Gravitational Potential Difference Between Optical-Atomic Clocks onboard China Space Station (CSS) and Ground Station via Optical Time Transfer links

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1. Introduction

Chinese Space station (CSS)

- On April 29, 2021, Tianhe, the core module of CSS, was successfully launched.
- In 2022, CSS will carry an optical clock and atomic clock with long-term stability **8×10^{-18}** and **2×10^{-16}**.
- Relativistic theory **up to order $1/c^4$**

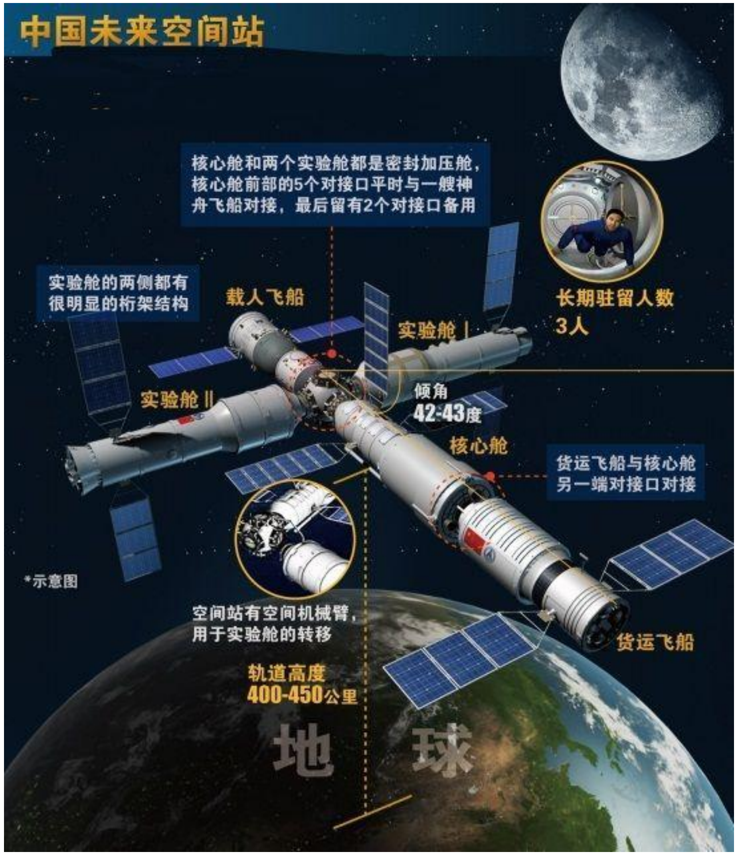

Fig. 1 Chinese Space station (CSS) , Orbit ~ 400 km)

1. Introduction

Chinese Space station (CSS)

Optical Laser Time Link
Transmit Wavelength = 0.532 μm
Time measuring precision (single shot, RMS) 60ps
Repetitions rate 1 kHz
Onboard Detector
Single photon
Detection precision (single shot, RMS) < 30ps
Gate mode and FOV: 128°
Onboard Timer
Single-shot RMS resolution: < 10ps
Average measurement rate: >1kHz
Laser Retroreflector Array
Size: Ø150mm×61mm & Dihedral offset: 14″±2″
Compact octahedral array, less spread effect

