Calibrating tropospheric errors on ground-based GNSS reflectometry: calculation of bending and delay effects

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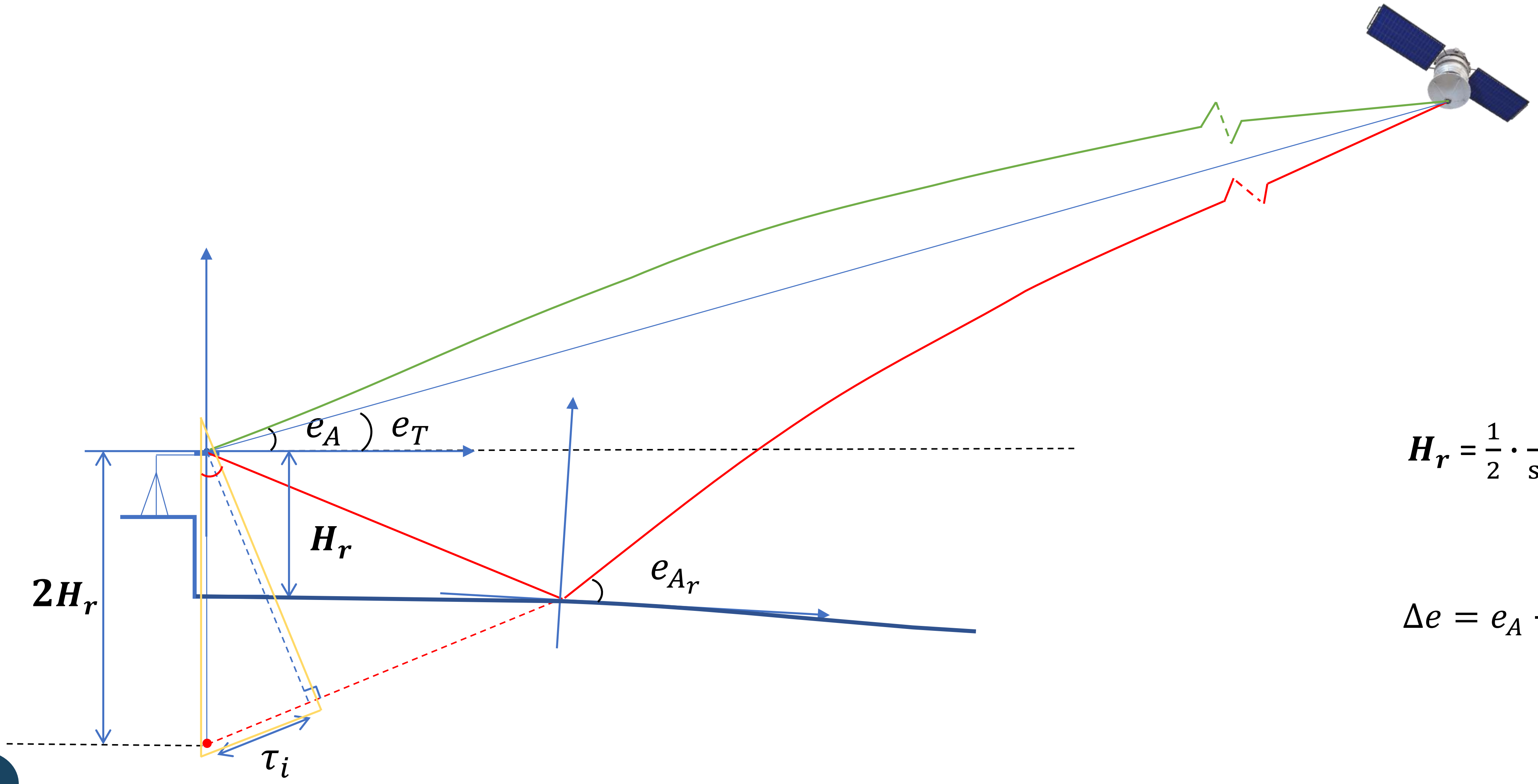
The Bending Angle Correction



