

A python package for interior modelling of rocky exoplanets using Gibbs free energy minimization

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Mass & Radius

→ mean density

→ 0th order interpretation of planet composition

In-depth characterization of interiors requires detailed mineralogy & structure

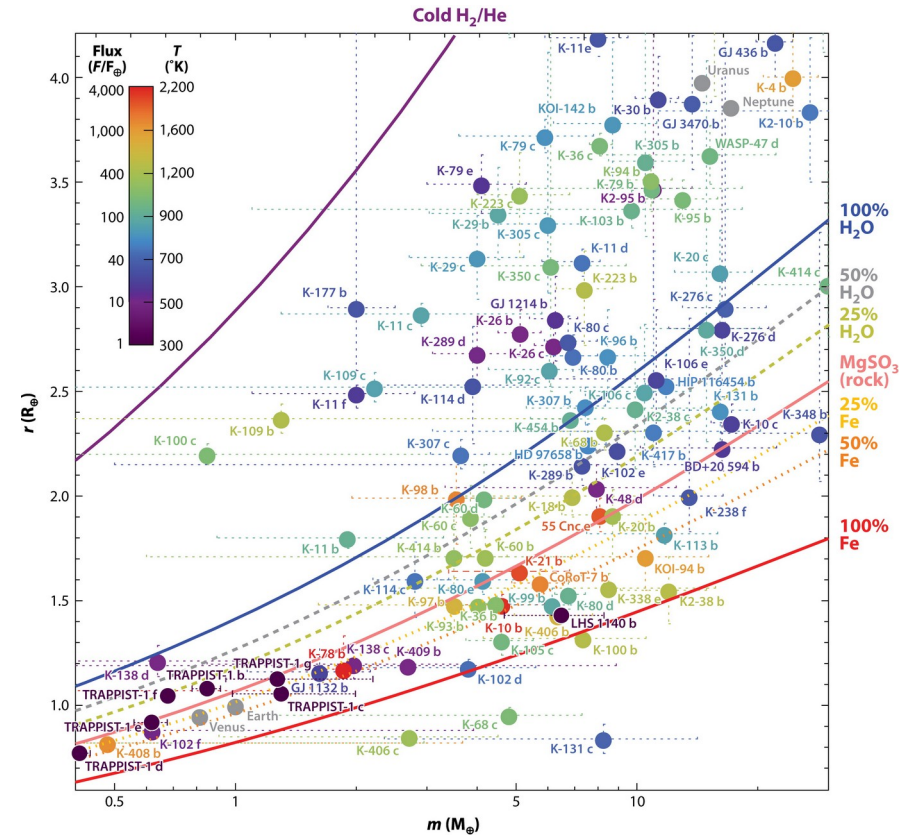


Figure from Kaltenegger 2017





Sharing is encouraged

Diversity of estimated exoplanet chemistry

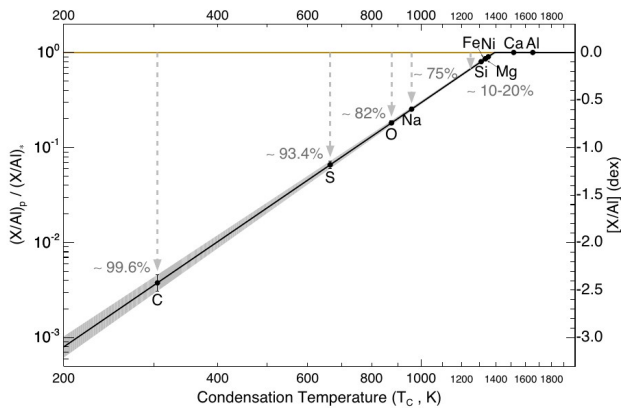
