A semi-analytical model for marine ice sheet dynamics

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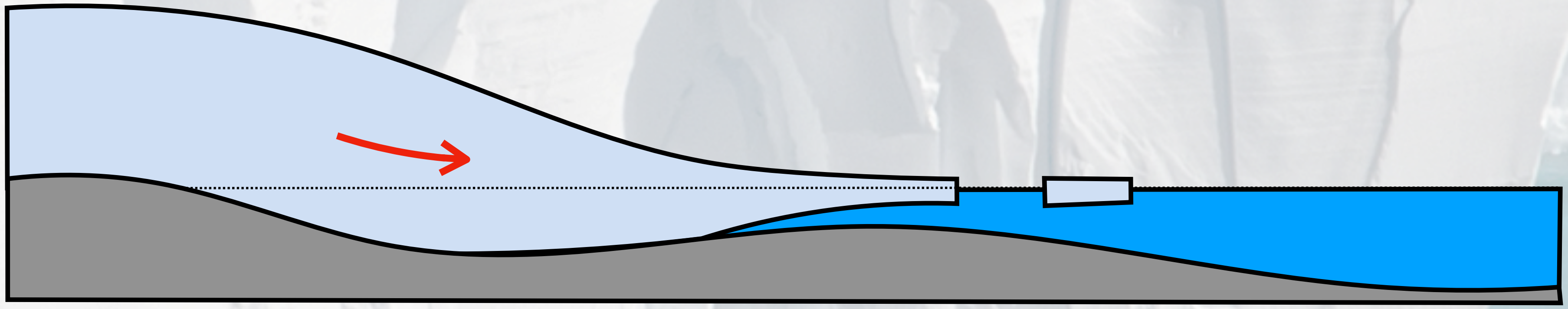
Marine ice sheets have potential to cause **rapid** and **large sea level change**. Their dynamics are only partially understood.

Simplified flow-line models have helped to illuminate fundamental aspects of the behaviour (e.g. instability of a grounding line on an upsloping bed).

e.g. Schoof 2007, Schoof 2012, Pegler 2018, Haseloff & Sergienko 2018, Sergienko & Wingham 2019

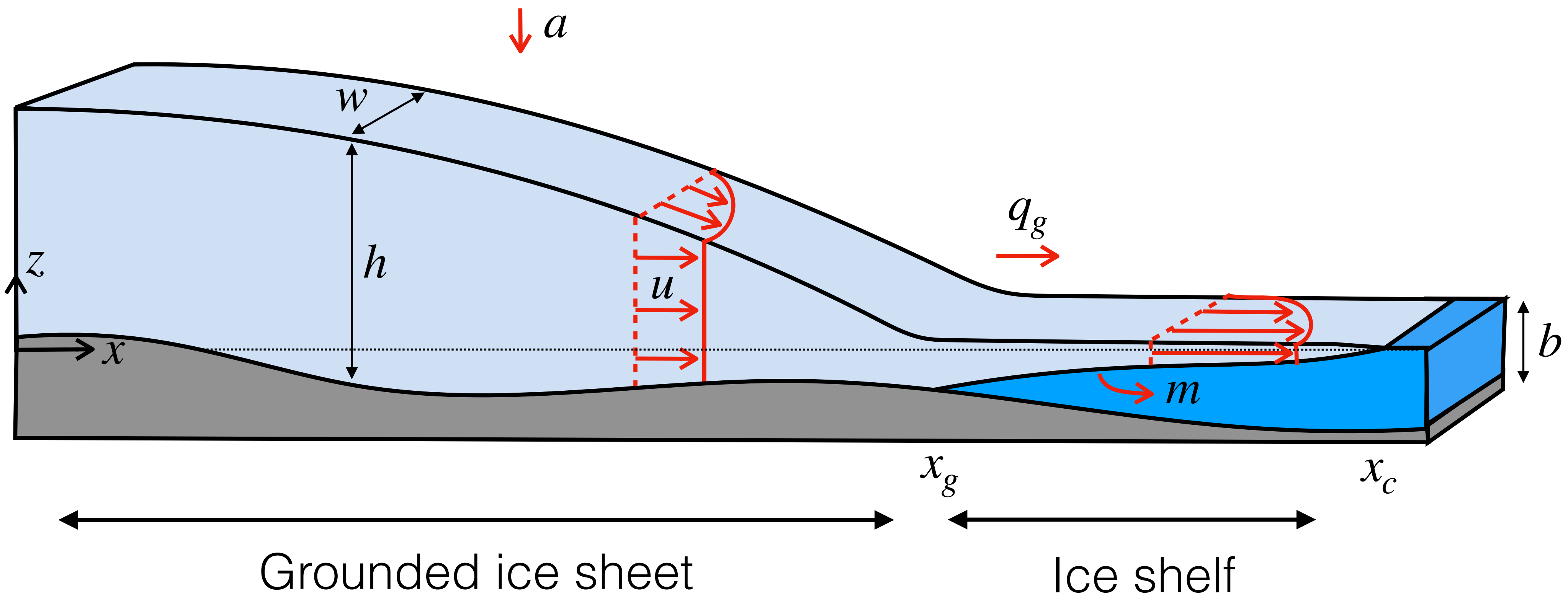


This work aims to build on such previous work in two ways:

- Provide a simplified description of the time-dependent **dynamics** (i.e. beyond just an analysis of steady states / stability)

- Present the key physics without too much complicated algebra

The starting point is a width- and depth-integrated flow-line model:



Accumulation

Basal melting

Mass conservation

$$\frac{\partial h}{\partial t} + \frac{\partial}{\partial x}(hu) = a - m$$

Force balance

$$\frac{\partial}{\partial x}\left(4h\mu\frac{\partial u}{\partial x}\right) - \rho_i g h \frac{\partial s}{\partial x} - \tau_b - \frac{3\mu h u}{w^2} = 0$$

Extensional stress

Driving stress

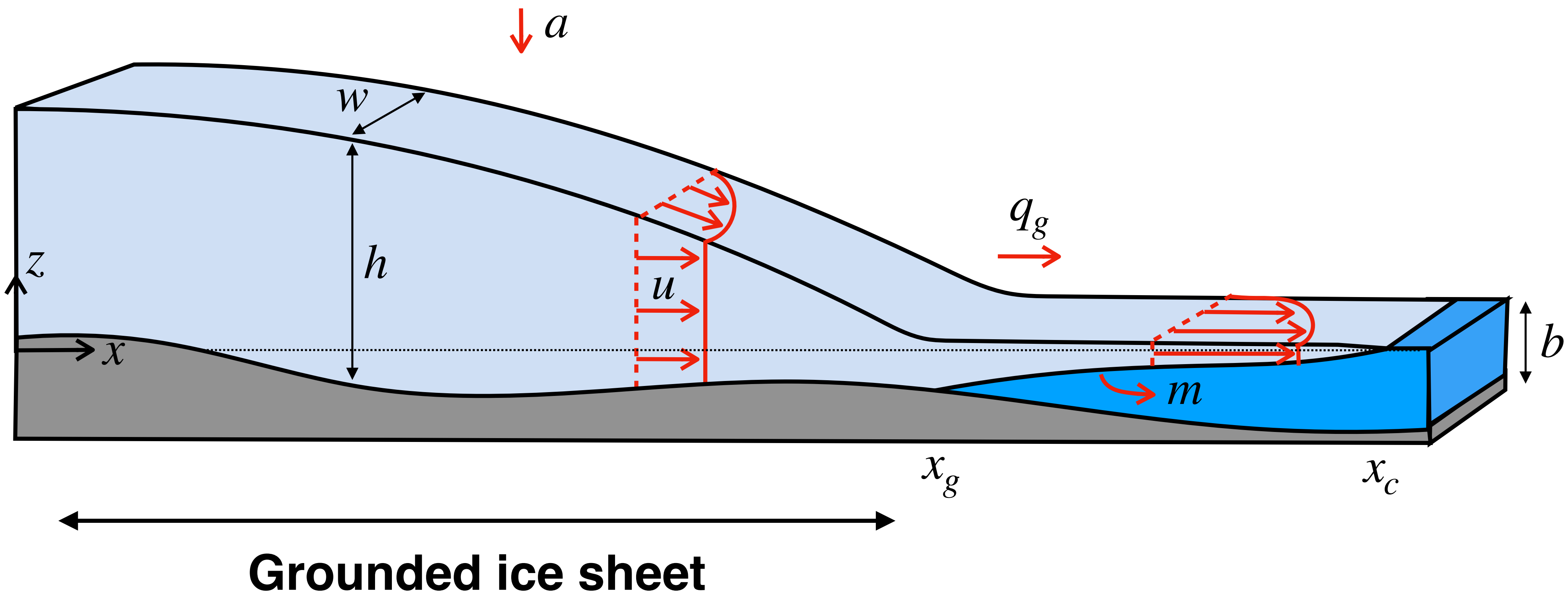
Basal drag

Side drag (buttressing)

$$s = \begin{cases} b + h & h > h_f \\ \left(1 - \frac{\rho_i}{\rho_o}\right) h & h \leq h_f \end{cases}$$

$$h_f(x) = -\frac{\rho_o}{\rho_i} b(x)$$

Simplifications: Newtonian ice rheology, plastic basal friction law



Accumulation

Basal melting

$$\underline{\frac{\partial h}{\partial t}} + \underline{\frac{\partial}{\partial x}(hu)} = \underline{a} - m$$

$$\frac{\partial}{\partial x}\left(4h\mu\frac{\partial u}{\partial x}\right) - \underline{\rho_i g h\frac{\partial s}{\partial x}} - \underline{\tau_b} - \frac{3\mu h u}{w^2} = 0$$