$$H_r = \frac{1}{2} \cdot \frac{\tau_i}{\sin e_A}$$

$$\Delta e = e_A - e_T$$

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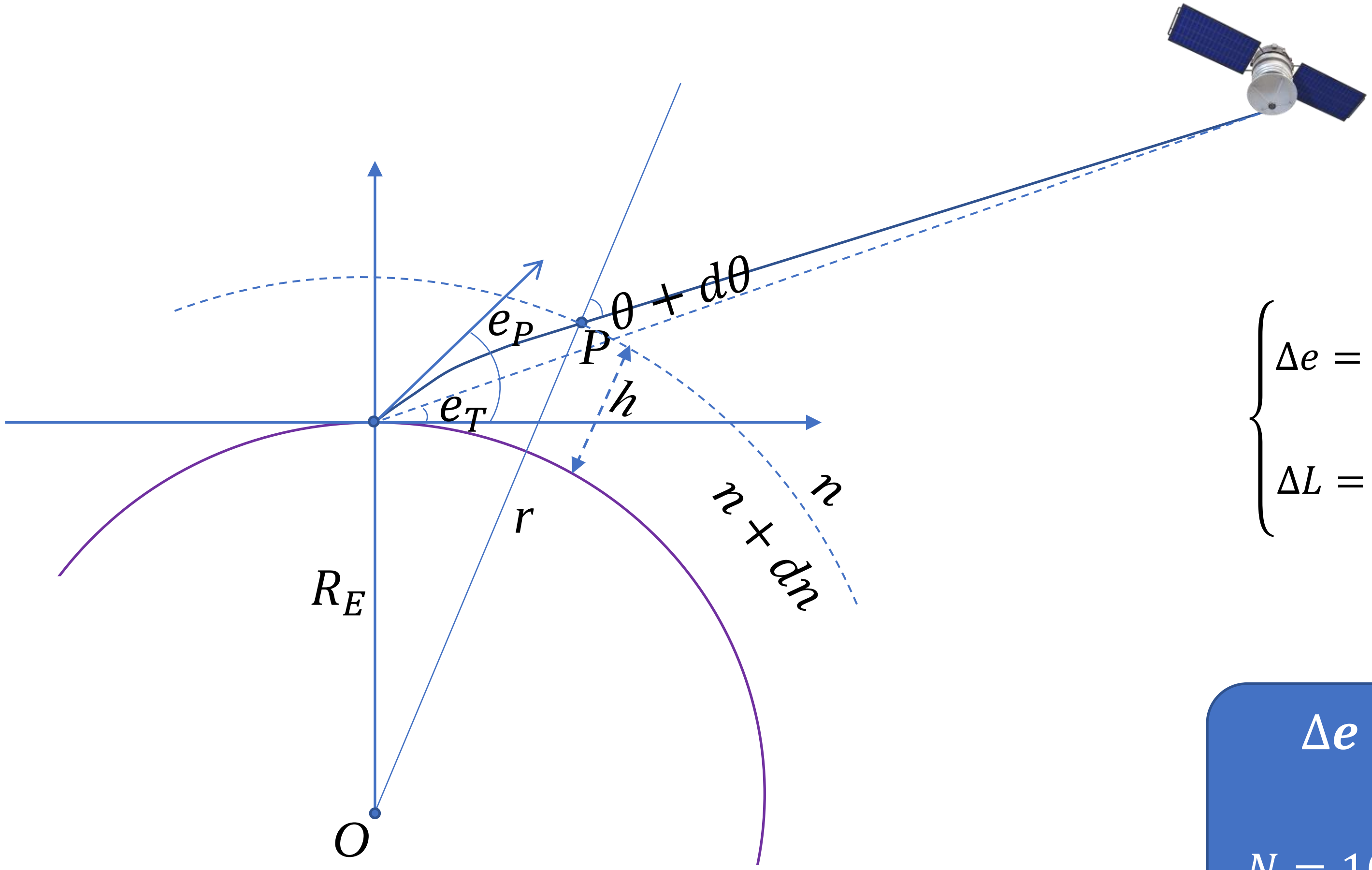
The Bending Angle Correction



$$(n-1) = (n_0 - 1) \cdot e^{\frac{h_0 - h}{H_{scl}}}$$

$$\begin{cases} \Delta e = \int_1^{n_0} \frac{tan\theta}{n}\, dn = 10^{-6} \int_{h_0}^{sat} \frac{N}{H_{scl}} \frac{tan\theta}{n}\, dh \\ \Delta L = \int_0^{sat} (n-1)\, dL = 10^{-6} \int_0^{sat} \frac{N}{cos\theta}\, dh \end{cases}$$

$$\boldsymbol{\Delta e = 10^{-6} N_0 \cdot cos\, e_T\, mpf(e_T)}$$

$$N = 10^6 (n-1) = K_1 \frac{P_d}{T} + K_2 \frac{P_w}{T} + K_3 \frac{P_w}{T^2}$$

(Saastamoinen, 1973), (H. J. Yan, 1998; H. Yan & Ping, 1995)

The Bending Angle Correction

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$$\Delta e = 10^{-6} N_0 \cdot \cos e_T \, mpf(e_T)$$

$$mpf(e_T) = \frac{\int_0^\infty (n-1)\, dL \big|_{(e_T)}}{\int_0^\infty (n-1)\, dL \big|_{(90°)}}$$