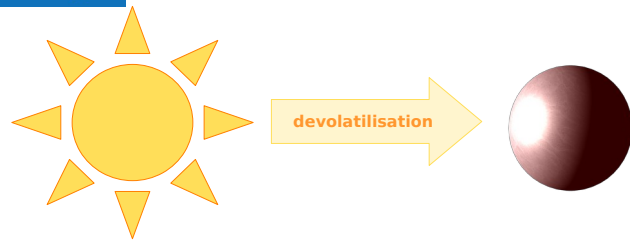
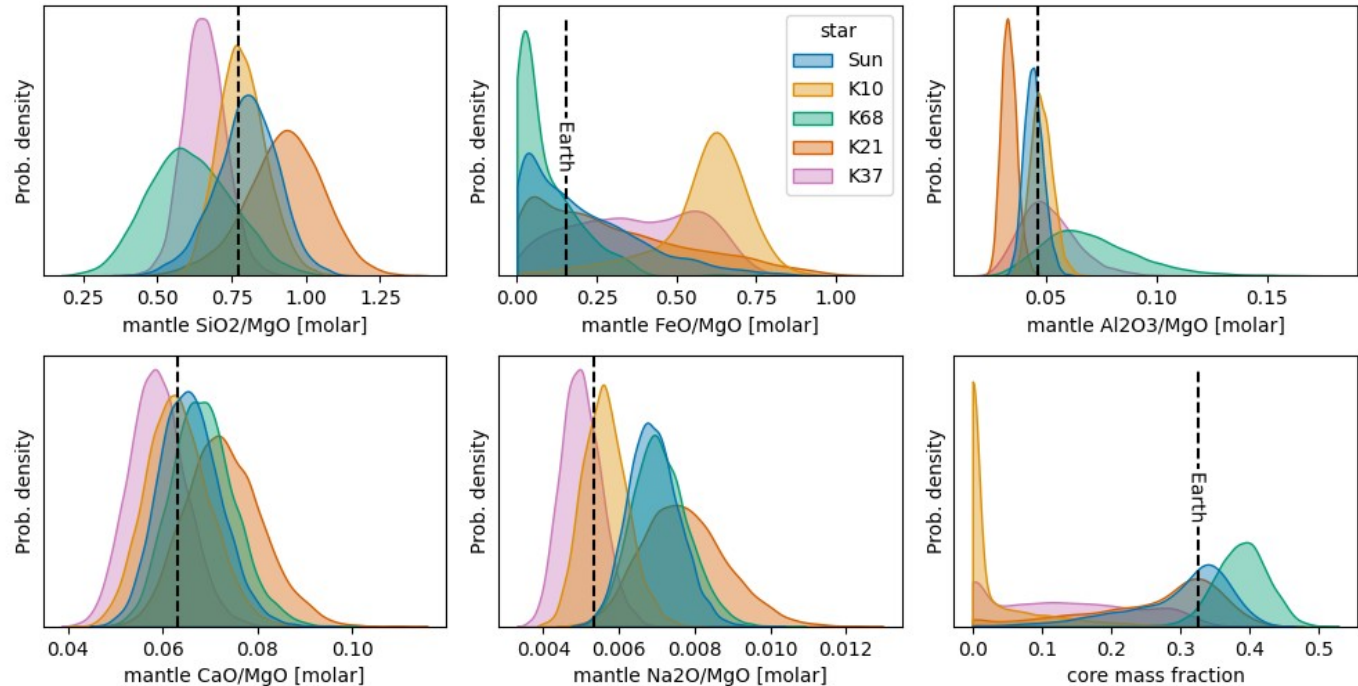


Figure from Wang et al. 2019

Estimated terrestrial planet interiors from stellar abundances



Terrestrial exoplanet composition estimated from abundances of planet-hosting stars using the method of Wang et al. 2019 (ExoInt, <https://github.com/astro-seanwhy/ExoInt>)



Existing interior models for terrestrial (exo-)planets

- **ExoPlex (python wrapper to Perple_X**

(Connolly 2009) [**Fortran77**])

