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Climate Advanced Forecasting of sub-seasonal Extremes

This project has received funding from the European Union's Horizon 2020 research and innovation programme under the Marie Skłodowska-Curie grant agreement No 813844

Extreme Value Analysis of Madden-Julian Oscillation

Mónica Minjares[1], Pascal Yiou[2], Isabel Serra[1], Marcelo Barreiro[3], Álvaro Corral[1]

1. Centre de Recerca Matemática, Complex Systems, Barcelona, Spain
2. Laboratoire des Sciences du Climat et de l'Environnement, UMR 8212 CEA-CNRS-UVSQ, Université Paris-Saclay and IPSL, Gif-sur-Yvette, France
3. Departamento de Ciencias atmosféricas, Facultad de Ciencias, Universidad de la Republica, Montevideo, Uruguay

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Motivation

Analyze MJO events with a peak over threshold approach

Perform different techniques for estimating thresholds.

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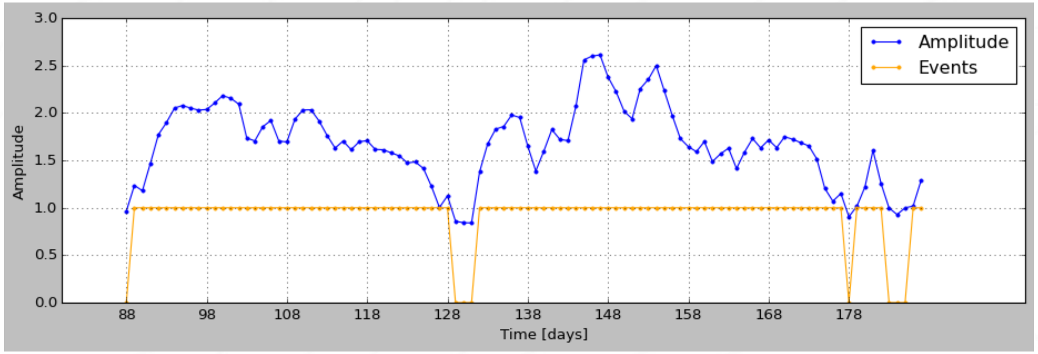
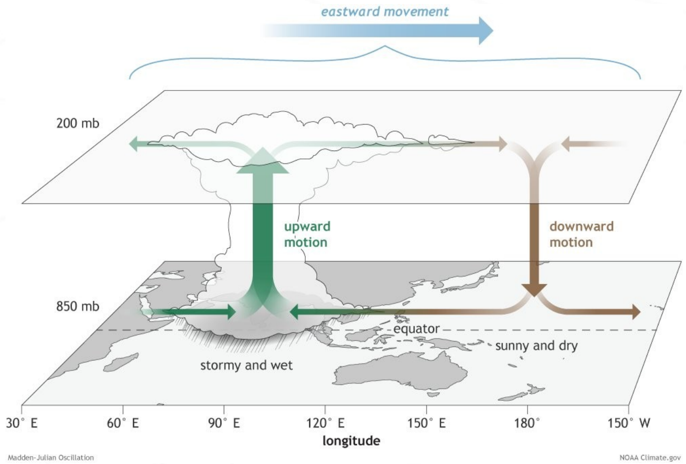
Madden-Julian Oscillation (MJO) event:

An event takes place when the Wheeler & Hendon index amplitude >1



Index amplitude in blue, the orange line mark the duration of an event



Size (energy):

$$S = \sum_{t=t_i}^{t_f} A(t)$$

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Generalized Pareto Distribution and Power-law tails

Assume there is a GPD above a threshold

$$G_{\xi,\beta} = \begin{cases} 1 - \left(1 + \frac{\xi x}{\beta}\right)^{-1/\xi} & \text{if } \xi \neq 0, \\ 1 - e^{-x/\beta} & \text{if } \xi = 1. \end{cases}$$

