Estimating States And Parameters in Earthquake Sequence Models in the Presence of a Parameter Bias

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Objective of Study

We investigate the efficacy of using data assimilation techniques for state estimation considering parameter uncertainty.

- *We address this objective by running a series of data assimilation experiments with a nonlinear model that mimics the evolution of earthquake cycles in two tectonic plates.*

Problem Statement:

Data assimilation has been used for forecasting across a wide range of applications. It involves combining the information from a numerical model as well as from observations to obtain the optimal estimate of the state of the system, referred to as the analysis. However, parameters determine the behavior of these numerical models and hence, an inaccurate representation of these model parameters may eventually affect the ability of a model to accurately estimate the true state of the system.

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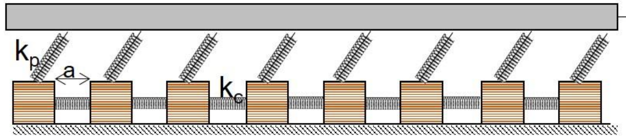
B. Forecast Ensemble

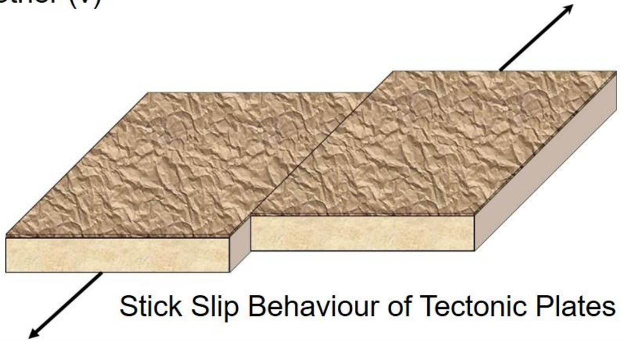
Numerical Model

<u>States of interest</u>: Shear stress acting on the plate (τ), the velocity with which the plates move with respect to each other (v)

A. Initial Ensemble



Burridge & Knopoff (1967) model for Stick-Slip Behaviour

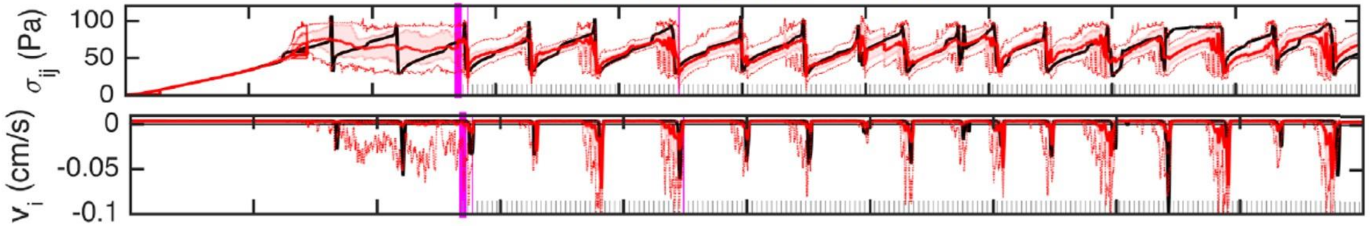


Stick Slip Behaviour of Tectonic Plates

Updated Ensemble

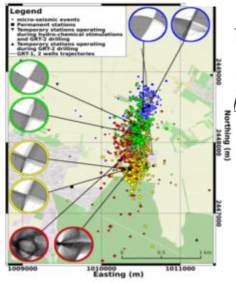
D. Data Assimilation

Combines observations with Model



Van Dinther, Y., Kunsch H. R., and Fichtner A. 2019: Ensemble data assimilation for earthquake sequences: Probabilistic estimation and forecasting of fault stresses. Geophysical Journal International, 217 (3), 1453-1478.