Extensional stress

Basal drag

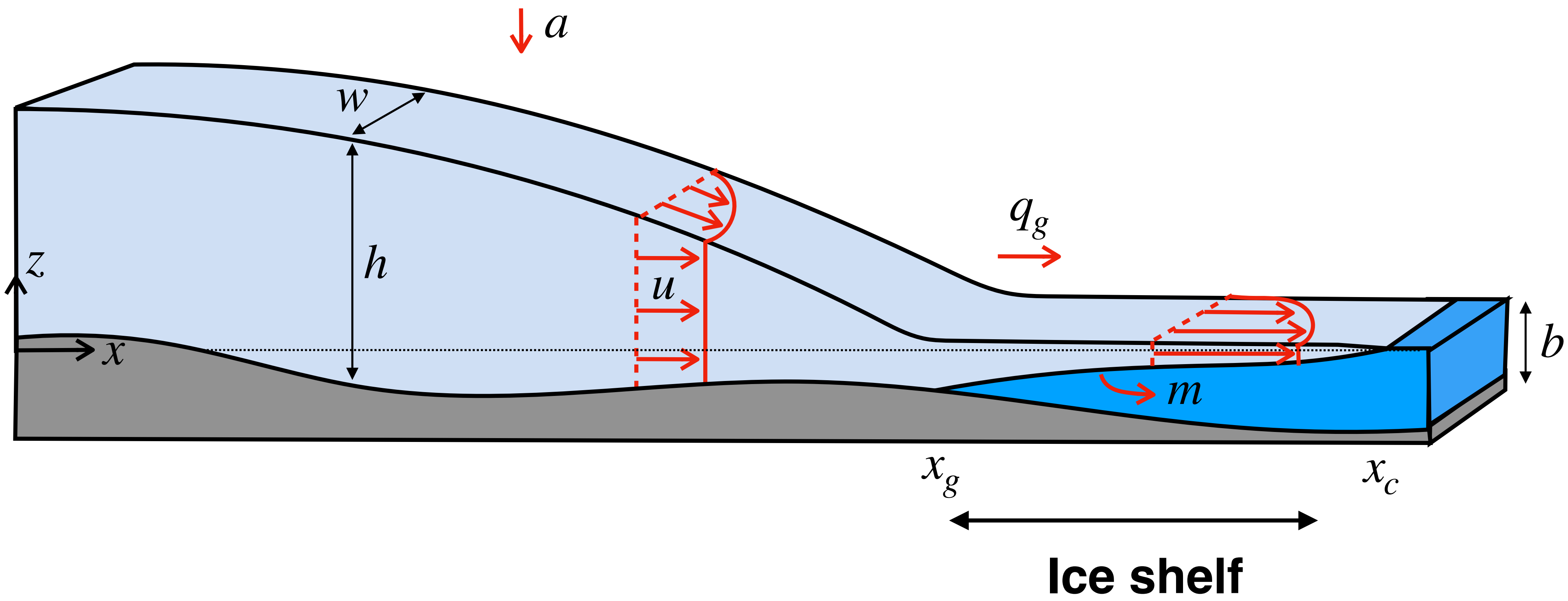
Driving stress

Side drag (buttressing)

⇨

$$h = \sqrt{\frac{2\tau_b}{\rho_i g}}(x_g - x)^{1/2}$$

$$\boxed{\sqrt{\frac{2\tau_b x_g}{\rho_i g}}\frac{\mathrm{d}x_g}{\mathrm{d}t} = a x_g - q_g}$$



Accumulation Basal melting

$$\frac{\partial h}{\partial t} + \underline{\frac{\partial}{\partial x}(hu)} = \underline{a} - \underline{m}$$

$$\frac{\partial}{\partial x}\left(4h\mu\frac{\partial u}{\partial x}\right) - \underline{\rho_i g h\frac{\partial s}{\partial x}} - \tau_b - \underline{\frac{3\mu h u}{w^2}} = 0$$

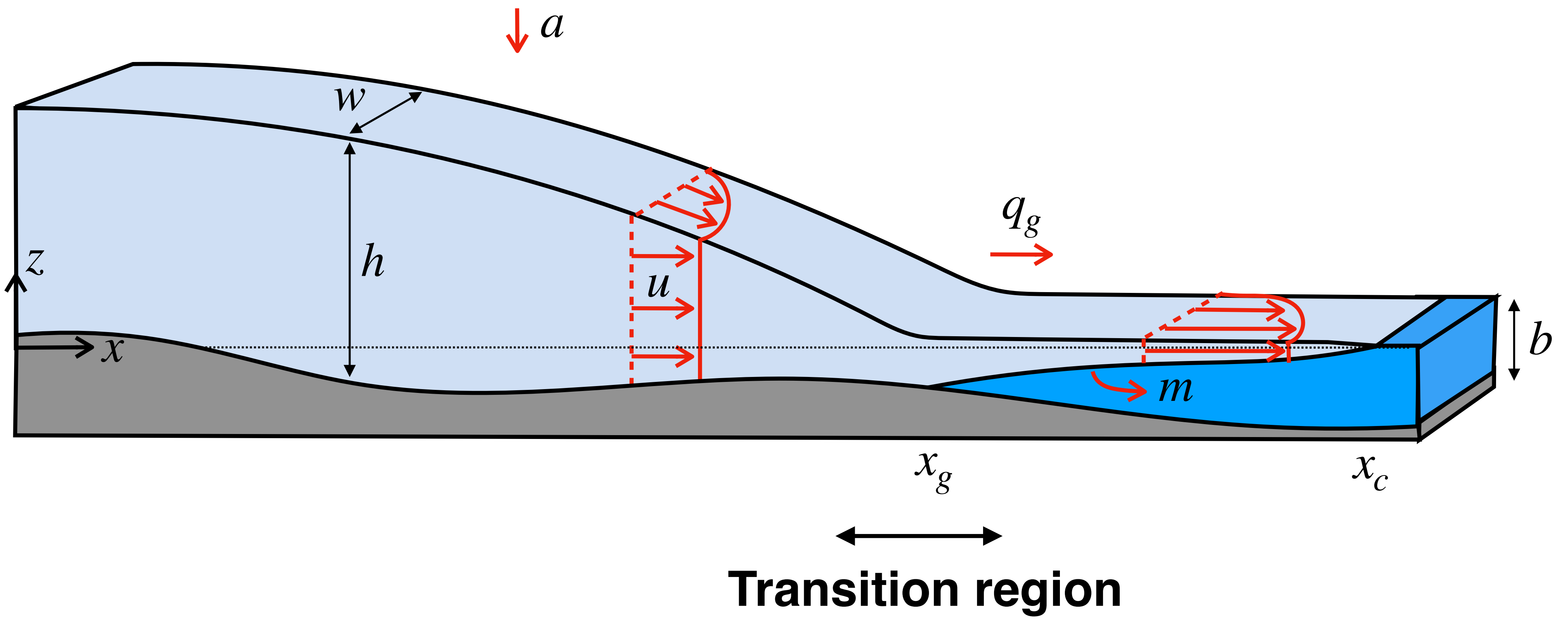
Extensional stress Basal drag

Driving stress Side drag (buttressing)