Lorenzo 2018, Hinkel & Unterborn 2018

<https://github.com/CaymanUnterborn/ExoPlex>

- **Magrathea (C++, tabulated MgSiO_3 EOS)**

Huang et al. 2022

<https://github.com/Huang-CL/Magrathea>

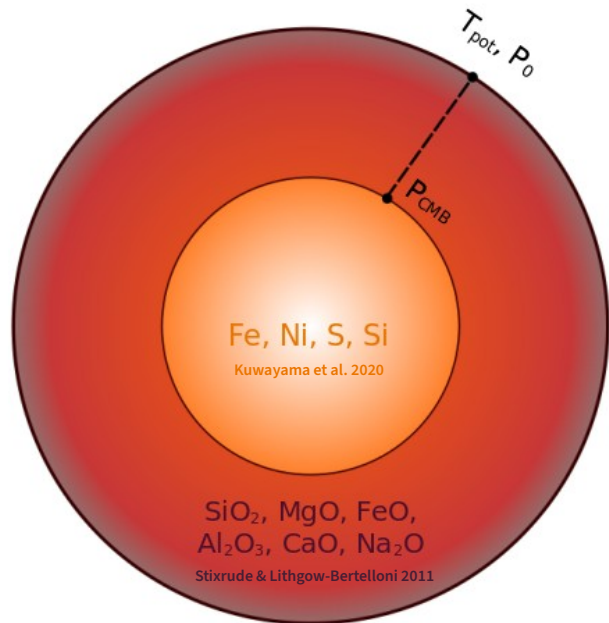
→ Want to have a reasonably fast software package for computing accurate mineralogies, implemented in a popular and easy-to-use language

ExoPlex (precomputed mineralogy):	~1.5s
ExoPlex (with Mineralogy computation):	>2min
Magrathea:	~0.69s



Interior structure model

Forward model: computes radius (or mass) as function of mass (or radius), mantle & core chemistry.



1. Initial guess on interior structure with MgSiO₃-perovskite mantle EOS (Seager et al. 2007)

→ obtain P_{CMB}

2. Integrate adiabatic gradient $P_{\text{surface}} \rightarrow P_{\text{CMB}}$

$$\frac{dT}{dP} = \frac{\alpha T}{C_p \rho}$$

compute mineralogy along the way by Gibbs free energy minimization, using linearized solid-solution series (White et al. 1958, Connolly 2009). Implementation in Cython.



→ obtain temperature, density and mineralogy profile

3. Refine interior structure with these updated profiles based on mineralogy



Testcase Earth

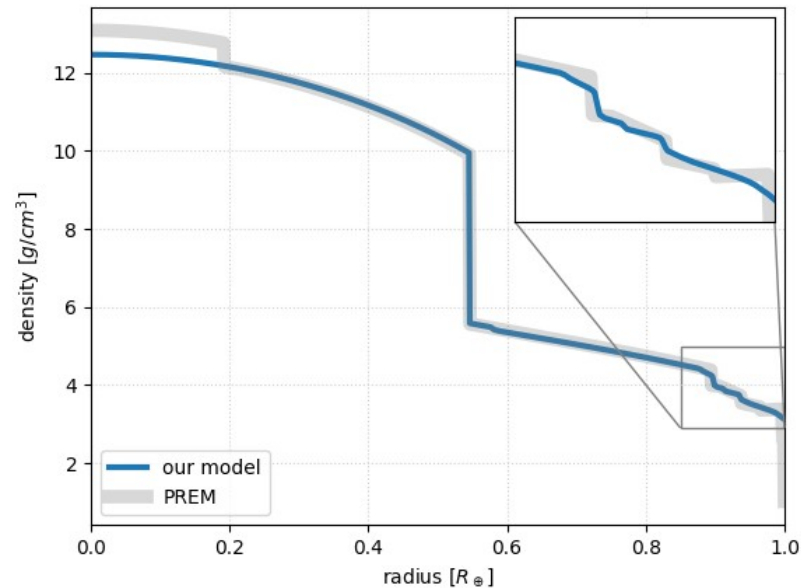
Earth

$R=1.0$

R_{earth} (computed)

$M=1.0$

M_{earth} (fixed)



Comparison of computed interior structure with PREM (Dziewonski & Anderson 1981).