C. Observations



Measured Ground Motion Data
- Seismometers
- InSAR

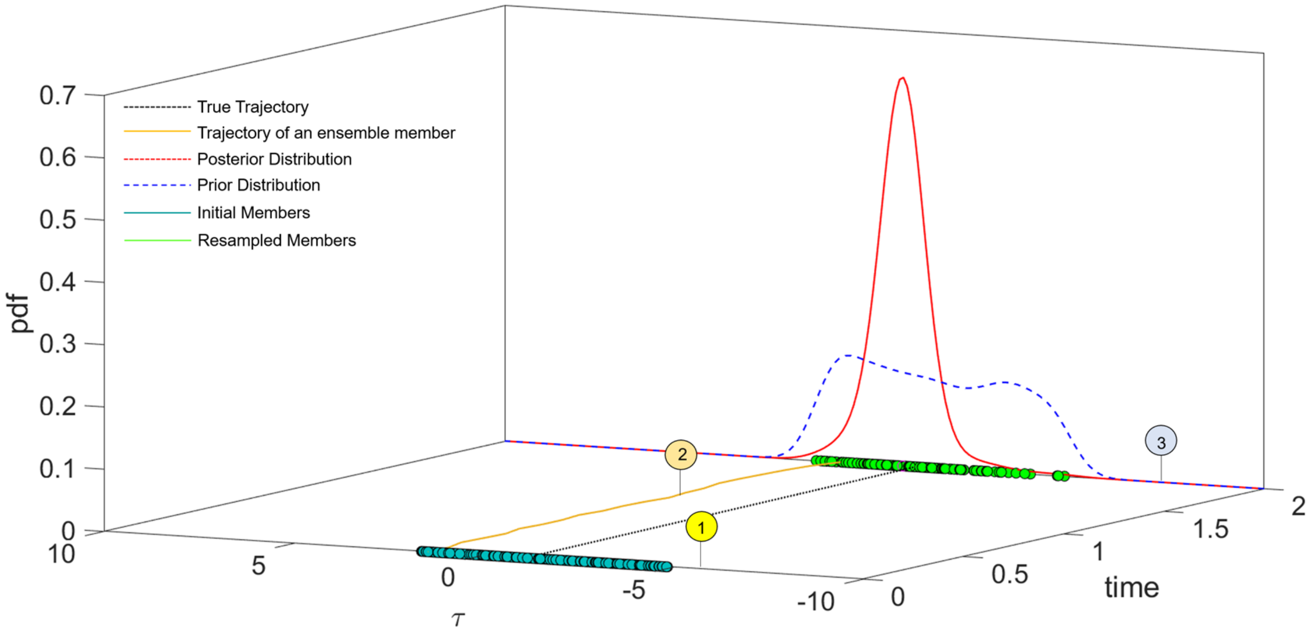
Maurer et al. (2020)

Burridge, R., & Knopoff, L. (1967). Model and theoretical seismicity. Bulletin of the Seismological Society of America, 57(3), 341–371. https://doi.org/10.1785/BSSA0570030341

Bayesian Inference: Particle Filter

Monte-Carlo Sampling: *The basic idea of the particle filter is to represent the prior pdf of the states by a set of N random members (referred to as particles) as:*

$$p(x) = \frac{1}{N}\sum_{i=1}^{N}\delta(x - x_i)$$



1. Initialisation:
2. Propagation:
3. Correction/Analysis:

(a) Calculate weights representing the likelihood

(b) Resampling

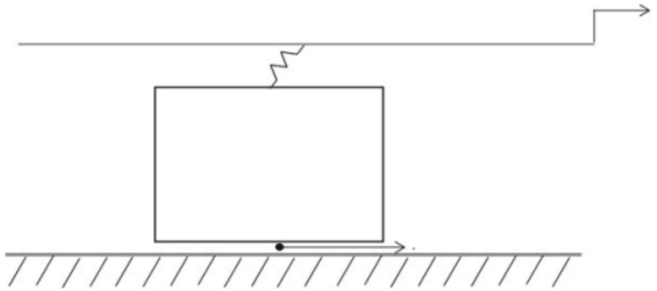
Model: Burridge-Knopoff Rate-and -State Friction