⇨

$$q = q_g - (m - a)(x - x_g)$$

$$h = \sqrt{\frac{3\mu}{(1 - \rho_i/\rho_o)\rho_i g w^2 (m - a)}\, q}$$



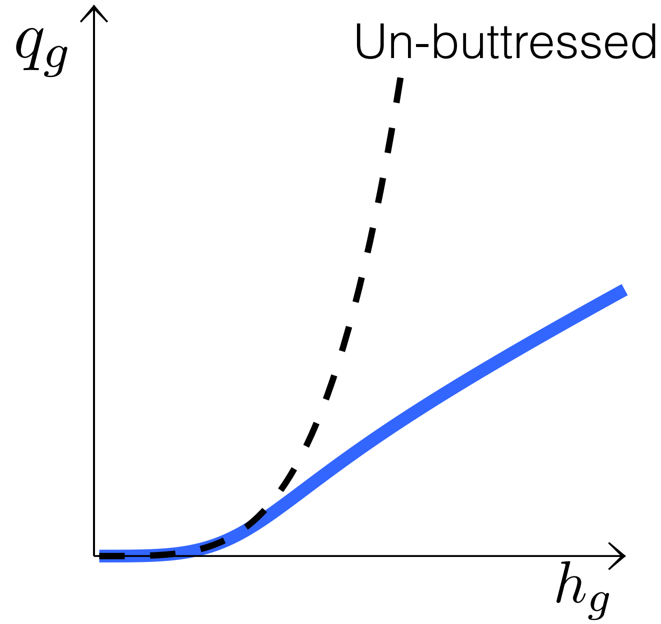
Ensuring continuity of ice thickness, velocity, and extensional stress, determines a relationship between grounding line ice flux q_g and ice thickness $h_g = h_f(x_g)$

$$\Rightarrow \boxed{\frac{4\mu\tau_b}{\rho_i g}\frac{q_g}{h_g^2} = \frac{1}{2}(1-\rho_i/\rho_o)\rho_i g h_g^2 - \frac{3\mu}{2w^2(m-a)}q_g^2}$$

