Studying multi-beam ion temperature inside a collisionless reconnection plasmoid by means of Gaussian Mixture Model

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Ion beams in space plasma

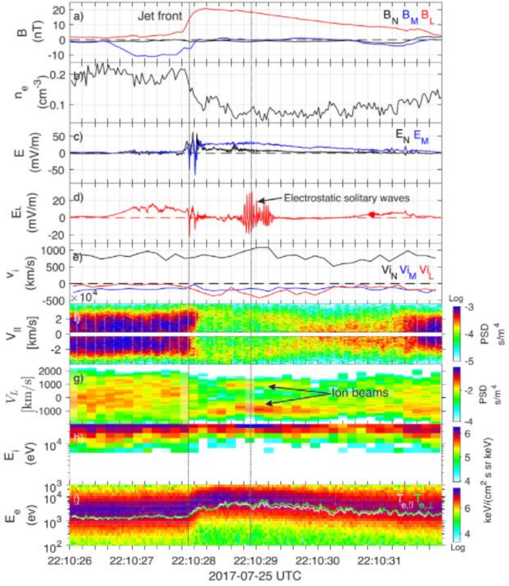
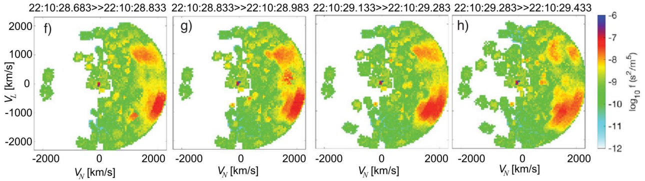


Figure 1. Jet crossing observed by MMS1. (a) Magnetic field **B**. (b) Electron density. (c) $\mathbf{E}_M$ and $\mathbf{E}_N$ components. (d) $\mathbf{E}_L$ component. (e) Ion velocity. (f) Electron phase space in the parallel direction (here electrons at energies <100 eV are excluded due to large uncertainty of electron measurements). (g) Ion phase space in the **L** direction. (h) Ion energy spectrum. (i) Electron energy spectrum.



[Liu, 2019]

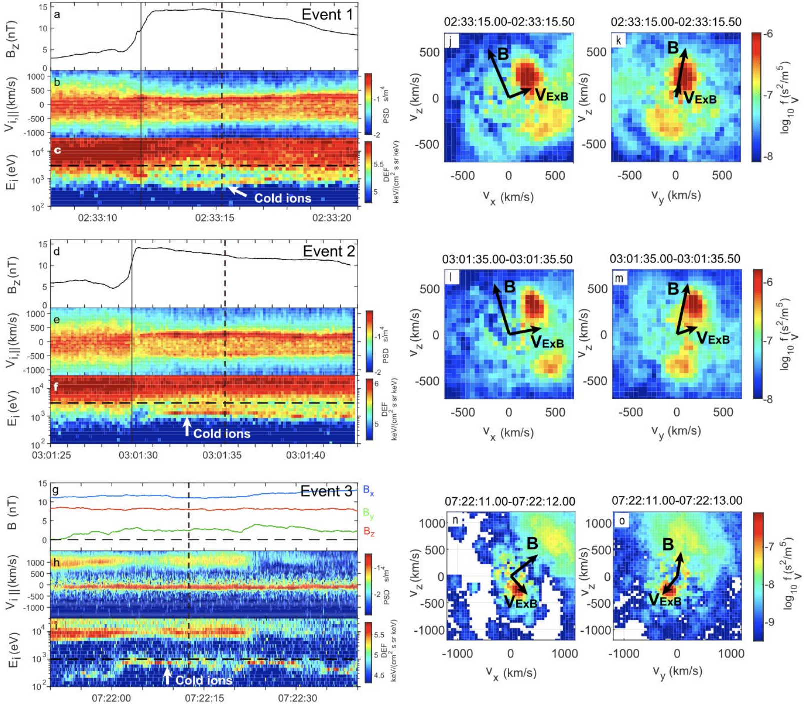
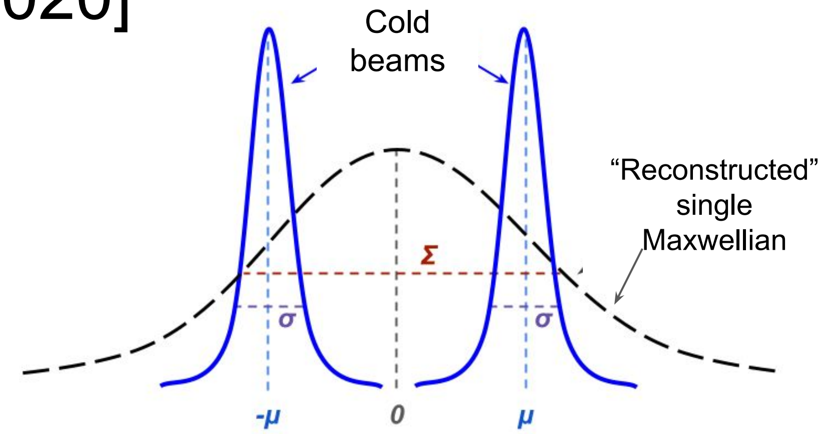


