Modeling the Depth Dependence of ^{137}Cs Concentration in Lake Onuma

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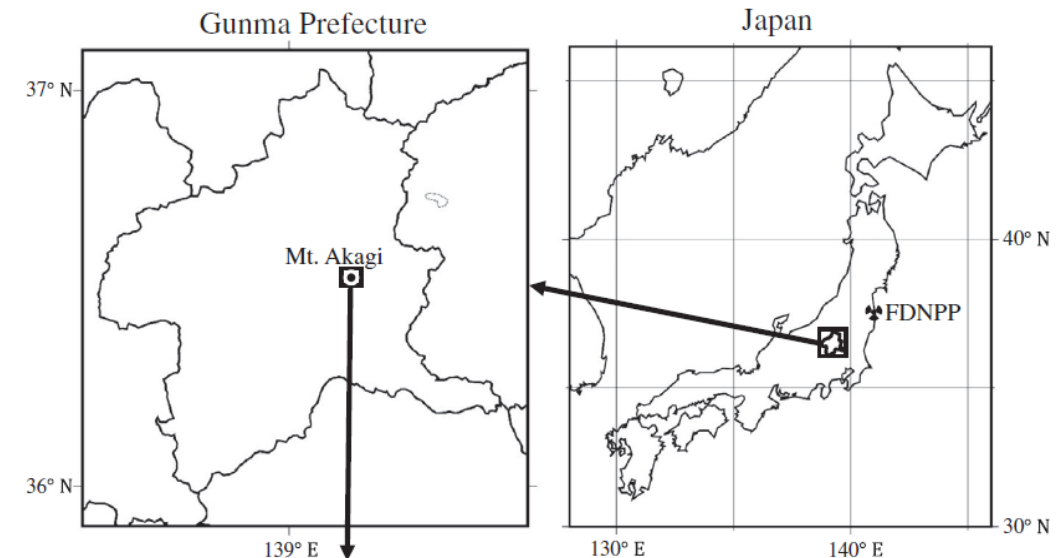
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Background

Lake Onuma

- Lake Onuma(Gunma Prefecture, Japan) 190km SW from FDNPP

- ^{137}Cs was detected from fish

- 1.5 years after the accident
→ activity concentration decrease slowed down

Location of Lake Onuma[1]

[1] Suzuki, K. *et al, Sci. Tot. Env*. (2018)

Objective

- Reproduce long-term temporal changes in the activity concentration

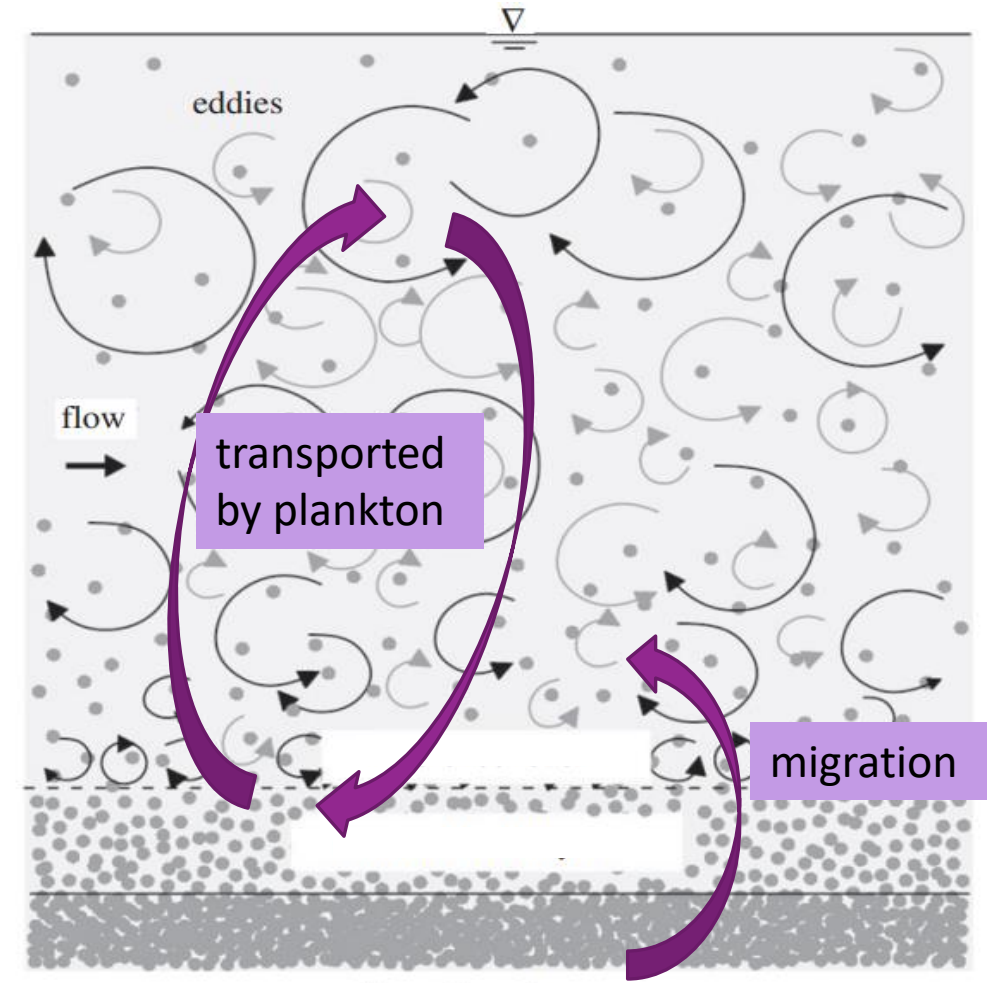- Reproduce vertical distribution of the activity concentration