The scaled and non dimensional equations of motion of the model coupled with Dietrich-Ruina rate and state dependent friction for a single block model are given by:

$$\dot{\bar{u}} = \bar{v} - 1$$

$$\dot{\bar{v}} = -\gamma^2[\bar{u} + (1/\xi)(\bar{f} + \bar{\Theta} + \ln(\bar{v}))]$$

$$\dot{\bar{\Theta}} = -\bar{v}(\bar{\Theta} + (1 + \epsilon)\ln(\bar{v}))$$



zero-dimensional version of the spring-block system by Burridge and Knopoff (1967)

<u>State Variables:</u>

$\bar{\Theta}$ is the scaled value of the already non-dimensional strength (the memory effect in the model),

$\bar{v}$ is the non-dimensional slip velocity

$\bar{u}$ the non-dimensional slip of the block relative to the driver plate,

$\bar{f}$ is the friction

<u>Static Parameters:</u>

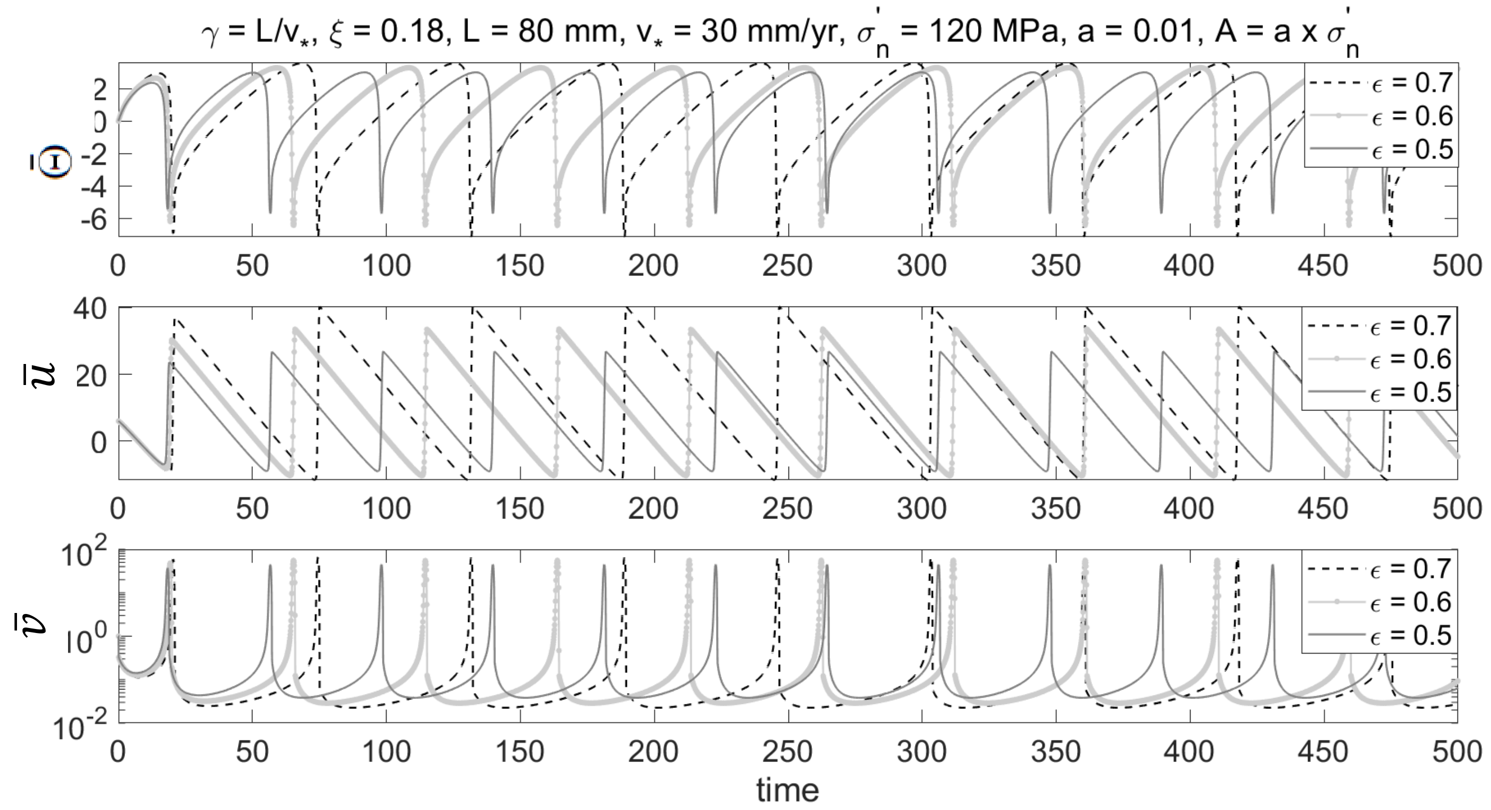
ε measures the sensitivity of the velocity relaxation and is a ratio of the stress parameters,

