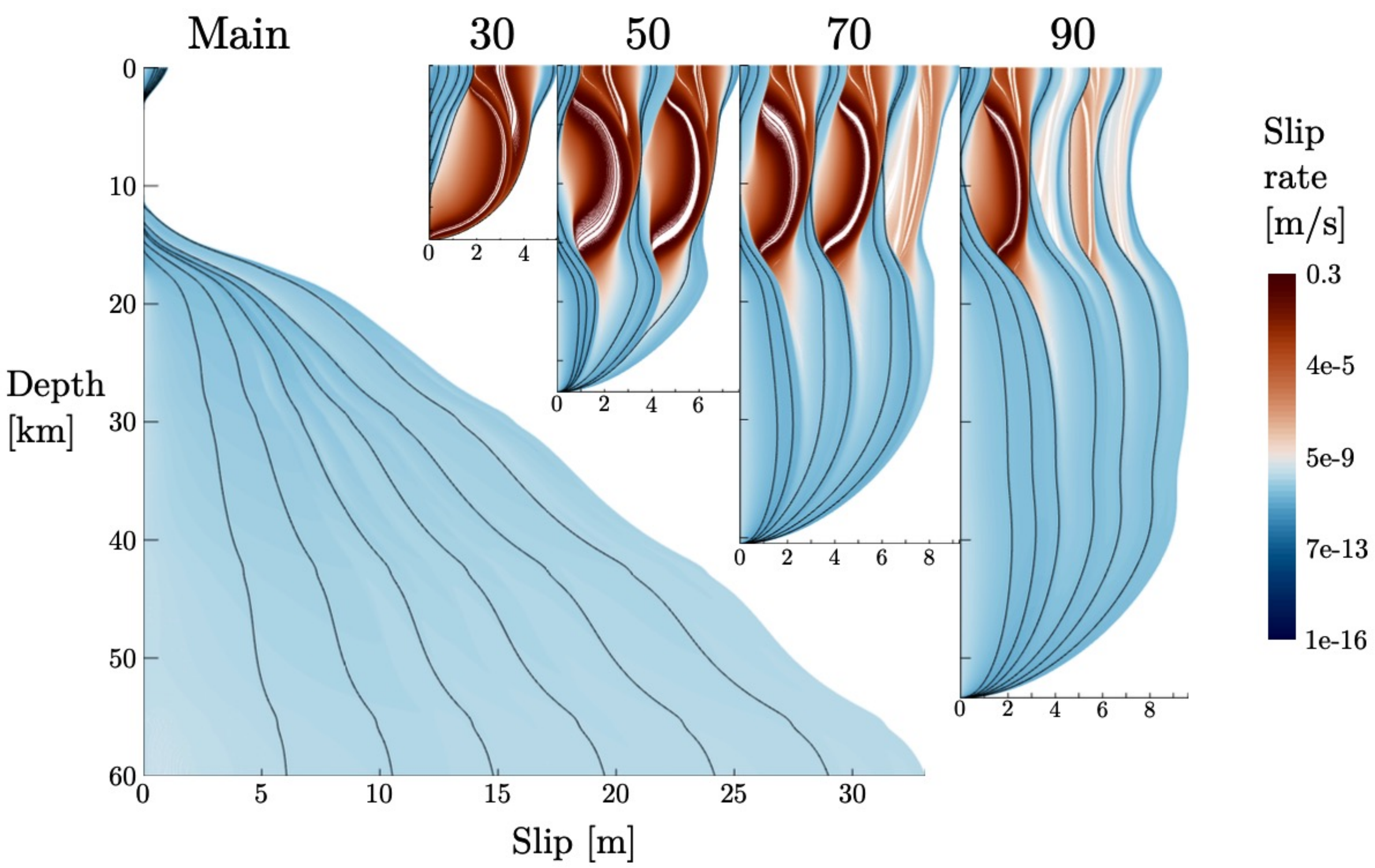
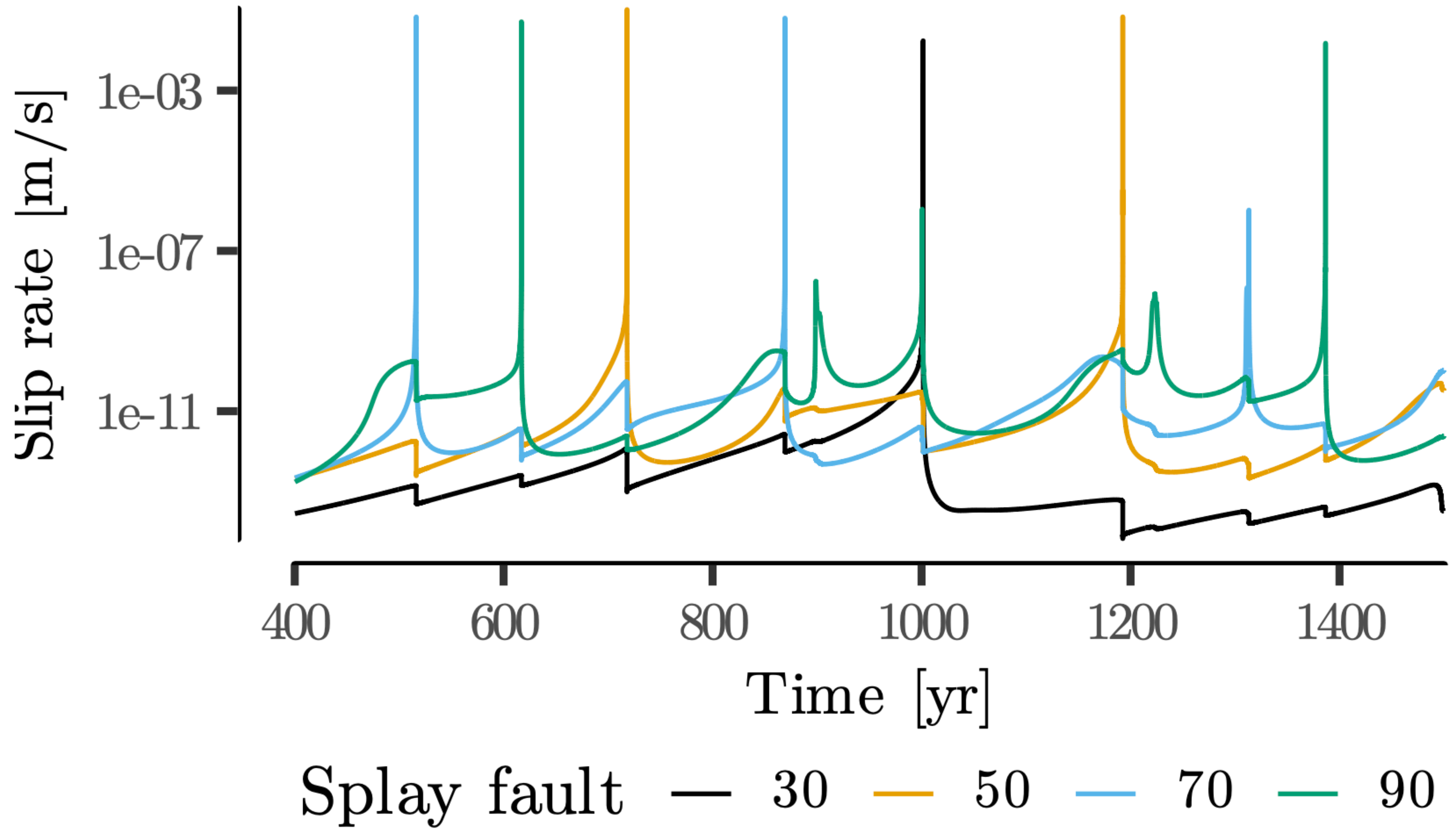