Definition: "delay only" total mapping function

$$\Delta L(e_T) = \int_0^{100km} (n-1)\, dL + [S - G]$$

$$\Delta L(e_T) = ZWD \cdot vmf_w(e_T) + ZHD \cdot vmf_h(e_T)$$

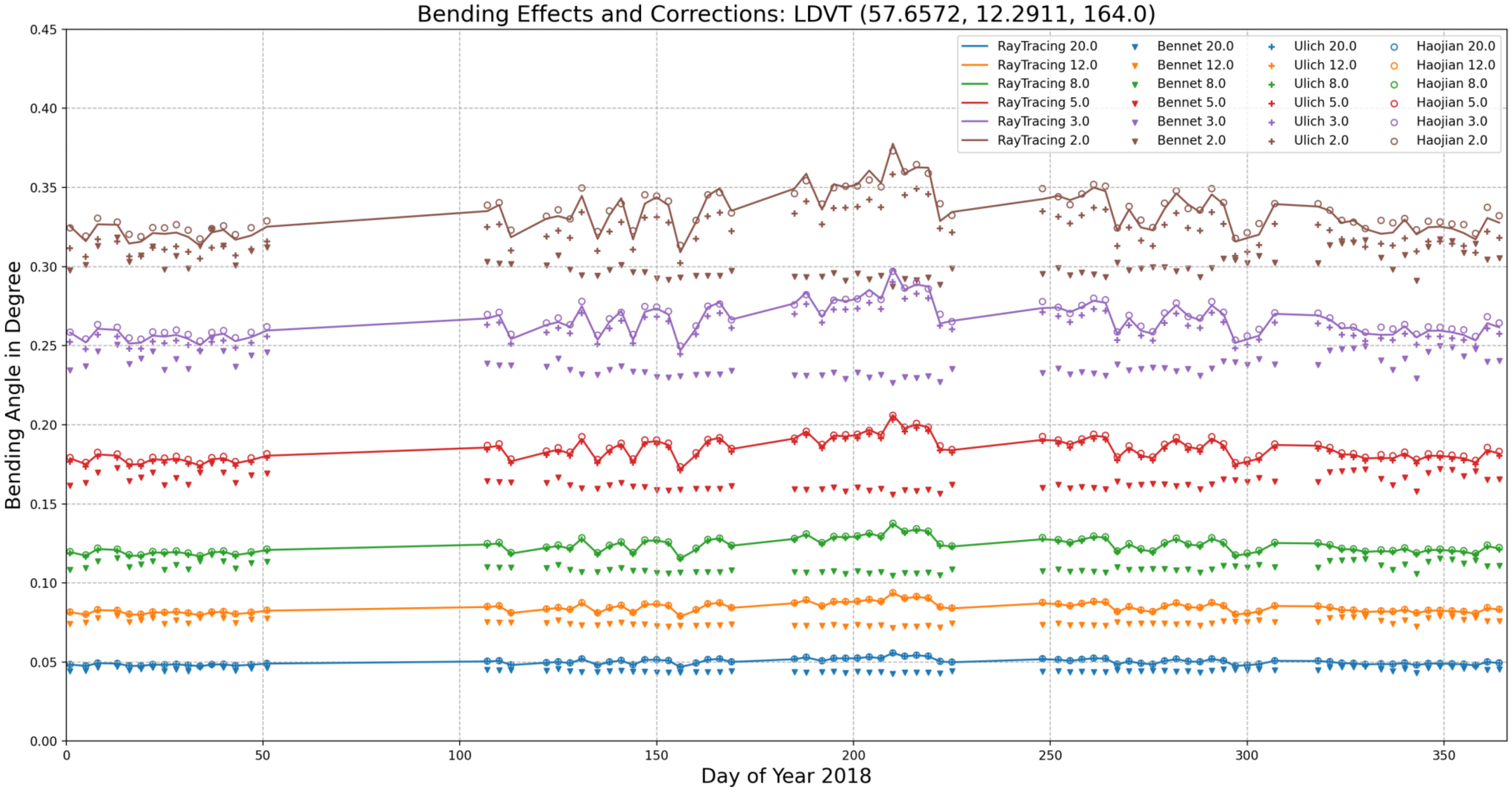
Practical calculation from existing mapping function

$$mpf(e_T) = \frac{ZHD}{ZTD} \cdot \left(vmf_h - \frac{10^{-6}}{4} (\cos e_T)^2 mpf_h{}^3 N_0\right) + \frac{ZWD}{ZTD} \cdot vmf_w$$

$$\begin{cases} \Delta L_G = \int_0^{100km} \left(\frac{1}{\cos\Delta e} - 1 \right) \cdot \frac{1}{\cos\theta} \, dL \\ \Delta L = \int_0^\infty (n-1)\, dL = \int_0^\infty \frac{(n-1)}{\cos\theta} dh \end{cases}$$

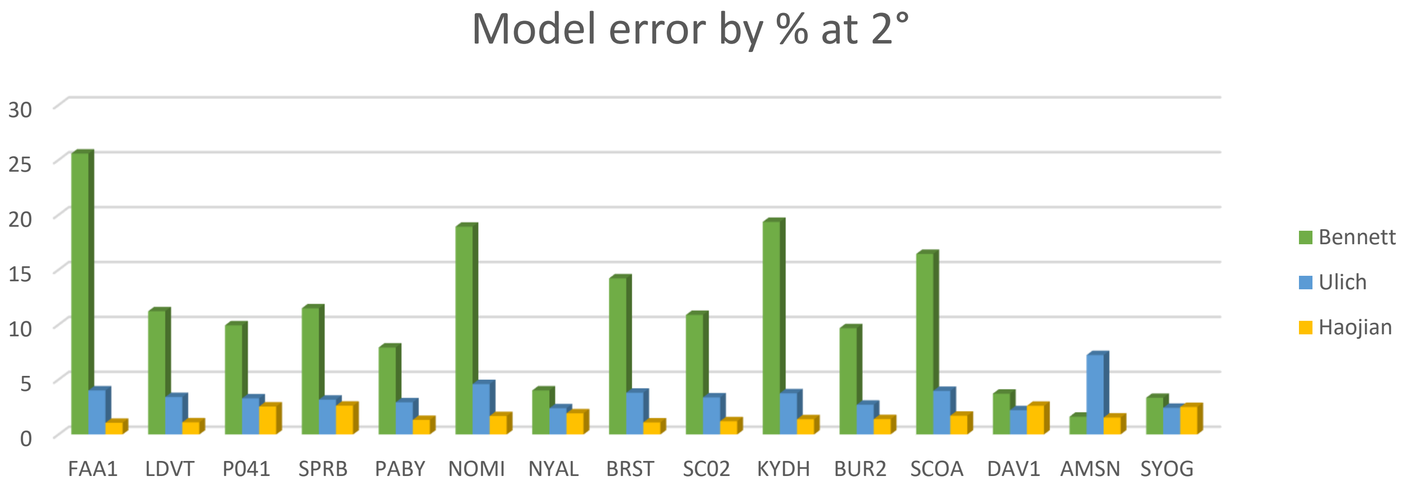
(Landskron & Böhm, 2018; Möller & Landskron, 2019)