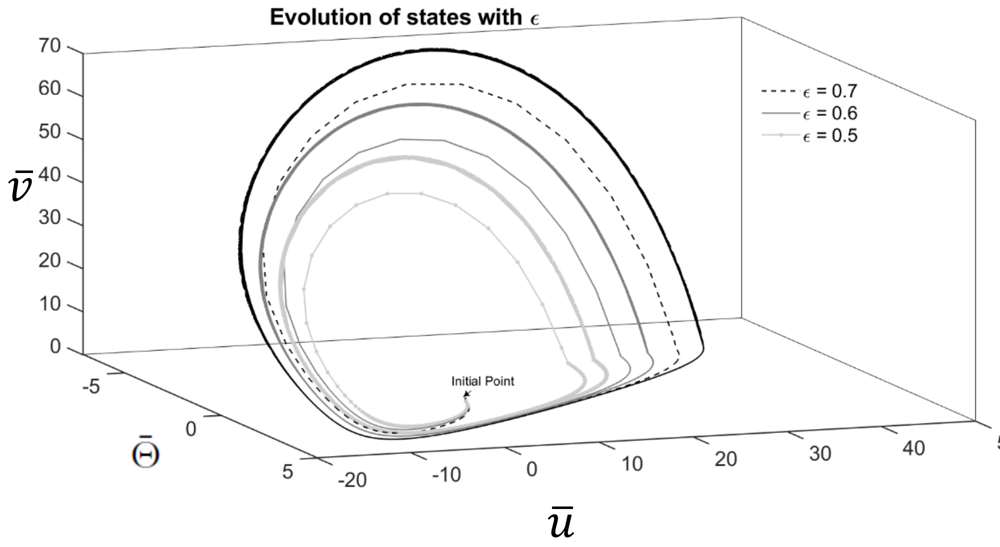
ɣ is the non-dimensional frequency,

ξ is the non-dimensional spring constant.

Erickson, B., Birnir, B., & Lavallée, D. (2011). Periodicity, chaos and localization in a BurridgeKnopoff model of an earthquake with rate-and-state friction. Geophysical Journal International, 187, 178–198. https://doi.org/10.1111/j.1365-246X.2011.05123.x