↑ Extensional stress

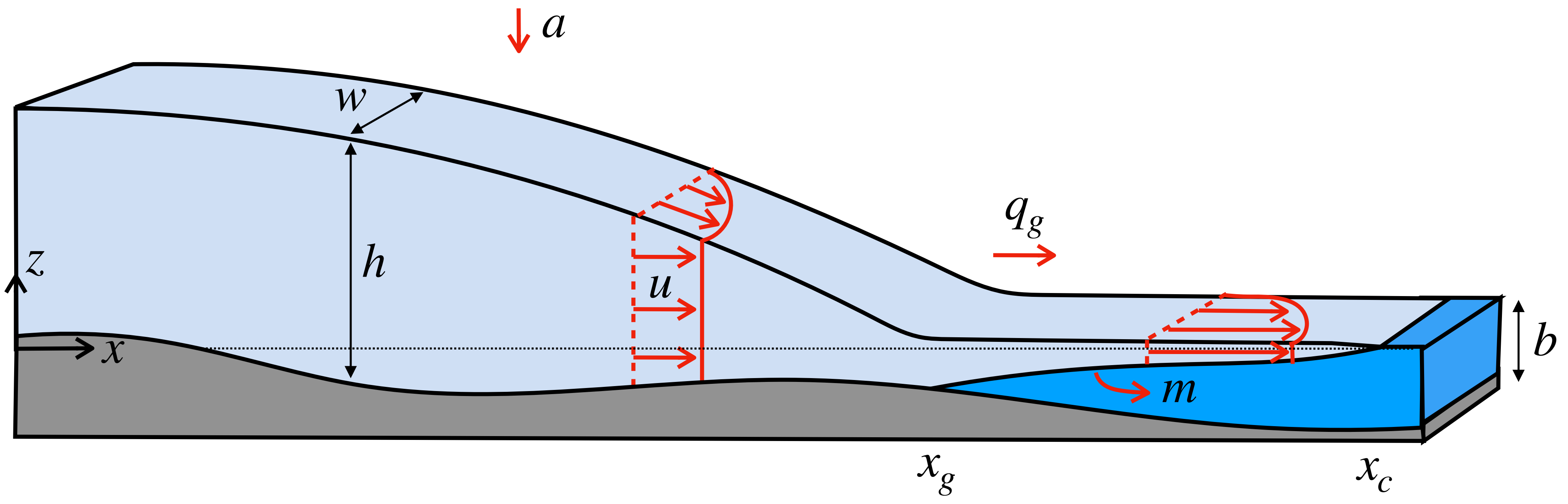
↑ Horizontal ‘pull’ due to buoyancy

↑ Buttressing



Schoof 2007, Pegler 2018, Haseloff & Sergienko 2018

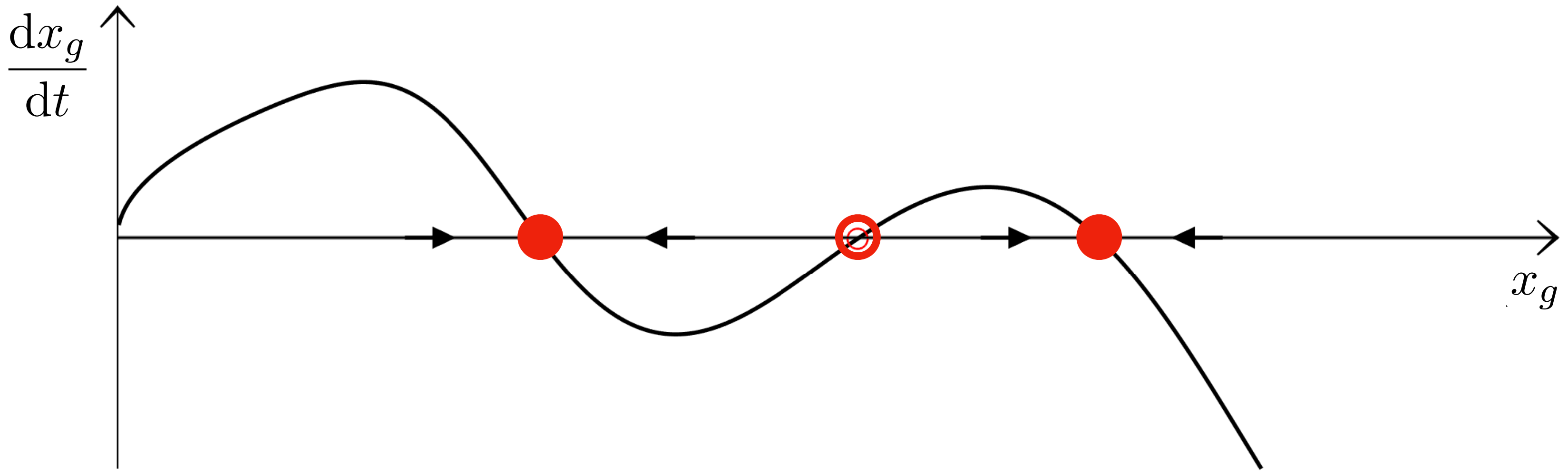
The reduced model:



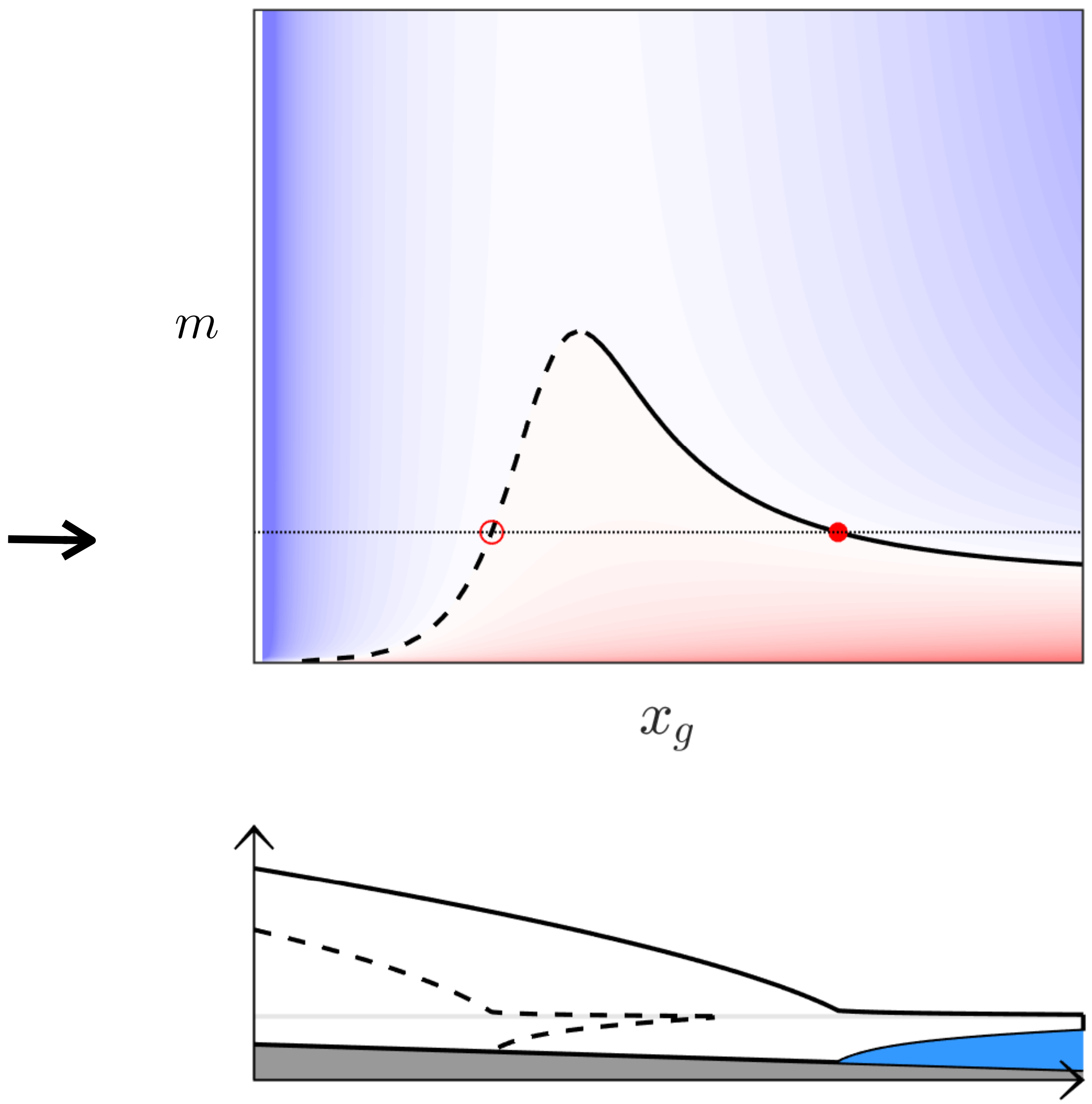
$$\boxed{\sqrt{\frac{2\tau_b x_g}{\rho_i g}}\frac{\mathrm{d}x_g}{\mathrm{d}t} = a x_g - q_g}$$

$$\boxed{\frac{4\mu\tau_b}{\rho_i g}\frac{q_g}{h_g^2} = \frac{1}{2}(1-\rho_i/\rho_o)\rho_i g h_g^2 - \frac{3\mu}{2w^2(m-a)}q_g^2 \qquad h_g = -\frac{\rho_i}{\rho_o}b(x_g)}$$