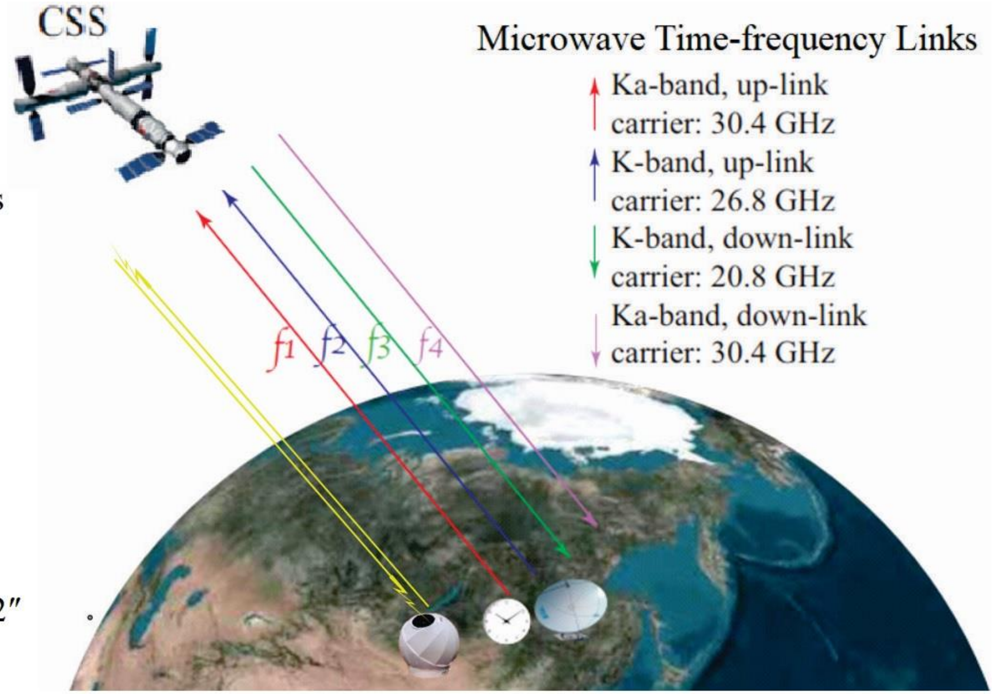

Fig. 2 Microwave and Optical transmission links of CSS

- Microwave Time-frequency Link: 0.3 ps @ 300s and 6 ps @ 24h
- Optical Laser Time Link : 0.1 ps @ 300s and 1 ps @ 24h

2. Gravitational Potential Difference

Clocks Rate Differences up to order $1/c^4$

Taking the ratio of two clock readings A & B yields (Blanchet et al. 2001)

$$\left(\frac{d\tau_B}{d\tau_A}\right) = \left(\frac{u_A}{u_B}\right) \times \left(\frac{dt_B}{dt_A}\right) \qquad (1)$$

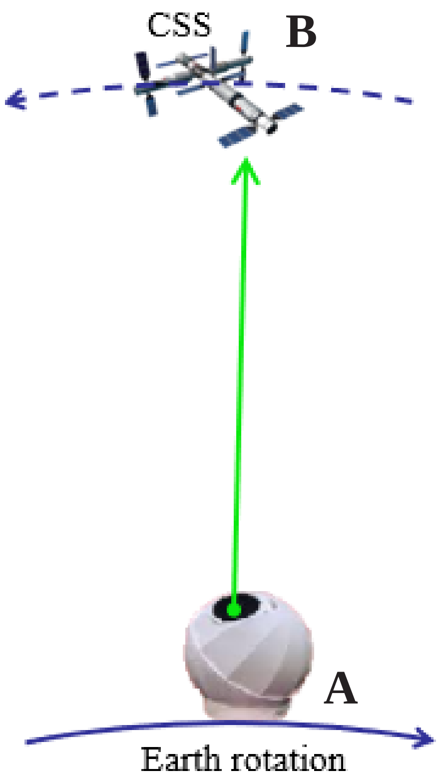

Fig. 3 One-way time transfer

Where :

$$\frac{u_A}{u_B} = \left(\frac{d\tau}{dt}\right)_B \Big/ \left(\frac{d\tau_A}{dt_A}\right)_A = 1 + \frac{1}{c^2}\left(U_A - U_B + \frac{1}{2}v_A^2 - \frac{1}{2}v_B^2\right)$$

$$+\frac{1}{c^4}\Big\{(\gamma+1)\left(U_A v_A^2 - U_B v_B^2\right) + \frac{1}{2}\left(U_A - U_B\right)\Big[U_A - U_B + v_A^2$$

$$+2(1-\beta)\left(U_A + U_B\right)\Big] + \frac{3}{8}v_A^4 - \frac{1}{4}v_A^2 v_B^2 - \frac{1}{8}v_B^4$$

$$-2(\gamma+1)\left(\mathbf{U_A}\cdot\mathbf{v}_A - \mathbf{U_B}\cdot\mathbf{v}_B\right)\Big\} + O(c^{-6}) \qquad (2)$$

2. Gravitational Potential Difference

Clocks Rate Differences up to order $1/c^4$

The second term of Eq.(8) are the derivatives of coordinate time transfer Function (Linet & Teyssandier 2002) :

$$\frac{dt_B}{dt_A}=1-\frac{1}{c}\frac{N_{AB}\cdot(\mathbf{v}_A-\mathbf{v}_B)}{1-N_{AB}\cdot\frac{\mathbf{v}_B}{c}}+\frac{1}{c^3}\left[\mathbf{l}_A^{(2)}\cdot\mathbf{v}_A-\mathbf{l}_B^{(2)}\cdot\mathbf{v}_B\right]+\frac{1}{c^4}\left[\mathbf{l}_A^{(3)}\cdot\mathbf{v}_A-\mathbf{l}_B^{(3)}\cdot\mathbf{v}_B\right]$$
$$+\frac{1}{c^4}\mathbf{N}_{AB}\cdot\left[(\mathbf{l}_B^{(2)}\cdot\mathbf{v}_B)(\mathbf{v}_A-2\mathbf{v}_B)+(\mathbf{l}_A^{(2)}\cdot\mathbf{v}_A)\mathbf{v}_B\right]+O(c^{-5}) \qquad (3)$$