Figure 2. The specific velocity of cold ions behind dipolarization fronts (events 1 and 2) and the comparison with typical cold ions in lobe (event 3). (a and d) Z component of the magnetic field; (b and e) 1-D reduced ion distribution; (c and f) ion spectrograms of energy flux; (j and l and k and m) 2-D reduced ion distributions in *x-z* and *y-z* planes for events 1 and 2, respectively. (g-i) Magnetic field, 1-D reduced ion distribution, and ion spectrograms for event 3; (n and o) 2-D reduced ion distributions in *x-z* and *y-z* planes for event 3.

[Y. Xu, H. Fu, 2019]

Multi-beam moments [Goldman, 2020]



$$\boxed{f(\vec{v}) = f_1(\vec{v}) + f_2(\vec{v}) + \ldots + f_K(\vec{v}) = \sum_{k=1}^{K} f_k(\vec{v}),}$$

Superposition of k distinct populations of particles

Multibeam energy moments

	standard moments (*undecomposed*)	**k-th beam moments**	**multibeam *decomposed***
0-th	$n = \int f(\vec{v}) d\vec{v}$	$n_k = \int f_k(\vec{v}) d\vec{v}$	$n = \sum n_k$
1-th	$\vec{u} = \frac{1}{n} \int \vec{v} f(\vec{v}) d\vec{v}$	$\vec{\mu}_k = \frac{1}{n_k} \int \vec{v} f_k(\vec{v}) d\vec{v}$	$\vec{u} = \frac{1}{n} \sum_{k=1}^{K} n_k \vec{\mu}_k.$
2-th	$E_{therm} = \frac{m}{2} \int (\vec{v} - \vec{u})^2 f(\vec{v}) d\vec{v}$	$E_{therm}^{(k)} = \frac{m}{2} \int (\vec{v} - \vec{\mu}_k)^2 f_k(\vec{v}) d\vec{v}$	$E_{therm}^{(K)} = \sum_{k=1}^{K} E_{therm}^{(k)}$
Kinetic energy	$E_{kin} = mnu^2/2$	$E_{kin}^{(k)} = n_k m \mu_k^2/2$	$E_{kin}^{(K)} = \sum_{k=1}^{K} E_{kin}^{(k)}$

Gaussian Mixture Model (GMM)

Represents the distribution function as a superposition N of Maxwellian-like beams. Main steps:

$$p(\mathbf{x}|\Phi) = \sum_{k=1}^{K} w_k \mathcal{N}(\mathbf{x}|\theta_k), \quad \sum_{k=1}^{K} w_k = 1$$

Multivariate normal (Gaussian) distribution:

$$\mathcal{N}(x|\mu_k, \Sigma_k) = \frac{1}{(2\pi)^{D/2}} \frac{1}{|\Sigma_k|^{1/2}} \exp\left[-\frac{1}{2}(\mathbf{x} - \mu_k)^T \Sigma_k^{-1} (\mathbf{x} - \mu_k)\right]$$

Likelihood:

$$l(\Phi|X, Z) = \sum_{i=1}^{n} \ln\left[\sum_{k=1}^{K} w_k \mathcal{N}(x_i|\theta_k)\right]$$