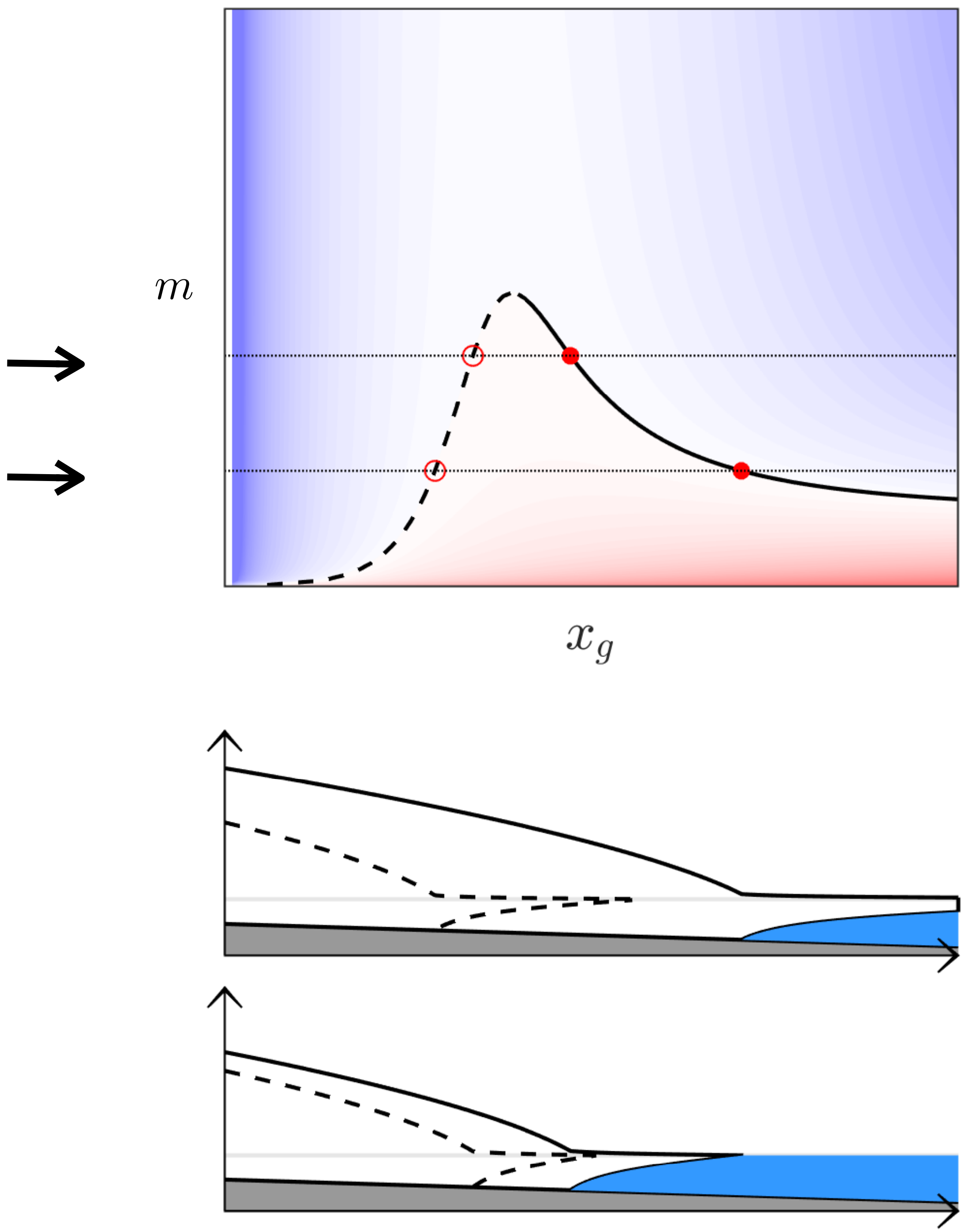
⇨ $\frac{\mathrm{d}x_g}{\mathrm{d}t} = F(x_g;\ldots)$