It follows from Eq. (2) & Eq. (3) that the clocks rate differences in one way time transfer is given by:

$$\frac{\delta T}{T}\equiv\frac{d\tau_B}{d\tau_A}-1=\left(\frac{\delta T}{T}\right)_c+\left(\frac{\delta T}{T}\right)_g \qquad (4)$$

Doppler effect

Contributions of Gravitational field

2. Gravitational Potential Difference

Clocks Rate Differences up to order 1/c^4

Special-Relativistic Doppler effect

$$\left(\frac{\delta T}{T}\right)_c = -\frac{1}{c} N_{AB}\cdot(\mathbf{v}_A-\mathbf{v}_B)$$

-1.731×10^-5

$$+\frac{1}{c^2}\left[\frac{1}{2}v_A^2-\frac{1}{2}v_B^2-\left[N_{AB}\cdot(\mathbf{v}_A-\mathbf{v}_B)\right](N_{AB}\cdot\mathbf{v}_B)\right]$$

-4.7656×10^-12

$$-\frac{1}{c^3}\left[[N_{AB}\cdot(\mathbf{v}_A-\mathbf{v}_B)]\left(\frac{1}{2}v_A^2-\frac{1}{2}v_B^2+(N_{AB}\cdot\mathbf{v}_B)^2\right)\right]$$

-1.312×10^-18

$$+\frac{1}{c^4}\left[\frac{3}{8}v_A^4-\frac{1}{4}v_A^2v_B^2-\frac{1}{8}v_B^4-\left[N_{AB}\cdot(\mathbf{v}_A-\mathbf{v}_B)\right](N_{AB}\cdot\mathbf{v}_B)\left(\frac{1}{2}v_A^2-\frac{1}{2}v_B^2+(N_{AB}\cdot\mathbf{v}_B)^2\right)\right]$$

$$+O(c^{-5})$$

-3.614×10^-25
Which will be ignored

2. Gravitational Potential Difference

Clocks Rate Differences up to order $1/c^4$

Contributions of gravitational field, mixed with kinetic terms:

-1.215×10^{-10}

2.53×10^{-14}

-2.13×10^{-18}

$$\left(\frac{\delta T}{T}\right)_g = \frac{1}{c^2}(U_A - U_B)$$
$$-\frac{1}{c^3}\left\{(U_A - U_B)\left[N_{AB}\cdot(\mathbf{v}_A - \mathbf{v}_B)\right] - \mathbf{I}_A^{(2)}\cdot\mathbf{v}_A + \mathbf{I}_B^{(2)}\cdot\mathbf{v}_B\right\}$$
$$+\frac{1}{c^4}\Big((\gamma+1)\left(U_A v_A^2 - U_B v_B^2\right) + \frac{1}{2}(U_A - U_B)$$
$$\times\left\{U_A - U_B + 2(1-\beta)(U_A + U_B) + v_A^2 - v_B^2 - 2\left[\mathbf{N}_{AB}\cdot(\mathbf{v}_A - \mathbf{v}_B)\right](\mathbf{N}_{AB}\cdot\mathbf{v}_B)\right\}$$
$$+\mathbf{N}_{AB}\cdot\left[\left(\mathbf{I}_B^{(2)}\cdot\mathbf{v}_B\right)(\mathbf{v}_A - 2\mathbf{v}_B) + \left(\mathbf{I}_A^{(2)}\cdot\mathbf{v}_A\right)\mathbf{v}_B\right]$$
$$+\left[\mathbf{I}_A^{(3)} - 2(\gamma+1)\mathbf{U}_A\right]\cdot\mathbf{v}_A - \left[\mathbf{I}_B^{(3)} - 2(\gamma+1)\mathbf{U}_B\right]\cdot\mathbf{v}_B\Big) + O(c^{-5}) \quad (13)$$

$$\frac{1}{c^4}\times\left[\mathbf{N}_{AB}\cdot\left[\left(\mathbf{I}_B^{(2)}\cdot\mathbf{v}_B\right)(\mathbf{v}_A - 2\mathbf{v}_B) + \left(\mathbf{I}_A^{(2)}\cdot\mathbf{v}_A\right)\mathbf{v}_B\right]\right] \approx 1.418\times10^{-20}$$
$$\frac{1}{c^4}\times\left(\left[\mathbf{I}_A^{(3)} - 2(\gamma+1)\mathbf{U}_A\right]\cdot\mathbf{v}_A - \left[\mathbf{I}_B^{(3)} - 2(\gamma+1)\mathbf{U}_B\right]\cdot\mathbf{v}_B\right) \approx 2.212\times10^{-22}$$