Method

Time Fractional Diffusion Equation

$$\frac{\partial p}{\partial t} = {}_0\mathscr{D}_t{}^{1-\alpha}\left[K\frac{\partial^2 p}{\partial x^2}\right]$$

$${}_0\mathscr{D}_t{}^{\beta}f(t) := \frac{1}{\Gamma(1-\beta)}\frac{d}{dt}\int_0^t\frac{f(t')}{(t-t')^{\beta}}dt'$$
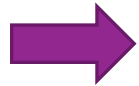
fractional derivatives(Riemann-Liouville type)

eddies

flow

transported by plankton

migration

[2]

[2] Sk Zeeshan, A., *et al.*, *Proc. R. Soc. A.* (2016)

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Solving Equation

$$\frac{\partial p}{\partial t} = {}_0\mathscr{D}_t^{1-\alpha}\left[K\frac{\partial^2 p}{\partial x^2}\right]$$

separation of variables

$p=X(x)\cdot T(t)$

➡️

$$\frac{d^2 X}{dx^2} = -\frac{\lambda}{K}X$$ (second-order ODE)

$$\frac{dT}{dt} = -\lambda \cdot {}_0\mathscr{D}_t^{1-\alpha}[T] \quad \cdots(*)$$

Laplace Transform of Eq.(*)

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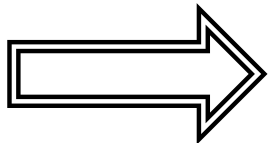$$\hat{T}(s) = T_0\frac{1}{s + \lambda s^{1-\alpha}}$$

$$= T_0\frac{1}{s}\frac{1}{1 + (\tau s)^{-\alpha}}$$ (λ=τ⁻ᵅ)

Inverse transform term-by-term

(R [α]>-1)