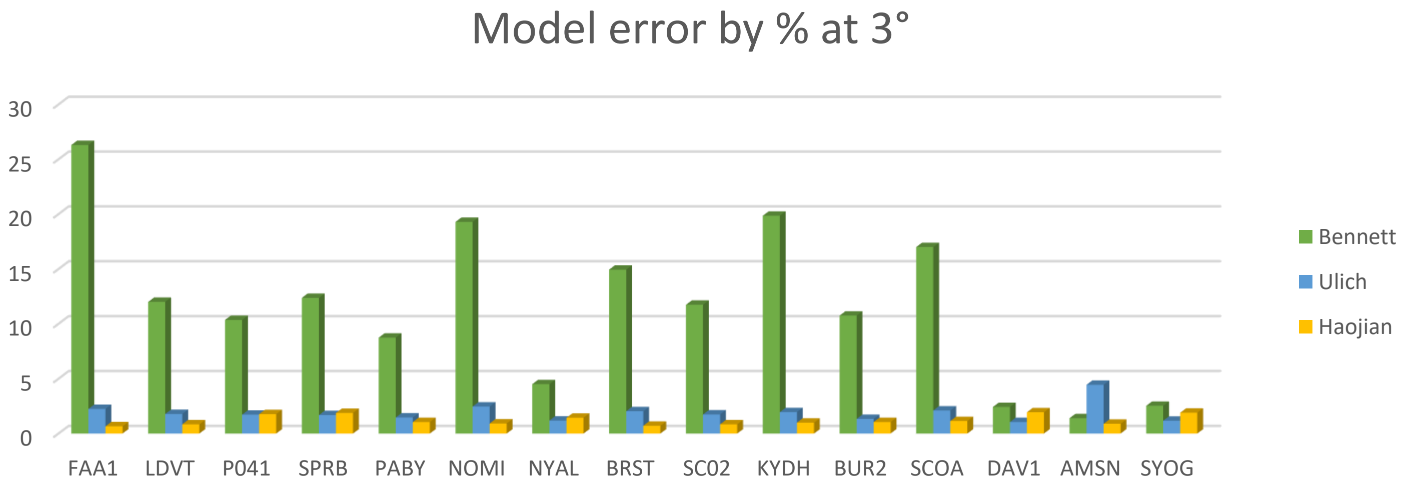
The Bending Angle Correction: Ray-tracing



(Bennett, 1982; Santamaría-Gómez et al., 2015), (Ulich, 1981)

The Bending Angle Correction: Ray-tracing

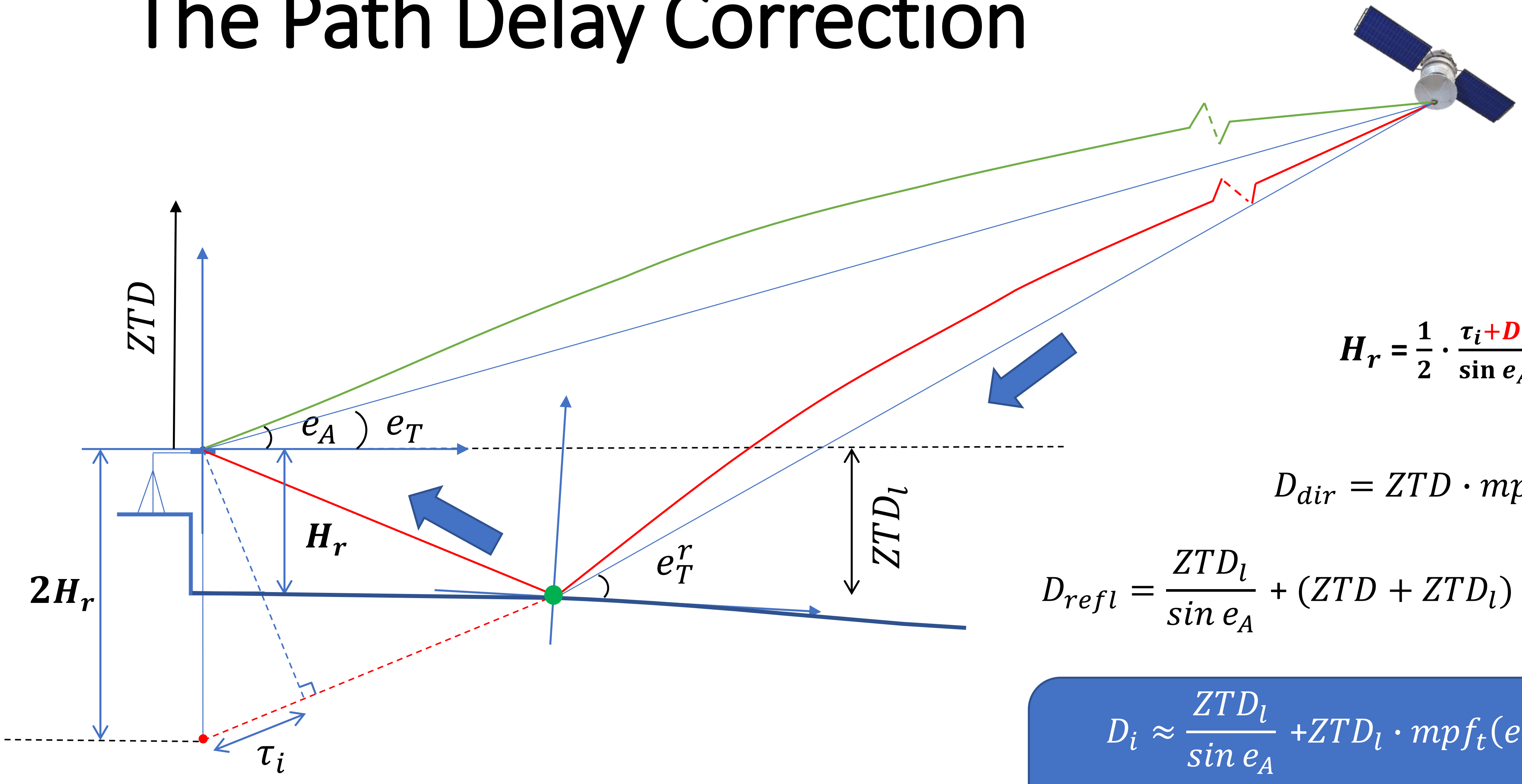




Mean RMS for 15 stations, 2018

Ray-tracing	Bennett (%)	Ulich (%)	Haojian (%)	
2° (0.327°)	11.2	3.6	1.7	0.0055°
3° (0.260°)	11.6	1.9	1.2	0.0031°
5° (0.181°)	11.9	0.93	0.75	0.0013°