- complex slip evolution and fault interaction, **irregular fast and slow events occurring on the splay faults**, including surface rupture, while **main fault creeps** at plate rate; complex **splay fault coupling**
- **Shortest/closest splay**: regular low-slip rate events & single fast earthquake.
- **Intermediate splays** in 50 km and 70 km distance: slip, slip rate and recurrence rate may correlate with splay fault length
- **Largest splay:** regular aseismic transients and seismic events which magnitude appears to decrease over time
- **Note**: only *a* varies, a more variable setup for example in normal stress may yield both main fault and splay fault rupture



Summary

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https://github.com/TEAR-ERC/tandem

- We introduce ***tandem***, an **open-source** code for volumetric, elliptic SEAS problems using a symmetric interior penalty discontinuous Galerkin (**SIPG-DG**) method to perform seismic cycle simulations on modern **HPC** infrastructure
- We demonstrate **high-order convergence** in space (and time) for both elasto-static and SEAS problems, also when material properties vary within a cell / the fault or exterior boundary is curved: provided both material coefficients and geometry are discretised with the same polynomial degree as the displacement
- We exploit the curvilinear geometry representation to **accurately represent stresses or displacement gradients on-fault**
- **We address the high computational demands of SEAS problems** by using efficient libraries for the DG kernels, linear algebra, solver and preconditioner support. We also **optionally** allow to use **discrete Green's function**, evaluated and checkpoint once in an embarrassingly parallel pre-computation step using algorithmically optimal and scalable sparse parallel solvers and preconditioners.
- We envision that for problems in which no analytical Green's function is available (complex fault systems, 3D subsurface, topography), **HPC empowered DG SEAS methods may be competitive with BEM** (which then involves the solution of a large and dense system of equations)