Model: Parameter Uncertainty under study





Physical Significance of ε

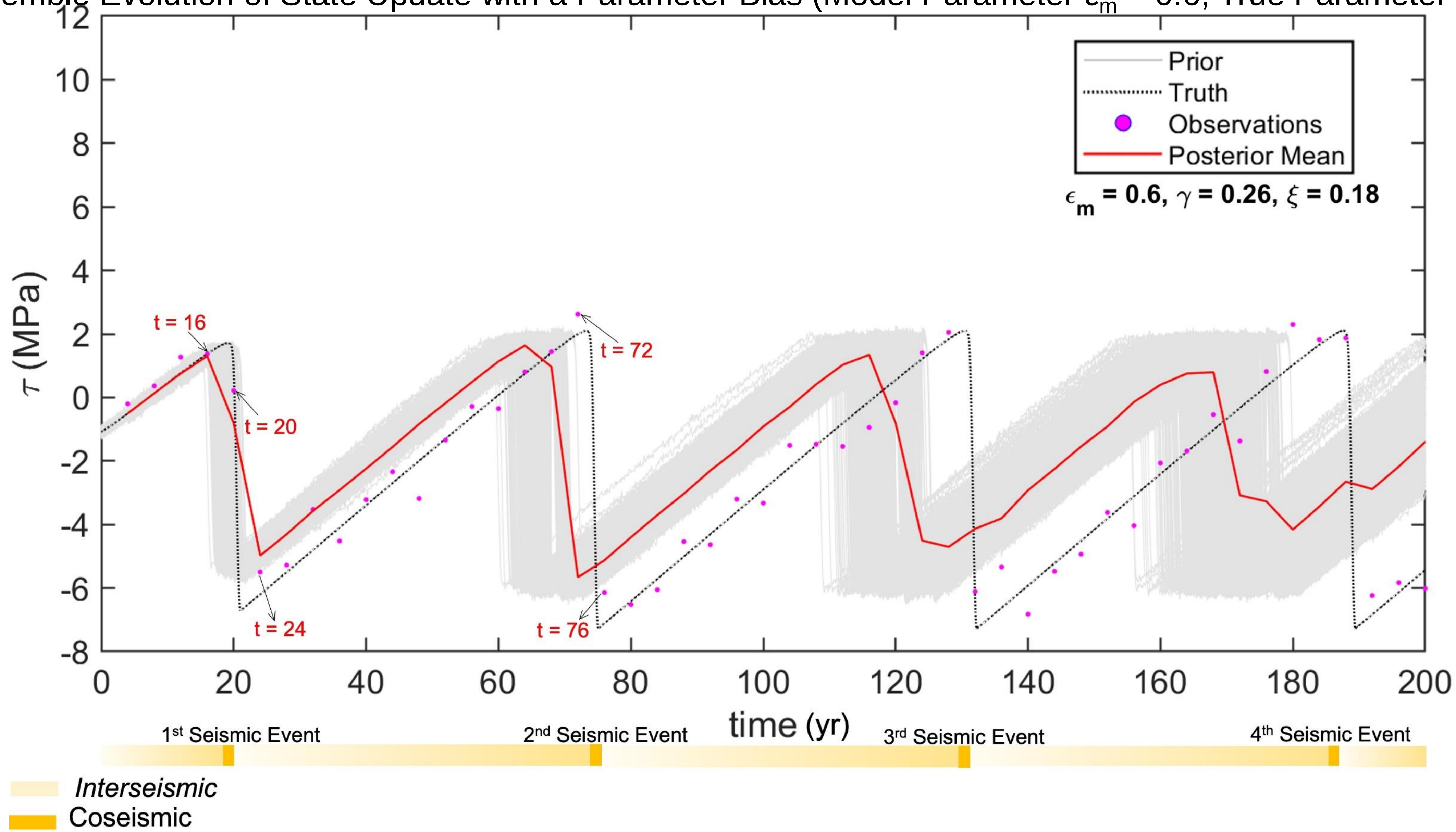
$$\epsilon = \frac{B - A}{A}$$

(B − A) plays the role of a stress drop while A corresponds to the strength excess.