The Path Delay Correction



$$H_r = \frac{1}{2} \cdot \frac{\tau_i + D_i}{\sin e_A}$$

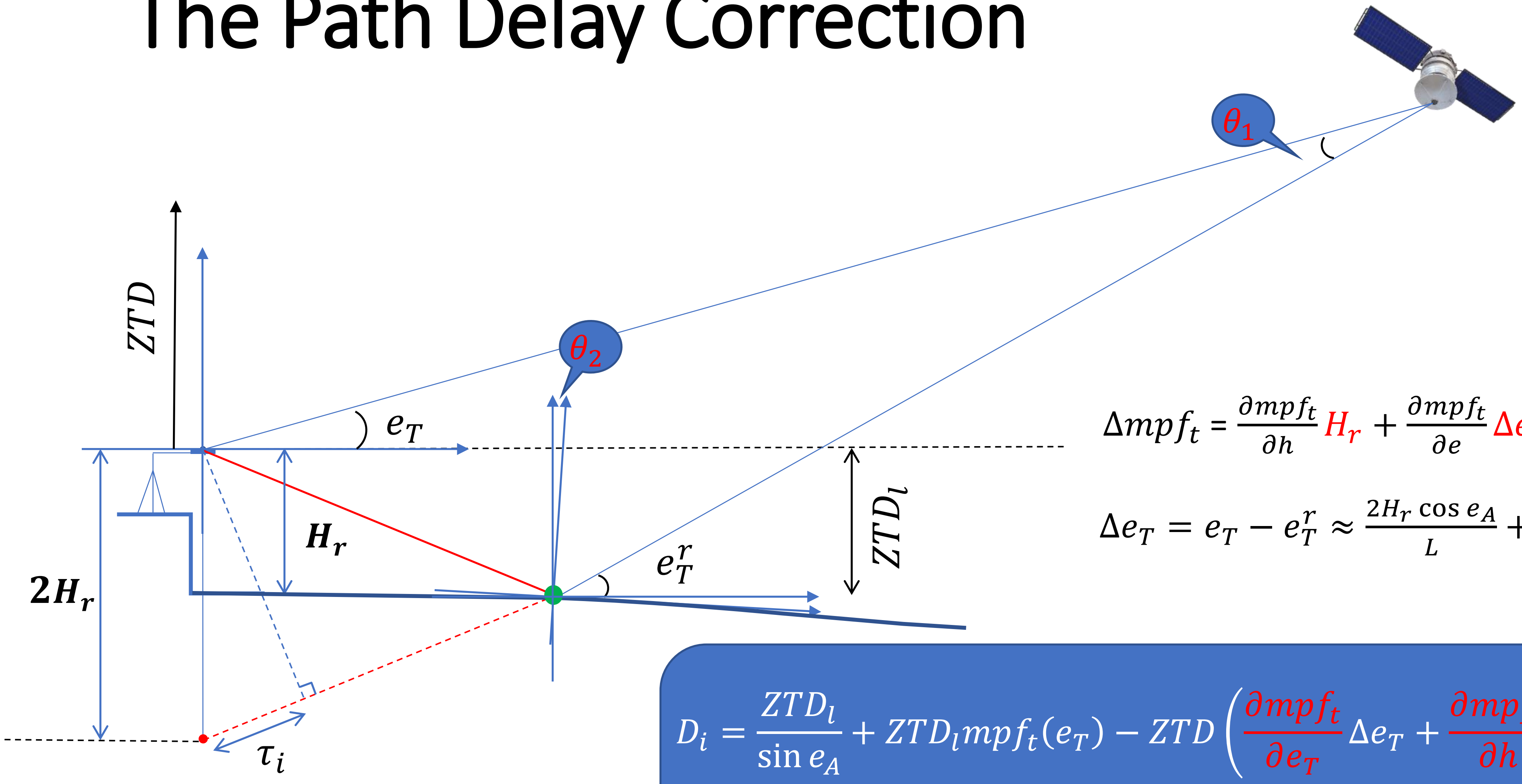
$$D_{dir} = ZTD \cdot mpf_t(e_T)$$

$$D_{refl} = \frac{ZTD_l}{\sin e_A} + (ZTD + ZTD_l) \cdot (mpf_t(e_T) - \Delta mpf_t)$$

$$D_i \approx \frac{ZTD_l}{\sin e_A} + ZTD_l \cdot mpf_t(e_T) - ZTD \cdot \Delta mpf_t$$