Example: a linearly sloping bed below sea level



Example: a linearly sloping bed below sea level



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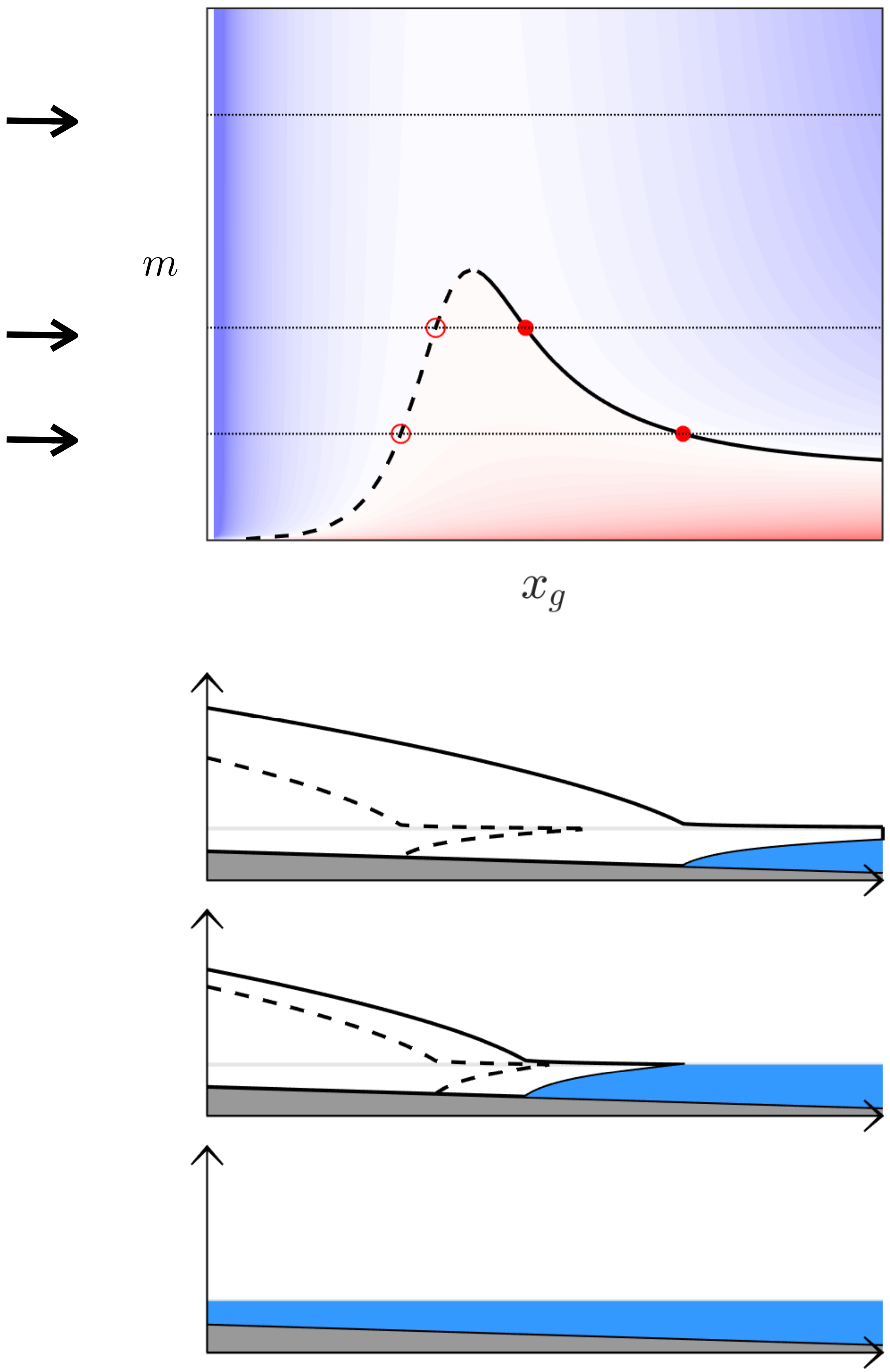