Mantle chemistry (wt%)

McDonough & Sun 1995

$\text{SiO}_2 = 45.0$

$\text{MgO} = 37.8$

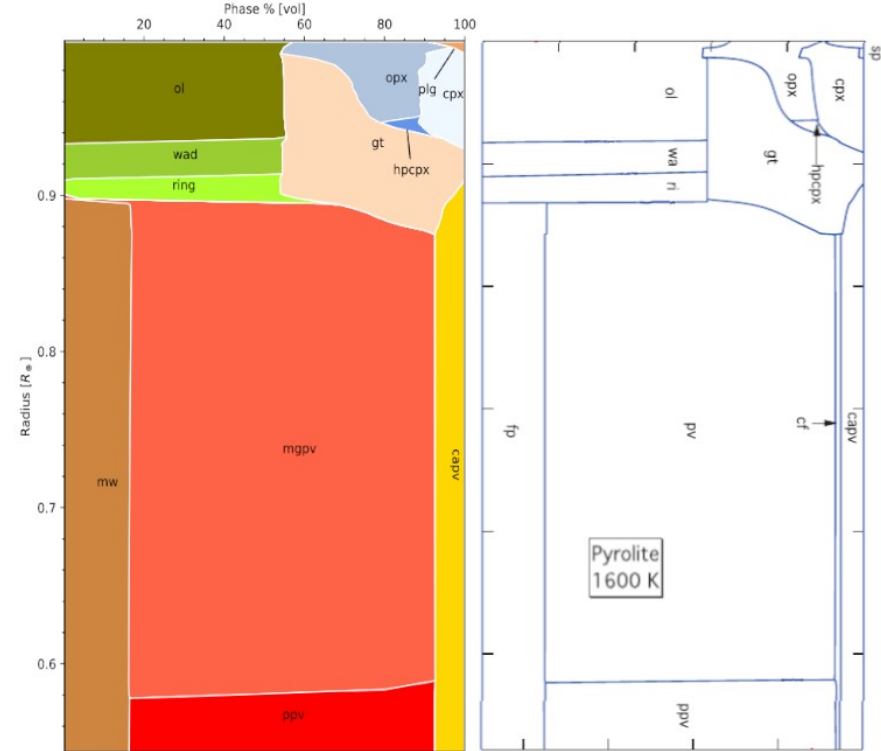
$\text{FeO} = 8.05$

$\text{CaO} = 3.55$

$\text{Al}_2\text{O}_3 = 4.45$

Average runtime
for entire
calculation ~10-13
sec., but depends
on:

- Mineral complexity
(less, i.e. FMS-system → faster)
- Size of planet



Comparison of mineralogies inferred with our model (left) and literature (right). Modified from Stixrude & Lithgow-Bertelloni 2011