$$ZTD_l = 10^{-6} \cdot N_l \cdot H_r$$

The Path Delay Correction



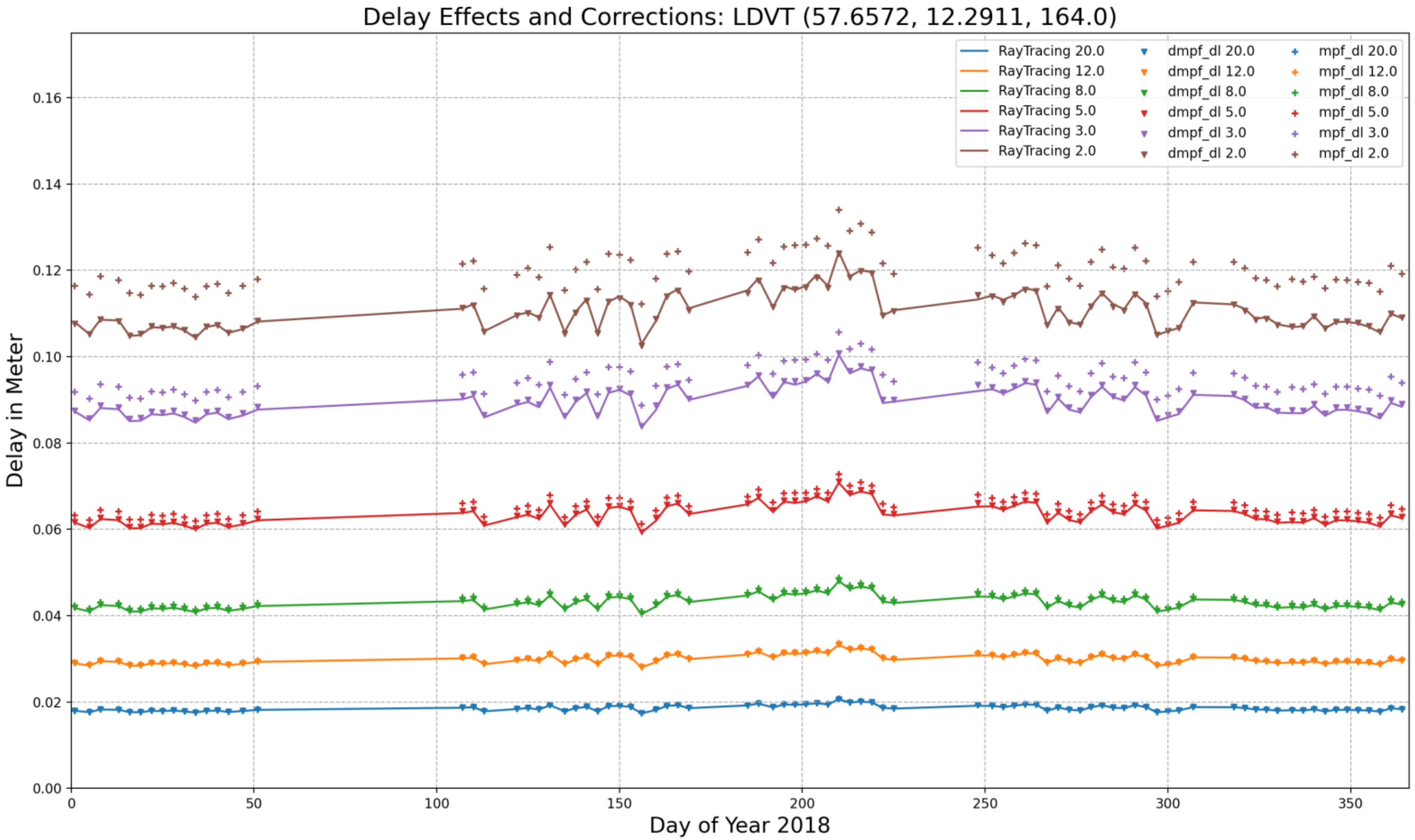
$$\Delta mpf_t = \frac{\partial mpf_t}{\partial h} H_r + \frac{\partial mpf_t}{\partial e} \Delta e_T$$

$$\Delta e_T = e_T - e_T^r \approx \frac{2H_r \cos e_A}{L} + \frac{H_r}{R \tan e_A} \approx \frac{H_r(1+\sin e_A)}{R \tan e_A}$$

$$D_i = \frac{ZTD_l}{\sin e_A} + ZTD_l mpf_t(e_T) - ZTD\left(\frac{\partial mpf_t}{\partial e_T}\Delta e_T + \frac{\partial mpf_t}{\partial h} H_r\right)$$

$$= H_r\left[\frac{10^{-6}N_l}{\sin e_A} + 10^{-6}N_l mpf_t(e_T) - ZTD\left(\frac{\partial mpf_t}{\partial e_T}\frac{(1+\sin e_A)}{R \tan e_A} + \frac{\partial mpf_t}{\partial h}\right)\right]$$

Path Delay Corrections: Ray-Tracing



Mean RMS for 15 stations, 2018		
Ray-tracing (mm)	mpf (mm)	dmpf (mm)
2° (108)	9.7	0.57
3° (87.8)	5.3	0.68
5° (62.1)	2.1	0.43
5° (42.2)	0.75	0.22

The mapping function slant factor [*Williams, 2017*]:

$$D_i = 2 \cdot 10^{-6} H_r N_l mpf_t$$

Conclusion

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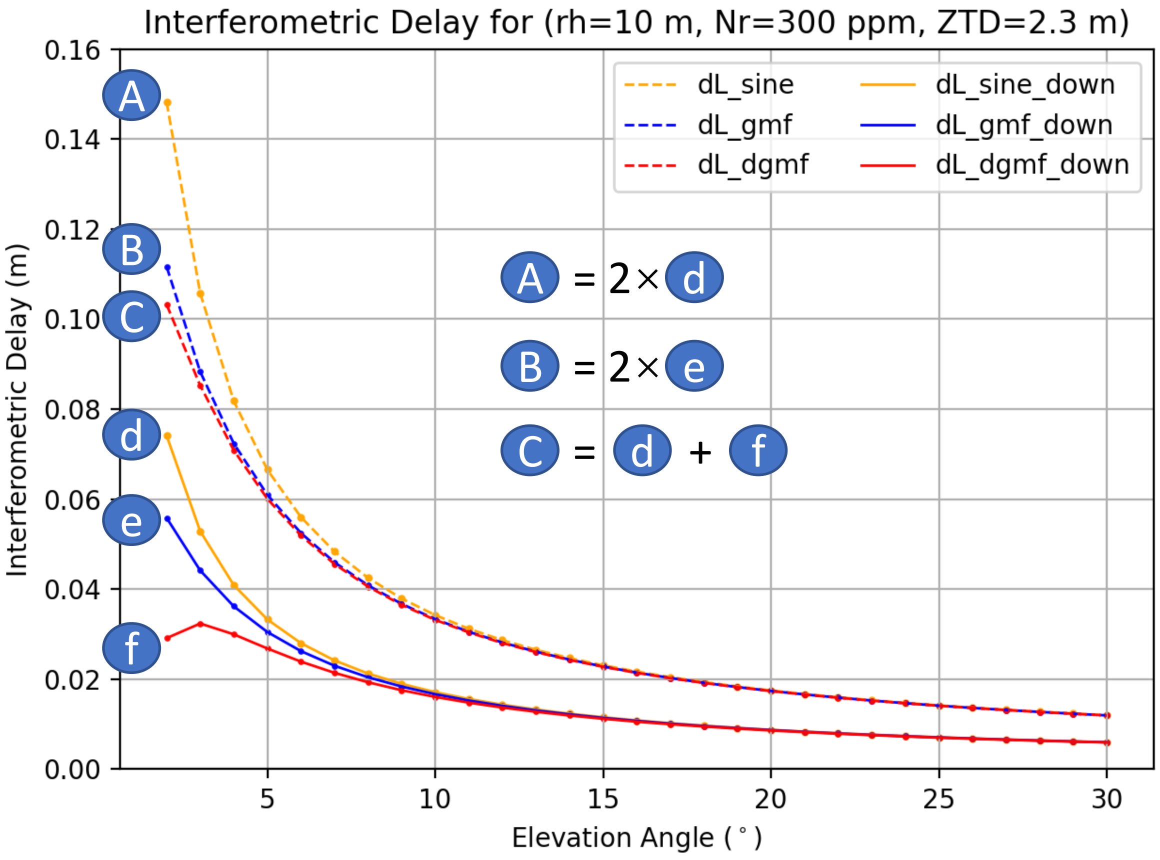
- New GNSS-R bending angle correction with model error around 1.7% (0.0055 °) at 2° (using ground refractivity and mapping function)

- New GNSS-R path delay correction with model error < 1% (<1mm) for a 10-meter reflector (using the mapping function and it’s derivative)

- More complicated but require “almost” no additional (ZTD) input

Backups

The Path Delay Correction: Characteristics



Ⓐ The sine slant factor:

$$D_i = 2 \cdot 10^{-6} H_r N_l \frac{1}{\sin e_A}$$

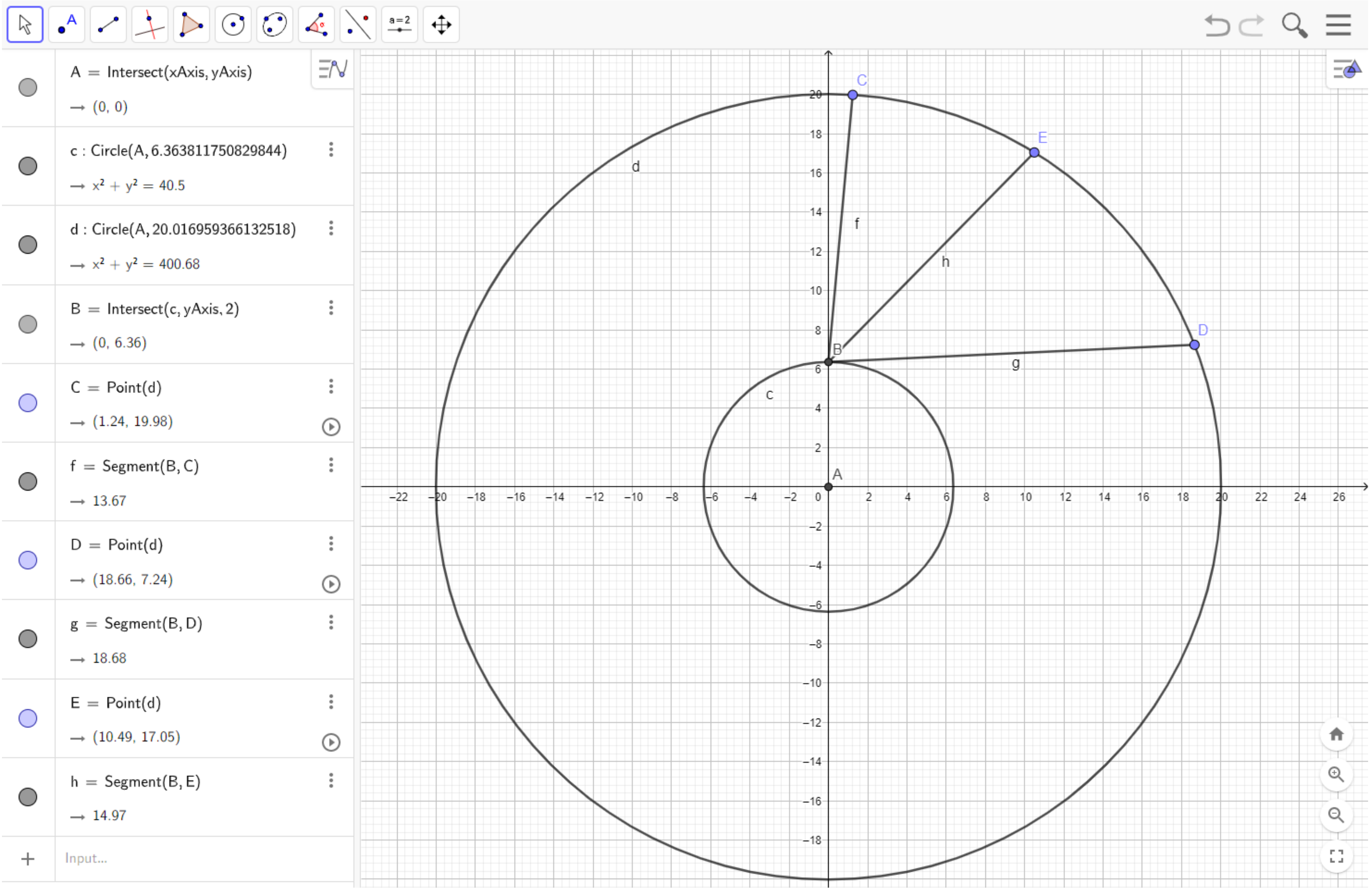
Ⓑ The mapping function slant factor [*Williams, 2017*]:

$$D_i = 2 \cdot 10^{-6} H_r N_l mpf_t$$

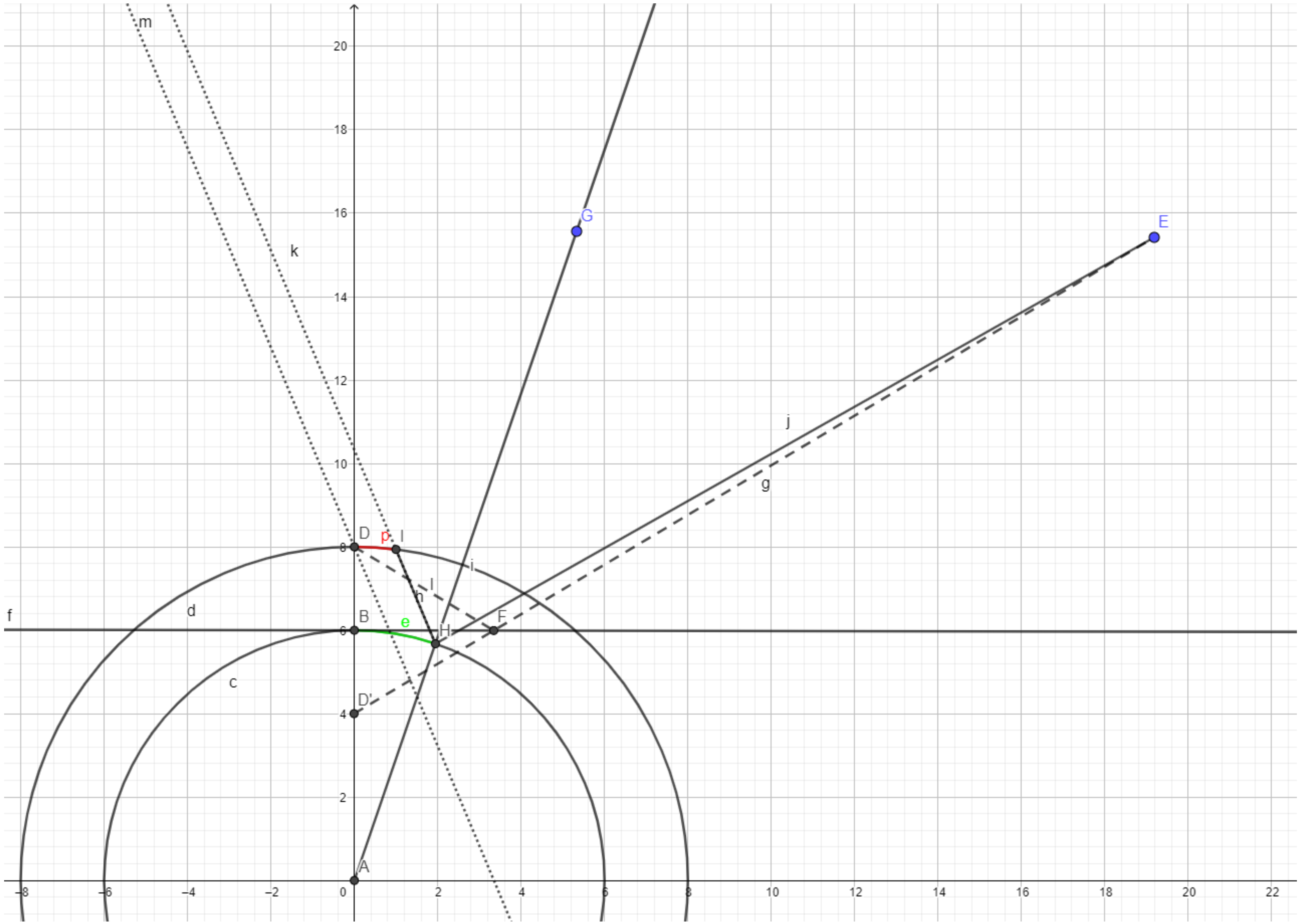
Ⓒ The mapping function derivative slant factor:

$$D_i = \frac{10^{-6} H_r N_l}{\sin e_A} + 10^{-6} H_r N_l mpf_t(e_T) - ZTD\left(\frac{\partial mpf_t}{\partial e_T}\Delta e_T + \frac{\partial mpf_t}{\partial h} H_r\right)$$

Approximation: receiver to satellite range



Ray-tracing reflected signal (Spherical Earth



Experiment at Onsala Space Observatory

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