Which will be ignored

2. Gravitational Potential Difference

It follows from Eq. (1) to Eq. (5) that the gravitational potential difference is given by

$$\Delta U_{AB}^{One-way} = U_B - U_A = \frac{\frac{\delta T_{AB}}{T} - \left(\frac{\delta T}{T}\right)_c^{(3)} + \frac{1}{c^3}\left\{\mathbf{l}_A^{(2)} \cdot \mathbf{v}_A - \mathbf{l}_B^{(2)} \cdot \mathbf{v}_B\right\} - \left(\frac{\delta T}{T}\right)_g^{(4)}}{\left[\frac{1}{c^2} - \frac{1}{c^3}\mathbf{N}_{AB} \cdot \left(\mathbf{v}_A - \mathbf{v}_B\right)\right]} \quad (5)$$

Previous equations hold only in free space. In a real space with medium, we have other time delay functions :

- **Sagnac delay**
- **Atmospheric Refraction**
- **Solid Earth tide**

2. Gravitational Potential Difference

Laser Time Transfer Links

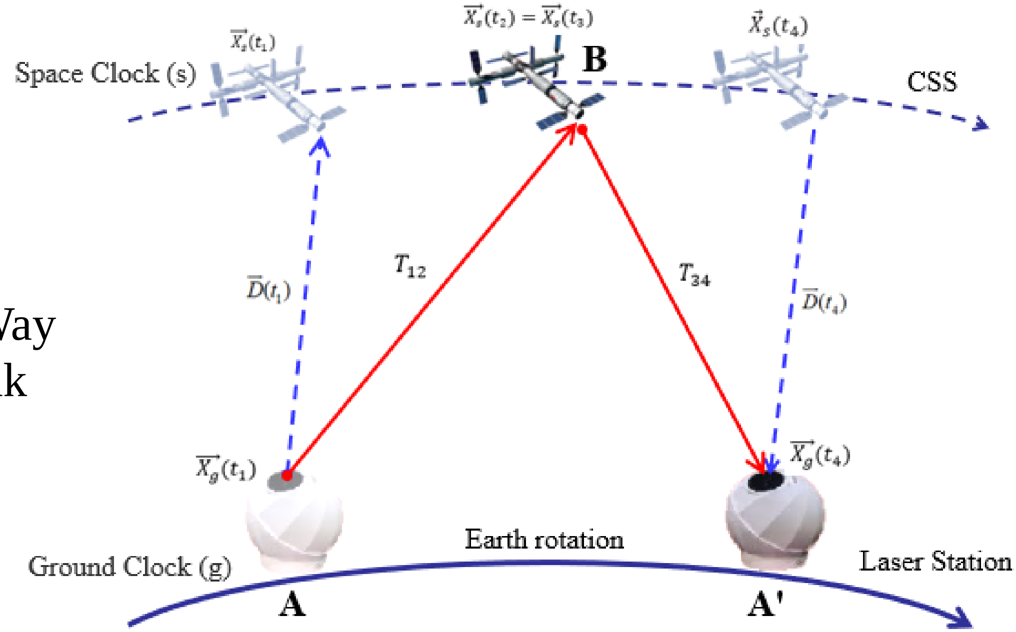

Fig. 3 Principle of Two-Way Laser Time Transfer Link

$$\Delta T_{AB} = \left(\tau^g + \tfrac{1}{2}\Delta\tau^g - \tau^s\right) + \tfrac{1}{2}\left(\Delta_1^g + \Delta_1^s - \Delta_2^g\right) + \tfrac{1}{2}\left(T_{12} - T_{34}\right) \quad (6)$$

- **Emission** time ground timescale
- **Received** time space timescale.
- **Round-Trip** ground timescale

- Instrumental delays (cables, electronic devices, etc.)

- Corrections (atmospheric, Shapiro delay, Sagnac delay and Relativity)

3. Discussion

- With quick development of time and frequency science and technology, the approach proposed in this study for time comparison is prospective.
- The most promising applications for future optical time/frequency links and optical clocks are fundamental physics and relativistic geodesy. For instance, testing gravitational redshift and determination of relativistic geoid.

- **Real Experiments are in Process Using Time Transfer by Laser Link (T2L2) Experiment On-board the Jason-2 Satellite.**

Reference

- Blanchet, L., Salomon, C., Teyssandier, P., & Wolf, P. (2001). Relativistic theory for time and frequency transfer to order1/c^3. Astronomy & Astrophysics, 370(1), 320-329.
- Linet, B., & Teyssandier, P. (2002). Time transfer and frequency shift to the order 1/c^4 in the field of an axisymmetric rotating body. Physical Review D, 66(2), 024045.

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