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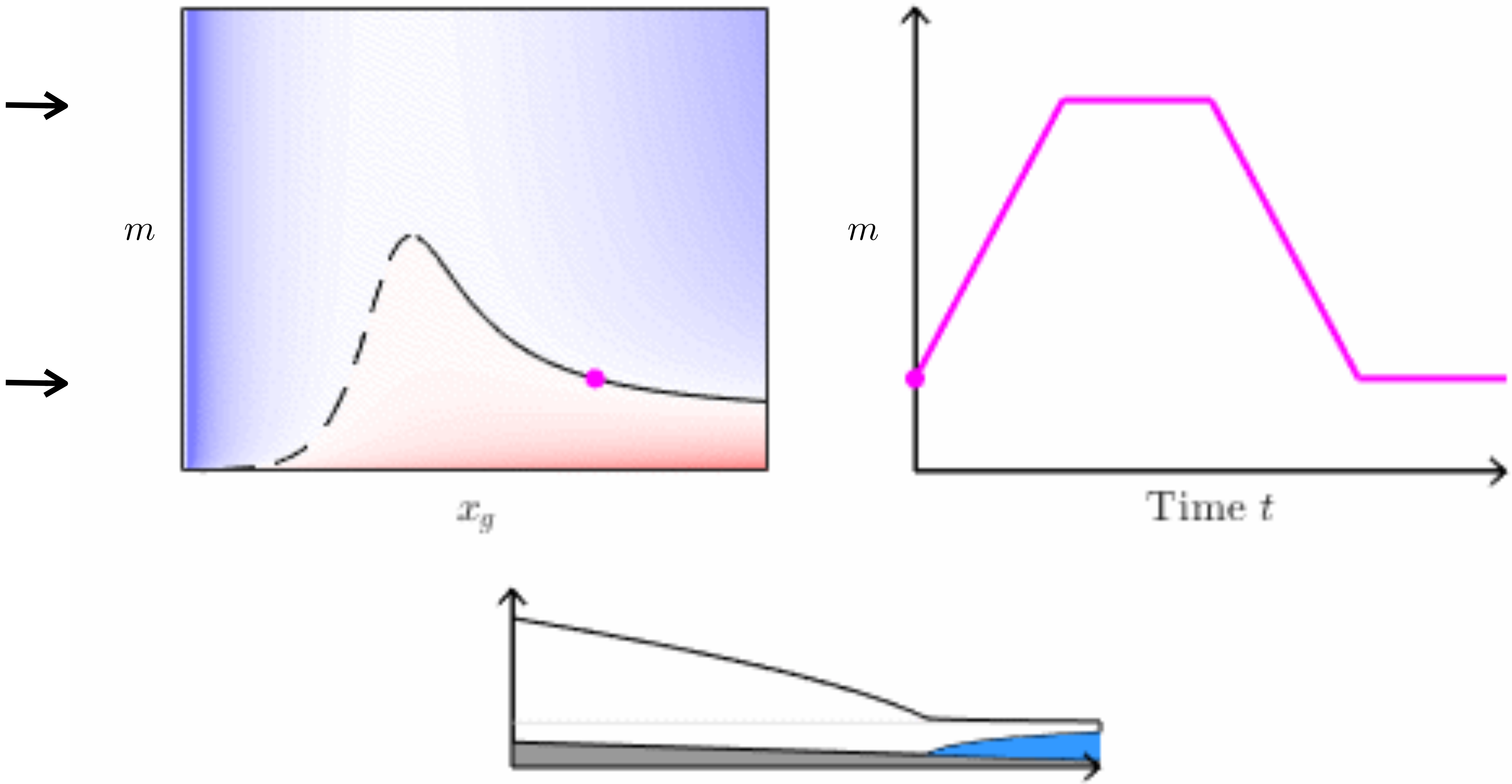
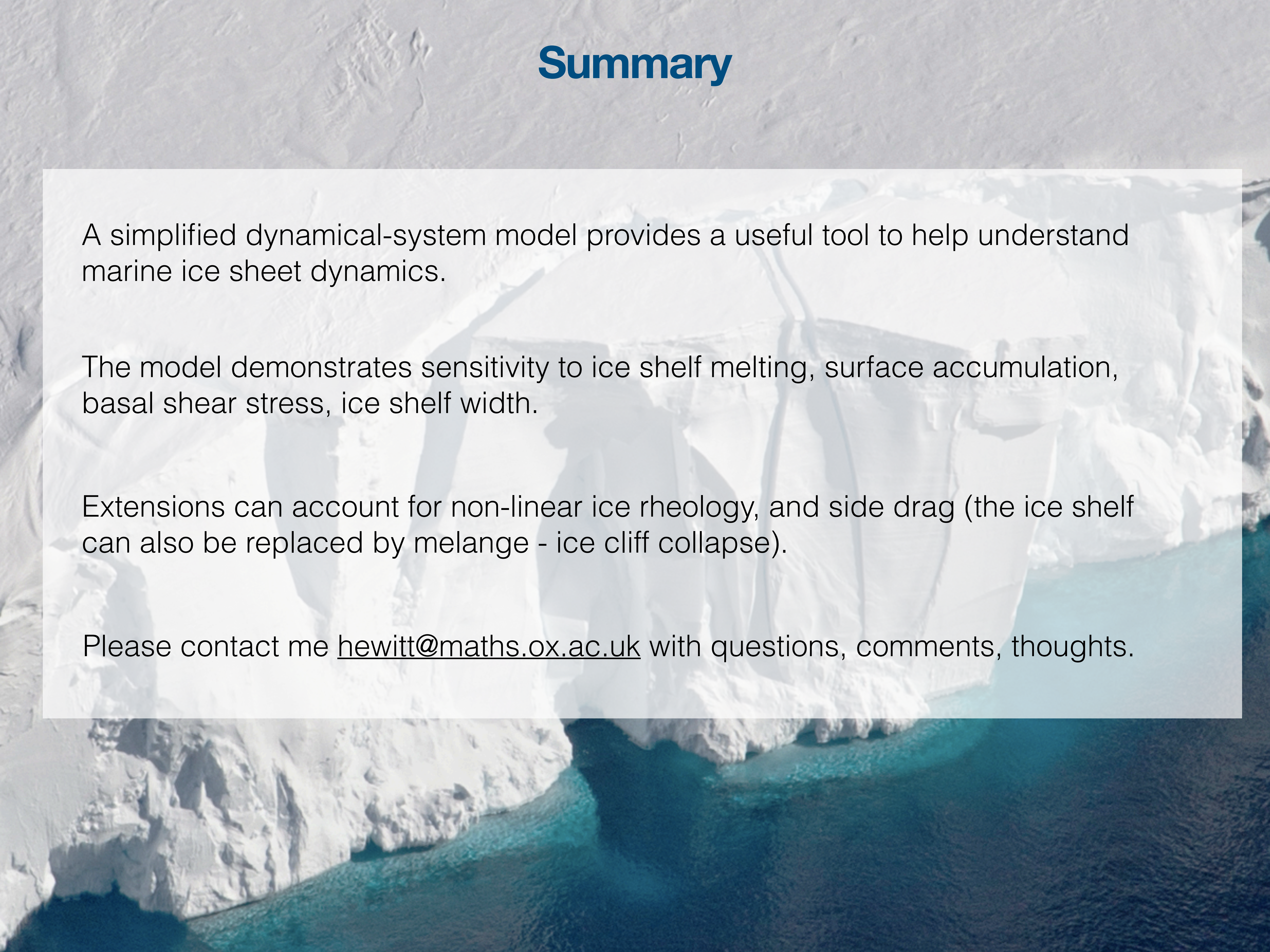


Summary



A simplified dynamical-system model provides a useful tool to help understand marine ice sheet dynamics.

The model demonstrates sensitivity to ice shelf melting, surface accumulation, basal shear stress, ice shelf width.

Extensions can account for non-linear ice rheology, and side drag (the ice shelf can also be replaced by melange - ice cliff collapse).

Please contact me hewitt@maths.ox.ac.uk with questions, comments, thoughts.