Bayesian information criterion:

$$\mathrm{BIC} = \ln(n)k - 2\ln(L)$$

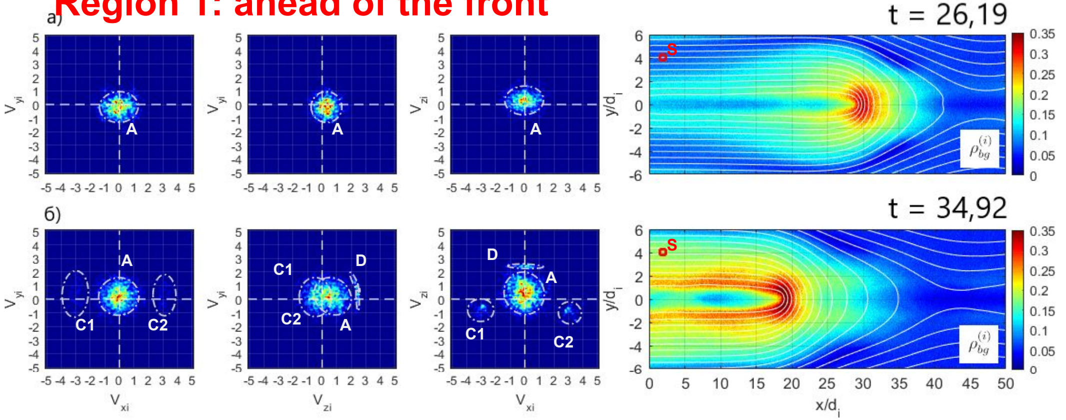
EM algorithm:

1. Init parameters
2. Aposterior probability
3. Mixture: likelihood maximization
4. Check if extremum reached
5. Repeat cycle

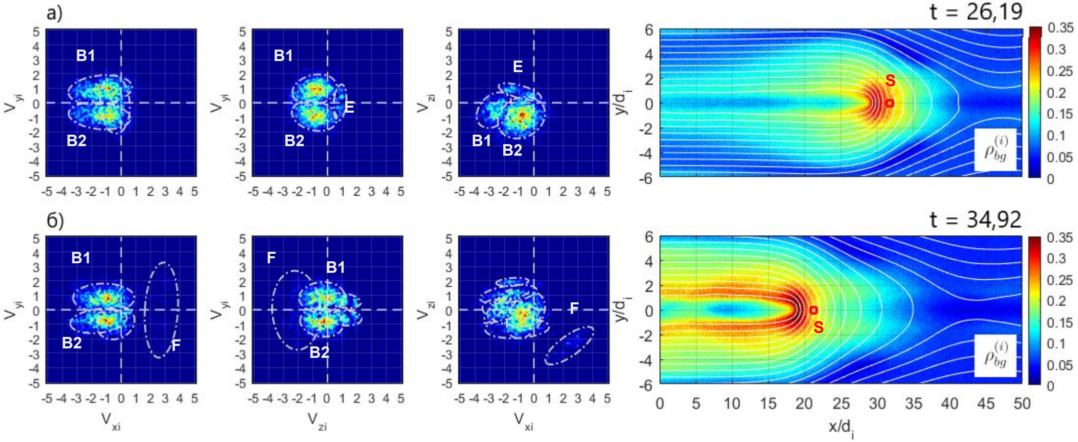
The method provides optimal number of beams and parameters (density, velocity, temperature) of each beam.

GMM: application to 2D symmetric reconnection (example)

Region 1: ahead of the front



Region 2: neutral plane, near the front.



iPIC3D:

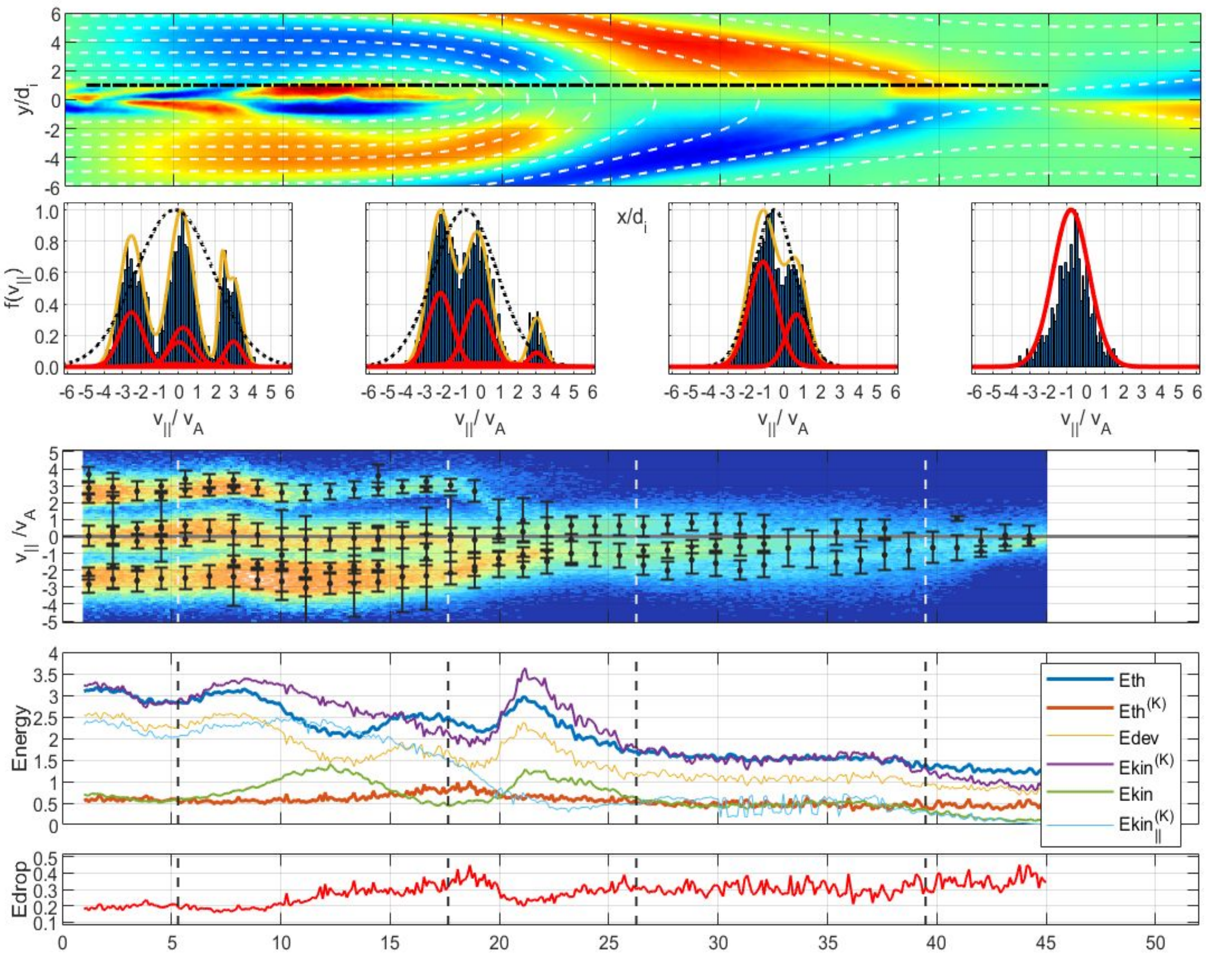
- implicit Particle-in-Cell code
- 2D/3D geometry, Cartesian grid
- Distribution function is approximated by “macroparticles”

Examples of ion distribution functions at different times:

- ahead of the front
- near the flux pile-up, neutral plane

A - background plasma
B1,2 - counter-streaming beams in exhaust;
C1,2 - parallel beams (ions trapped inside plasmoid);
D, E, F - weak accelerated populations revealed by GMM

GMM: temperature inside plasmoid



B_z (out-of-plane)

Sample ion VDF:
$f(v_{||}, x)$, several beams

Optimal GMM fit

E(thermal), **E(kinetic)**,
E(beam-thermal), **E(beam-kinetic)**,
"excessive thermal energy",
"parallel kinetic energy".