Or assume a power-law tail

$$f(x) \propto \frac{1}{x^{\alpha+1}}$$

With x the size, f(x) the probability density and $\alpha = (1/\xi)$

Key issue: determine the threshold in x

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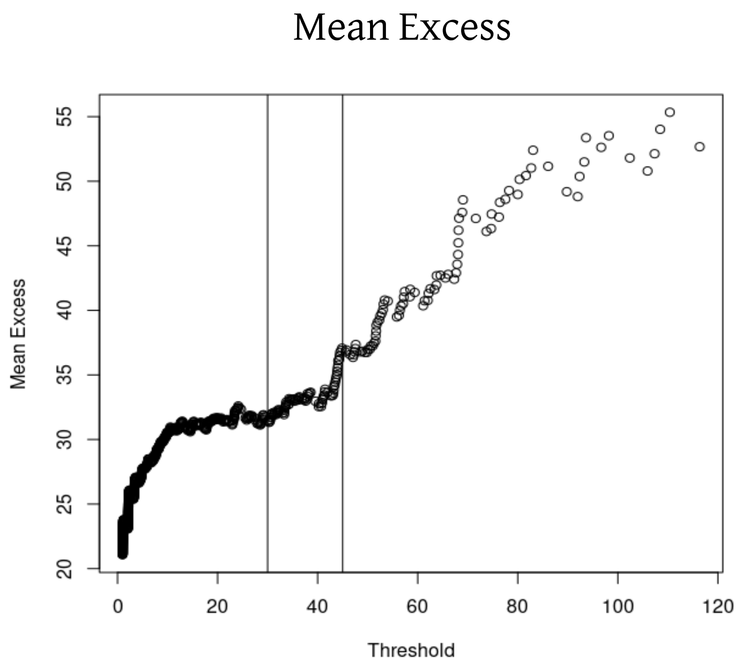
Methods for estimating the threshold



The mean excess function represents the conditional mean of the exceedance size over threshold (given that and exceedance ocurred.)

Upward trend- heavy tailed behavior (a straight line with positive gradient above some threshold is a sign of Pareto behaviour in tail)

Downward trend light-tailed behavior

Zero gradient – exponential tail

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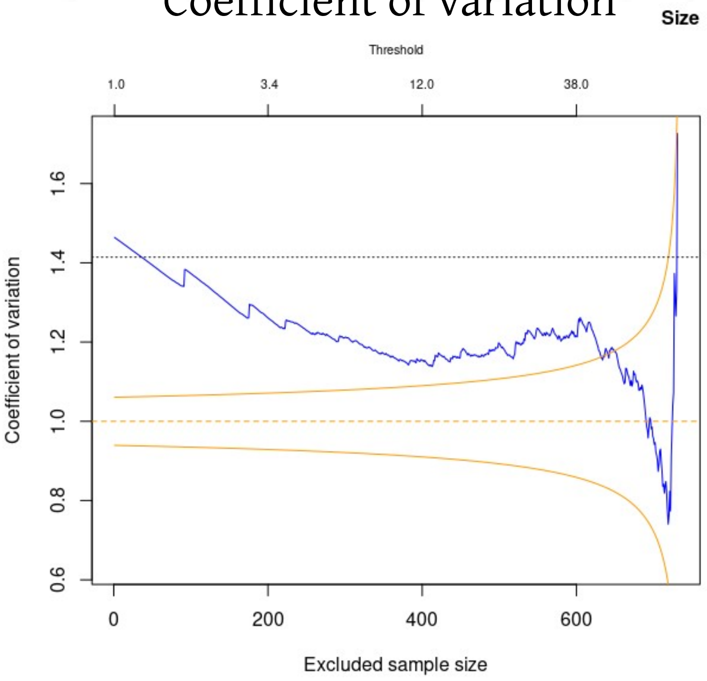
Coefficient of variation



Given a sample $\{x_k\}$ of size n of positive numbers, we denote the ordered sample $\{x_{(k)}\}$, so that $x_{(1)} \leq x_{(2)} \leq \cdots \leq x_{(n)}$. The CV-plot is the function cv(t) of the sample coefficient of variation of the threshold excesses $(x_j - t)$ for the exceedances $\{x_j : x_j > t\}$ given by

$$t \to cv(t) = \frac{sd\{x_j - t \mid x_j > t\}}{\text{mean}\{x_j - t \mid x_j > t\}}, \qquad t = x_{(k)}$$