➡️

$$T(t) = T_0 \cdot E_\alpha\left(-(t/\tau)^\alpha\right)$$

⇨
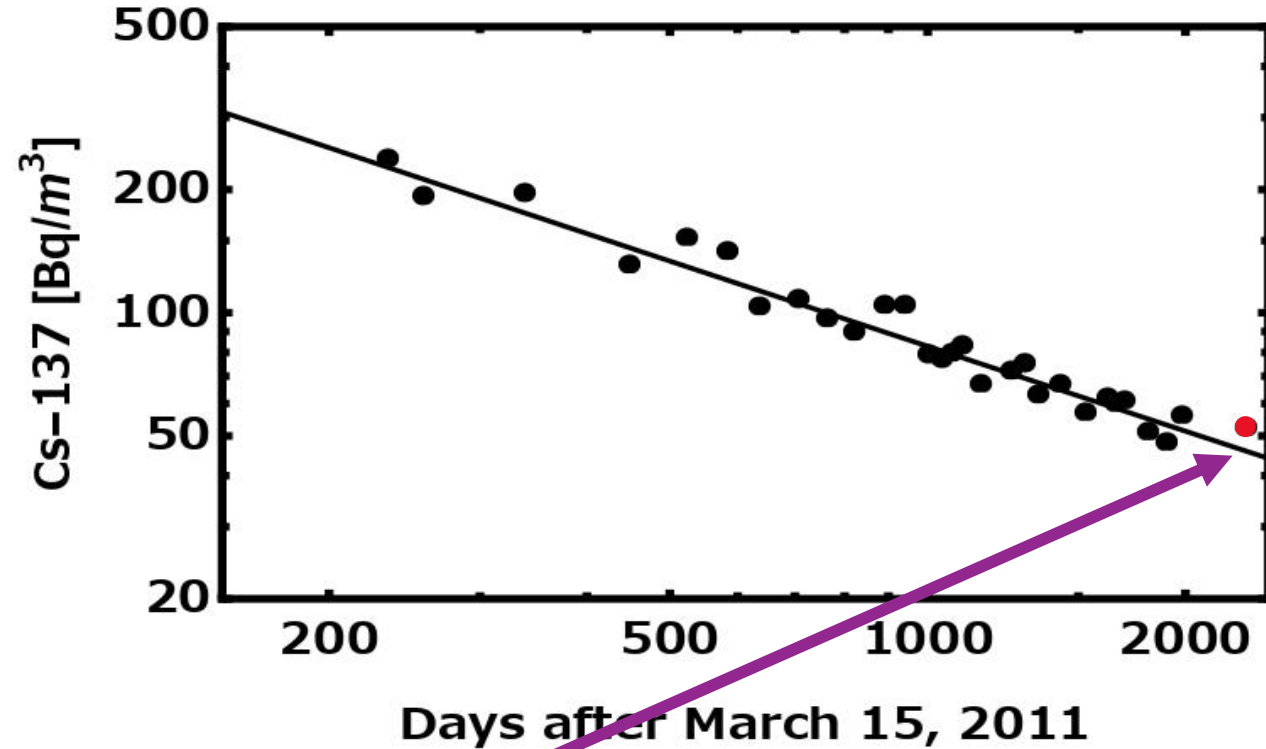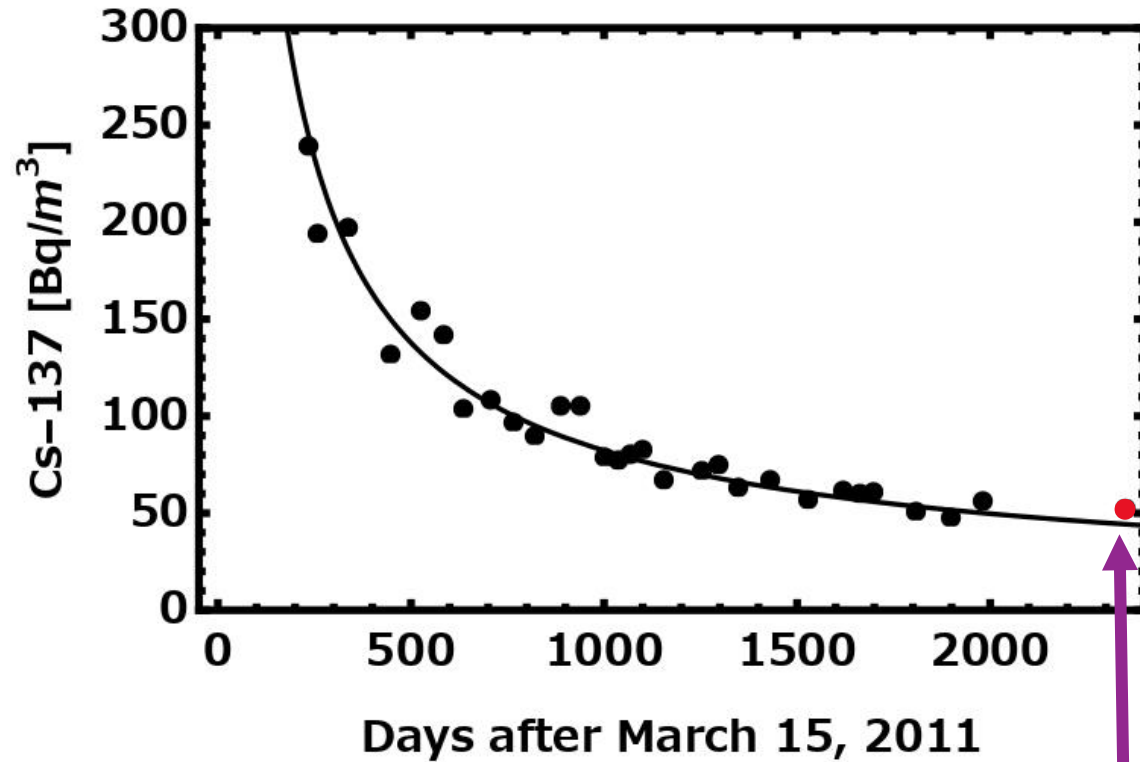
$$p(x,t) = \left\{C_1\cos\left(\sqrt{\frac{\lambda}