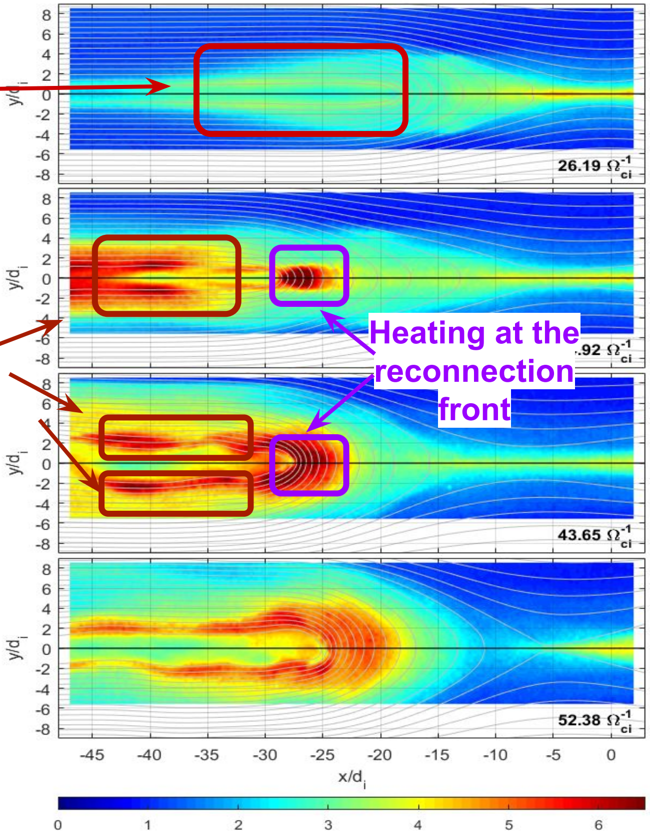
E_{drop} [Dupuis, Lapenta, 2020]:
E(beam-thermal) / **E(thermal)**

GMM: beam-temperature distribution

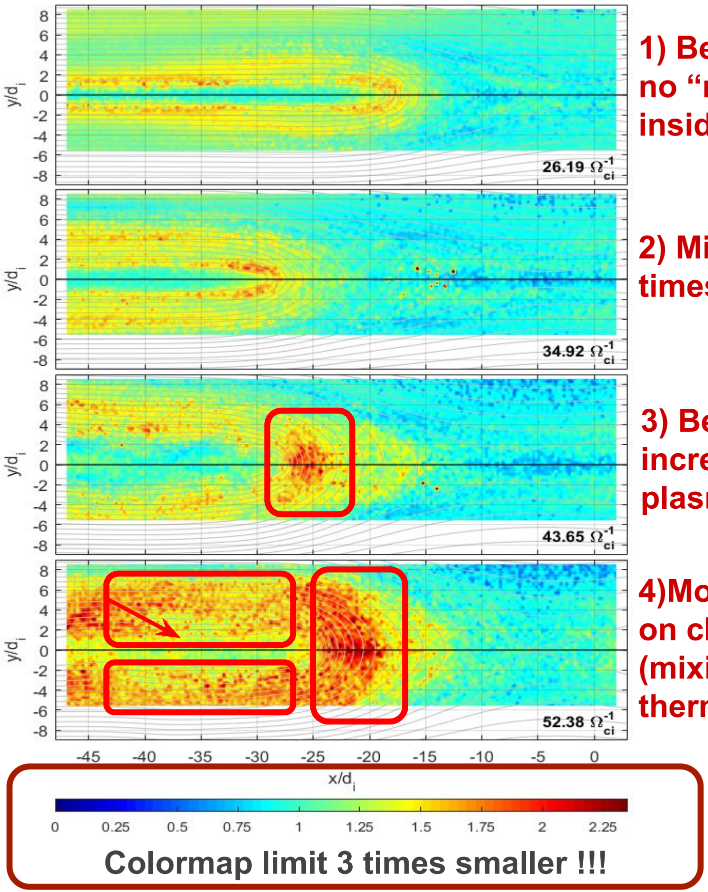
“Undecomposed” temperature

Streaming (reflected) ions, temperature increase

Trapped ions inside plasmoid at later times



Beam temperature



1) Beam temperature: no “real” heating inside diffusion region

2) Mild heating at earlier times

3) Beam temperature increase when plasmoid compressed

4)Most heating appears on closed field lines (mixing and thermalization)

Conclusions

1) We performed analysis of ion distribution functions during collisionless reconnection using Gaussian Mixture Model. The method computes the optimal number of Gaussian “beams” and calculates beam moments (weight, velocity, temperature).
2) Beam temperature appears several times smaller than “classical” temperature scalings predicted by [Drake, 2009], [Phan, 2014]. Beam temperature excludes kinetic contribution of counter-streaming beams and thus serves as a measure of **thermalization** of plasma as opposed to “classical” temperature.
3) Beam temperature scalings, energy partition, time evolution: work in progress!

Thanks!

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