Sharing is encouraged

Exoplanet mineralogies - examples

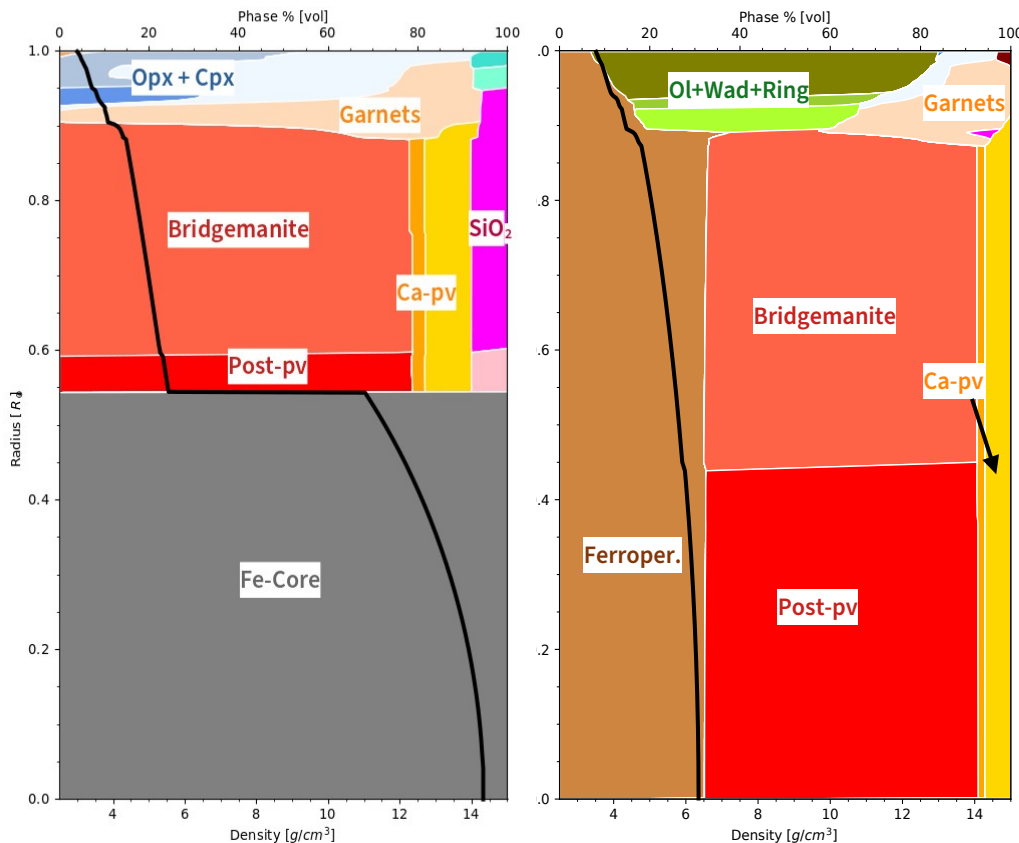
Extremely Si-rich mantle

$R=1$ $R_{\text{earth}} \text{ (fixed)}$
 $M=1.03$ $M_{\text{earth}} \text{ (computed)}$

Mantle chemistry (wt%):

$\text{SiO}_2 = 59.7$
 $\text{MgO} = 29.5$
 $\text{CaO} = 5.2$
 $\text{Al}_2\text{O}_3 = 5.2^*$
 $\text{Na}_2\text{O} = 0.5$

*assumed, necessary to absorb Na_2O in mineralogy



Inferred mineralogy based on chemical composition of hypothetical exo-Earths from Wang et al. 2019

Fe-rich mantle

$R=1$ $R_{\text{earth}} \text{ (fixed)}$
 $M=0.89$ $M_{\text{earth}} \text{ (computed)}$

Mantle chemistry (wt%):

$\text{SiO}_2 = 30.3$
 $\text{MgO} = 29.2$
 $\text{FeO} = 31.3$
 $\text{CaO} = 2.5$
 $\text{Al}_2\text{O}_3 = 3.28$
 $\text{Na}_2\text{O} = 0.23$



Mineralogical influence on planetary characteristics

- Oxygen fugacity → controls outgassed atmosphere (e.g. Wang et al. 2022, Noack et al. 2020, Sossi et al. 2020, Arculus 1985)
- Partial melting → outgassing (e.g. Noack et al. 2020) and crust formation (Brugman et al. 2021)
- (lower) mantle viscosity; sensitive to ferropericlasite (e.g. Thielmann et al. 2020) and Ca-pv (Immoor et al. 2022)
- Water storing capability: ol<px<ring&wad (e.g. Othani 2020)



- A software package for interior modelling with fast, detailed mineralogy evaluation
- Open source, written in Python 3 / Cython
- Will be released soon



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