Effects shear stress estimate τ and state variable $\bar{\Theta}$

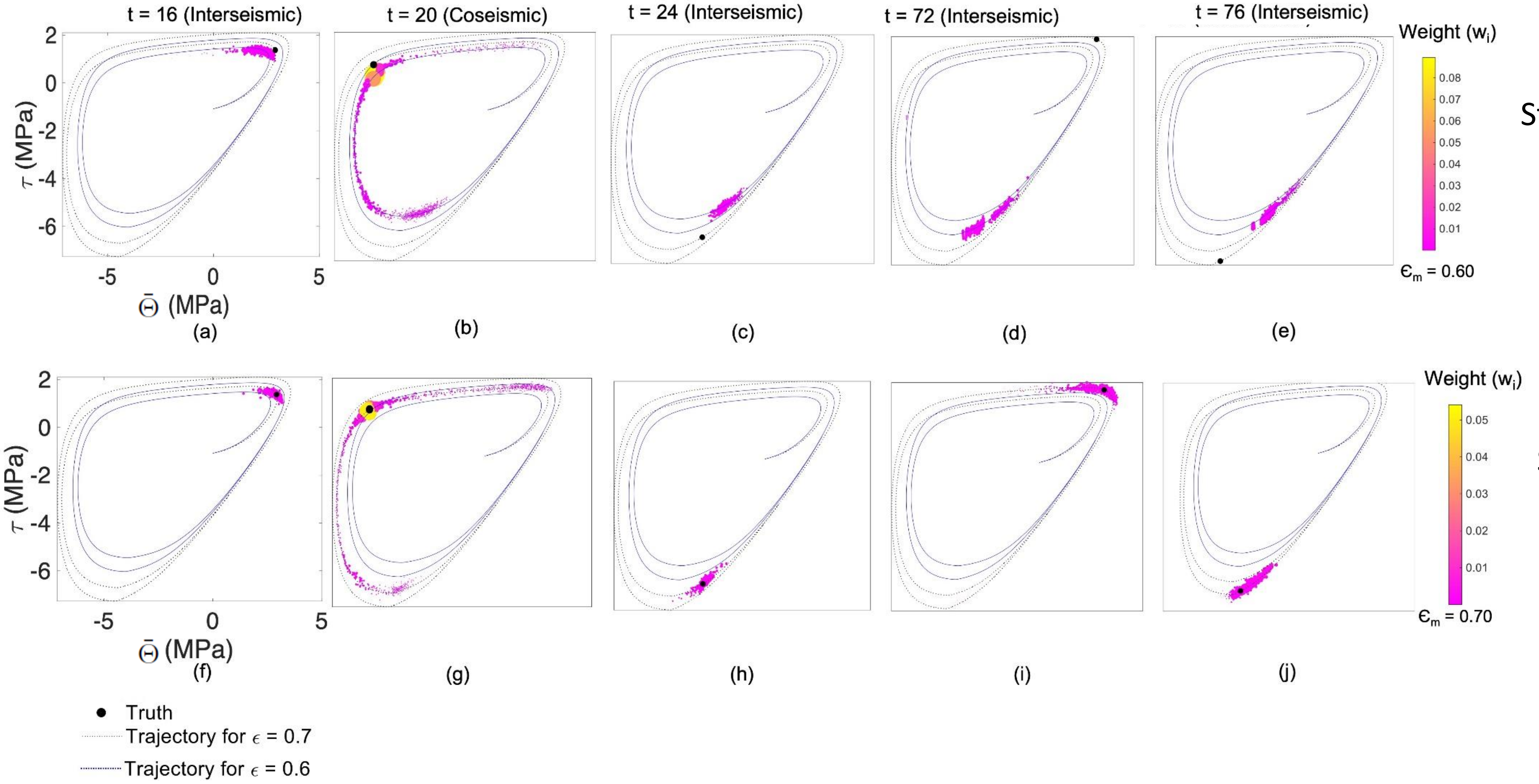
Results: Data Assimilation Without Parameter Uncertainty

Ensemble Evolution of State Update with a Parameter Bias (Model Parameter $\varepsilon_m = 0.6$, True Parameter $\varepsilon_t = 0.7$)



Results: Data Assimilation Without Parameter Uncertainty

<u>State Update</u>: Use the incorrect parameter value ε =0.6 in the model and <u>update states only</u>.