{K}}x\right) + C_2\sin\left(\sqrt{\frac{\lambda}{K}}x\right)\right\}$$
$$\cdot T_0 \cdot E_\alpha\left(-(t/\tau)^\alpha\right)$$
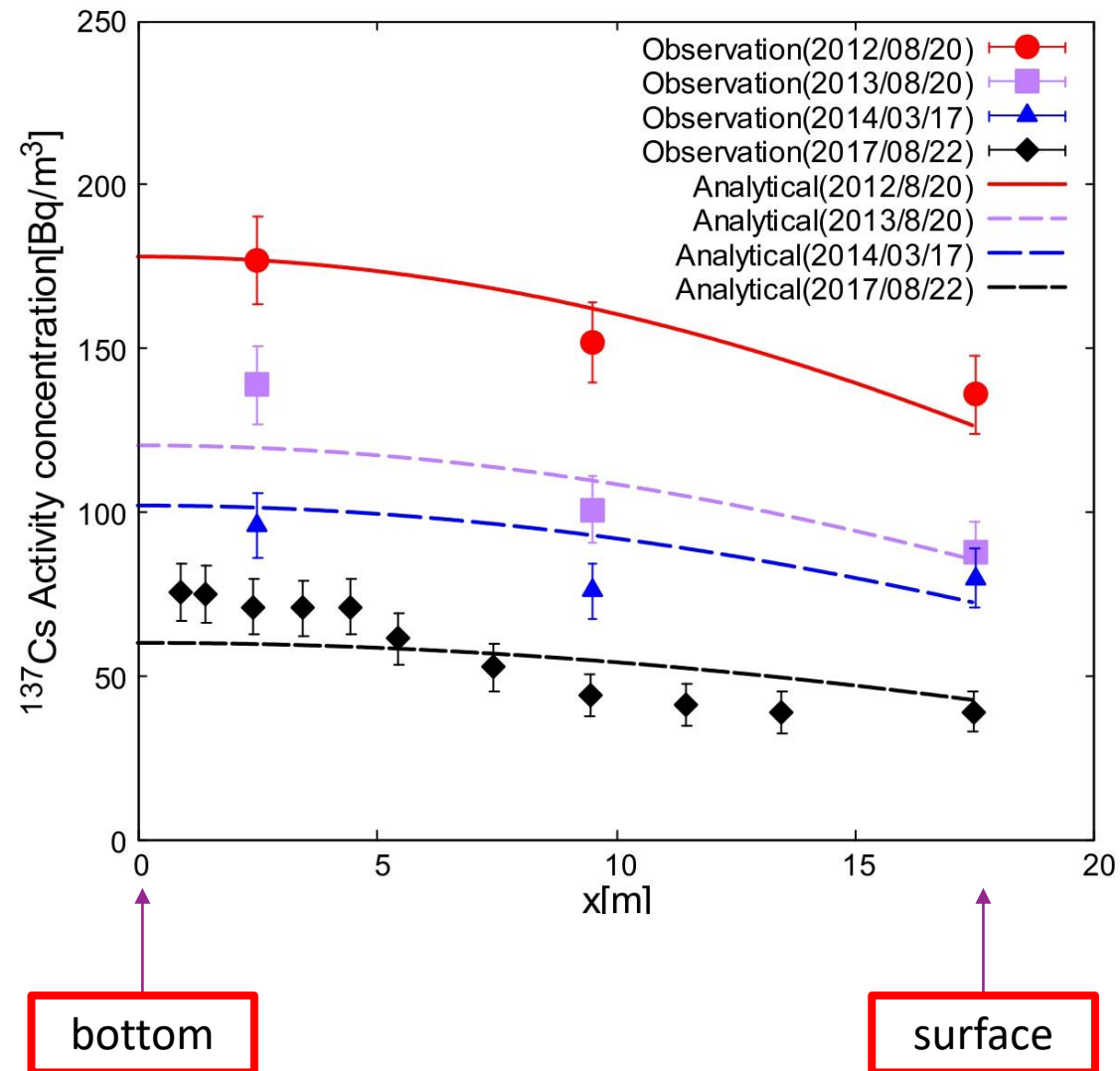
E: Mittag-Leffler function