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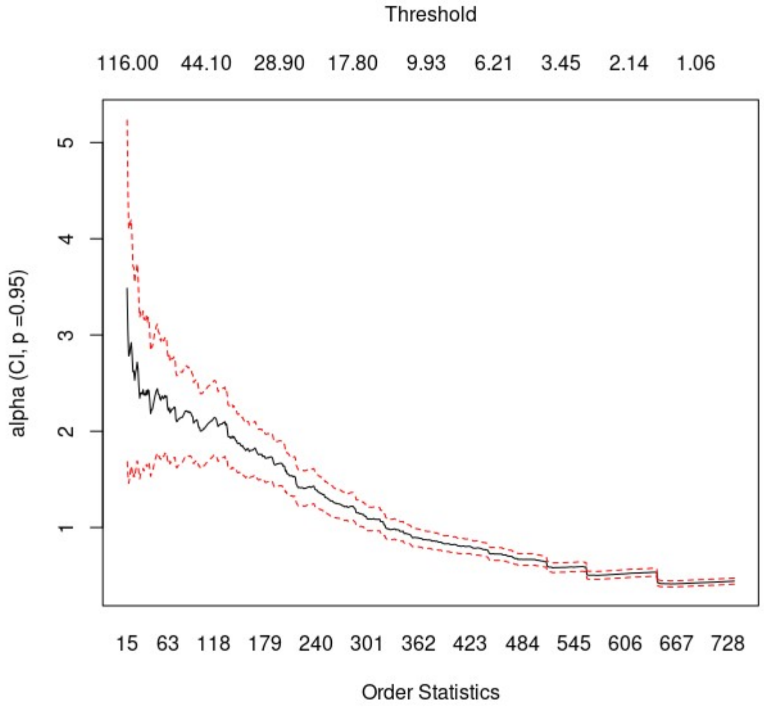
Hill plot



The Hill estimator is derived as the max. likelihood estimate of the power coefficient in the Power-law distribution.

where $\alpha=1/\xi$ is the tail index

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Estimate threshold

Tm test – multiple threshold test for a GPD, **u** is the lower value for which the GPD is not rejected (Castillo & Serra, 2015).

Clauset et al's and A. Deluca's method (power-law fit)
Fit by maximum likelihood and test goodness of fit by Kolmogorov-Smirnov, using Montecarlo Simulations.

the excesses are given by [Size>u]-u
We consider x extreme if x > t
t = 39.8 → 134 extremes
From a total of 736 events

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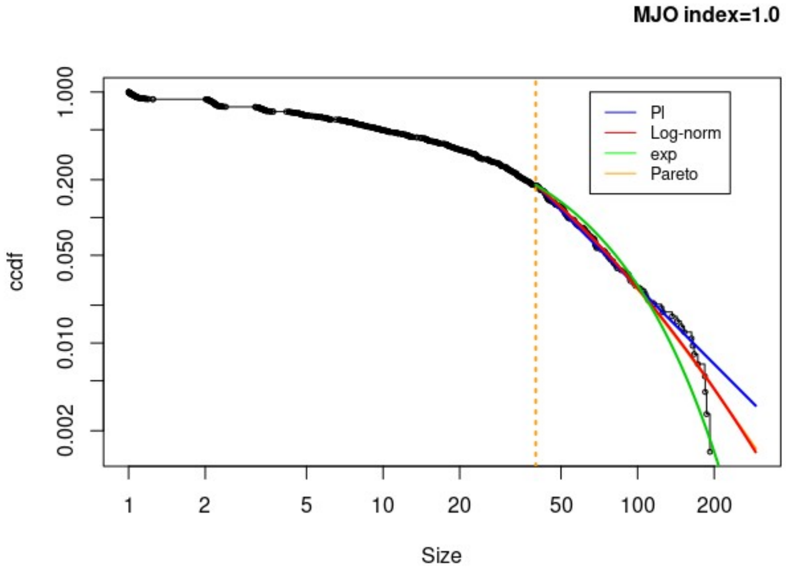
Models compared:
Log-normal, Weibull, Gamma, exponential, GPD, Power-law

We fit each of the models and compared the likelihoods on the extremes, we also computed the AIC.

$$\ell(\alpha) = \frac{1}{N} \ln L(\alpha) = \frac{1}{N} \sum_{i=1}^{N} \ln f(x_i)$$

$$\mathrm{AIC} = 2\mathrm{k} - 2\ln(\mathrm{L})$$



	log-normal	Weibull	Gamma	Exp	GPD	Power-law
log-lik	-595.2748	-576.4721	-577.5835	-581.2817	-574.0162	-574.8893
AIC	1194.5496	1156.9442	1159.167	1164.5634	1152.0324	1151.7786

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Conclusions

Estimate an optimal threshold is not an easy task with just one method but a combination of of them can get a better threshold.

Need to work on method to find the optimal threshold

The distribution of the size of MJO events is heavy-tailed with a better fit for a GPD and PL.

Work in progress

Explore different thresholds for the MJO index (0.90, 0.95, 1.0, 1.1, 1.2, 1.3, 1.4)
Threshold for the MJO phases

mminjares@crm.cat

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