State Update

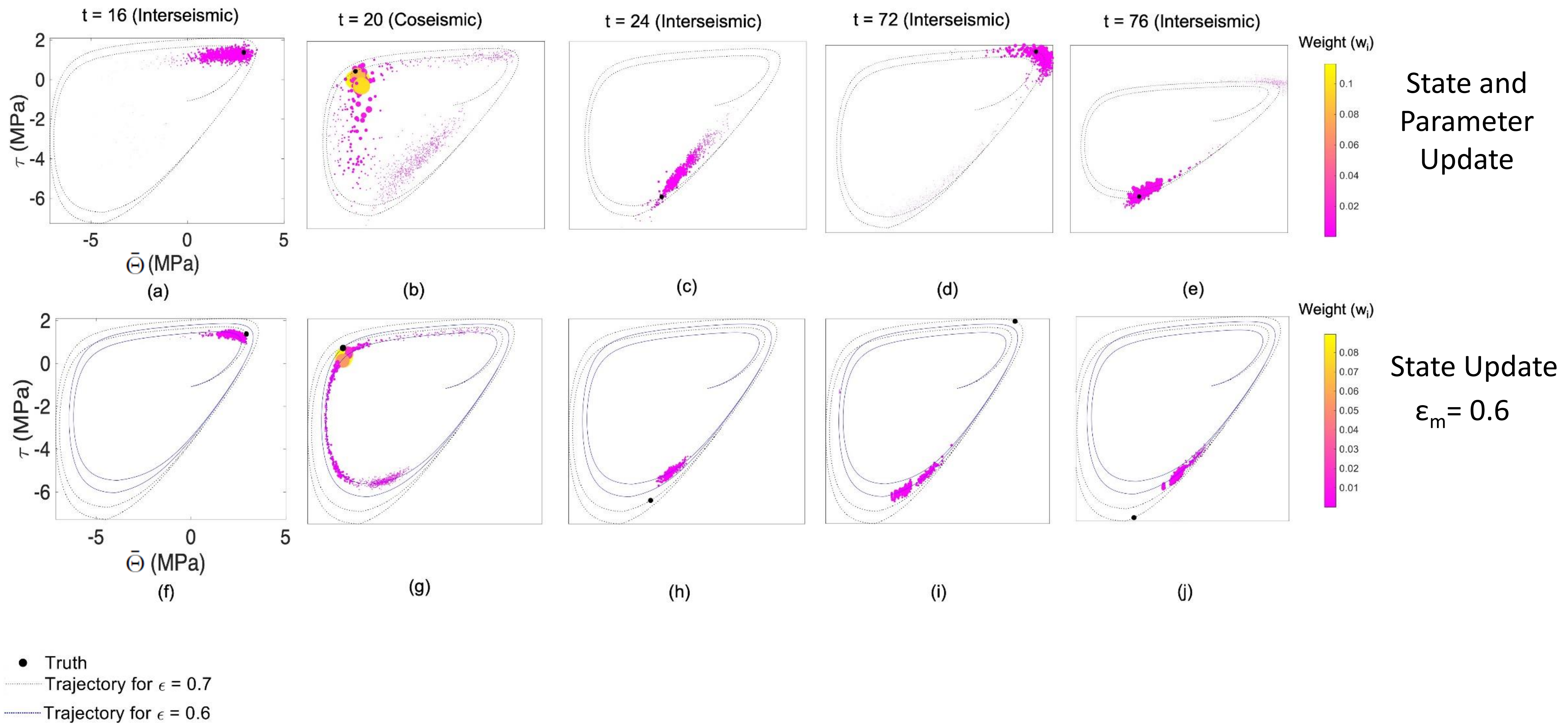
ε_m= 0.6

State Update

ε_m= 0.7

Results: Data Assimilation With Parameter Uncertainty

State and Parameter Update: Considers parameter uncertainty and update states as well as parameter ε.



Conclusion and Future Work

In the presence of parameter bias in a seismic cycle model, combined state- and parameter estimation provides a more favourable data-assimilation outcome than state estimation alone.

<u>Future Work</u>

- High-dimensional earthquake simulation
- Smoothers for joint state and parameter estimation
- Non-periodic earthquake cycles