$$E_\alpha(z) = \sum_{k=0}^{\infty}\frac{z^k}{\Gamma(\alpha k + 1)}$$

Comparison with time series

Model verification

Comparison with vertical distribution

Discussion

- Difference of distribution between **Particulate** and **Dissolved** ^{137}Cs

- Diffusion coefficient
  →varies with temperature

Should be considered in the model ?
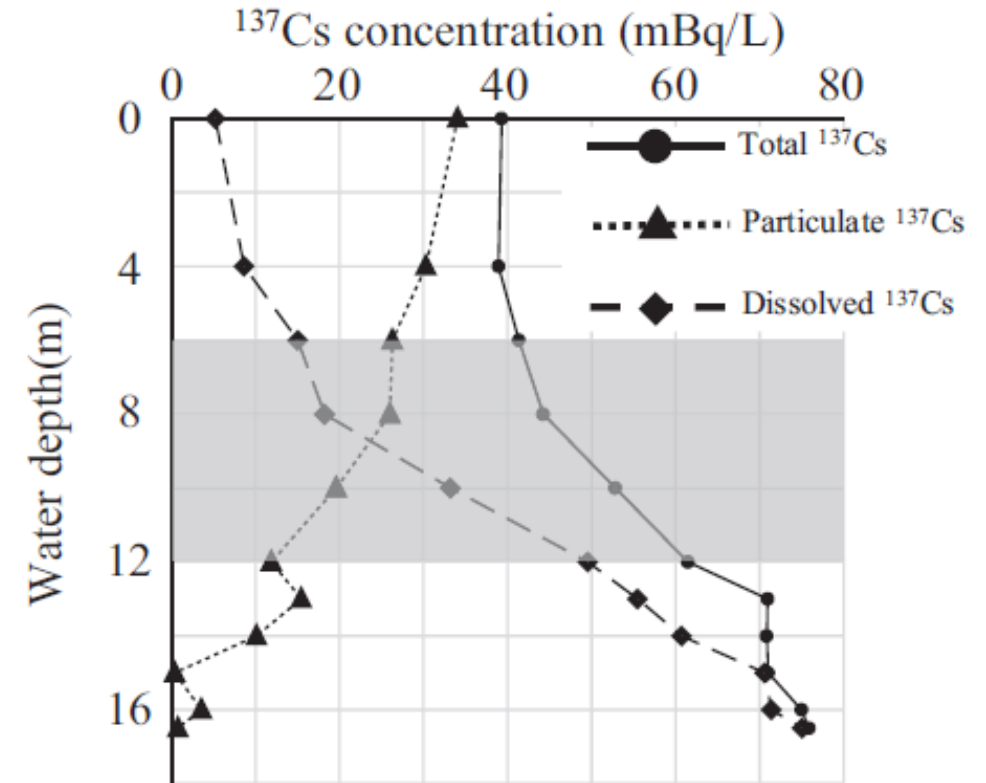
[3]Watanabe S., et al., *KEK Proc.*(2019)

Fig.2 Vertical profiles of ^{137}Cs concentration of the lake water in Lake Onuma on August 22, 2017. Gray zone indicates thermal stratification. [3]

Conclusion

- Reproduced both time series and vertical distribution with the Time Fractional Diffusion Equation.

- Depth profile suggested the migration of ^{137}Cs from bottom sediments.

References

[1] Suzuki, K. *et al*., Radiocesium dynamics in the aquatic ecosystem of Lake Onuma on Mt. Akagi following the Fukushima Dai-ichi Nuclear Power Plant accident, *Sci. Tot. Env.,* **622-623**, 1153-1164, (2018).

[2] Sk Zeeshan Ali, Subhasish Dey, Mechanics of advection of suspended particles in turbulent flow, *Proc. R. Soc. A,* **472**: 20160749 (2016).

[3] Watanabe, S., For elucidate of bottom stop phenomenon in ^{137}Cs concentration in wakasagi *hypomesus nipponensis* of Lake Onuma on Mt. Akagi, *KEK Proc*., **2019-2**(2019).

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