

# Analytical developments on 6C computation inspired by navigation algorithms

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# Summary

- Rotation sensors in seismology ?
- What does a rotation sensor measure ?
- How to use a rotation sensor ?
- Comparison with lab tests by F. Bernauer
- Algorithm and simulations !

# **Rotation sensors in seismology ?**

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*A quick introduction*

# Rotation sensors in seismology ?

- They allow to measure ground rotational motion
- Some, like the FOGs (typically the BlueSeis3A) are insensitive to ground translational motion

## Tilt correction

- OBS (Crawford and Webb, 2000)
- Wind noise (Lin et al, 2022)
- Strong motion (Bernauer et al, 2020)

## Back azimuth measurements

Seismic array with one station

Planetology ! (Bernauer et al, 2020b)

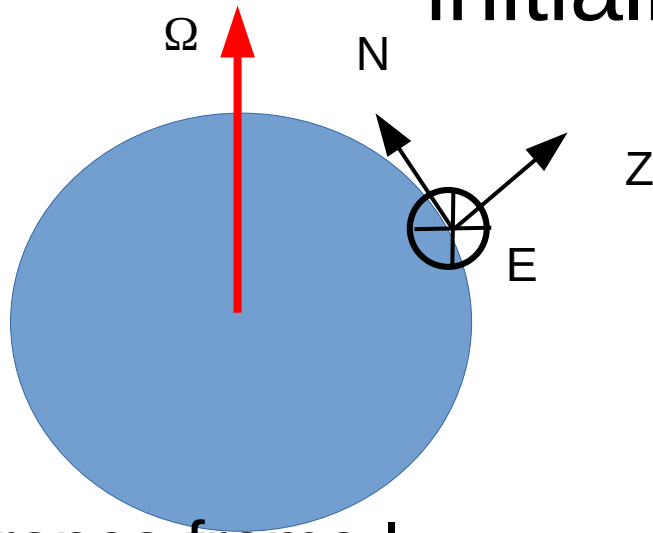
# **What does a rotation sensor measure ?**

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*Initialisation of the sensor heading*

# What does a rotation sensor measure ?

- It measures a rotation rate along its sensitivity axes
- It means it measures the Earth rotation – you can initialise the heading !



Inertial reference frame I

Earth reference frame E

Navigation (ENZ) reference frame N

Body (XYZ) reference frame B

Heading = difference between Y axis of Body frame and North

# What does a rotation sensor measure ?

Initialisation of

- Earth → Navigation (locally level)
- Navigation → Body (rotation sensors)

But also

- Rotation sensors → seismometer



# **How to use a rotation sensor ?**

*Integration and linear algebra !*



# How to use a rotation sensor ?

- The solution of this equation is the orientation of the sensors in the Navigation frame. Several approximations can be done :

$$\dot{C}_B^N = C_B^N [\omega_{IB}^B \times] - [\omega_{IE}^N \times] C_B^N$$

“direct”  
integration

(+ coning compensation)

Earth  
projection

## **Comparison with lab tests**

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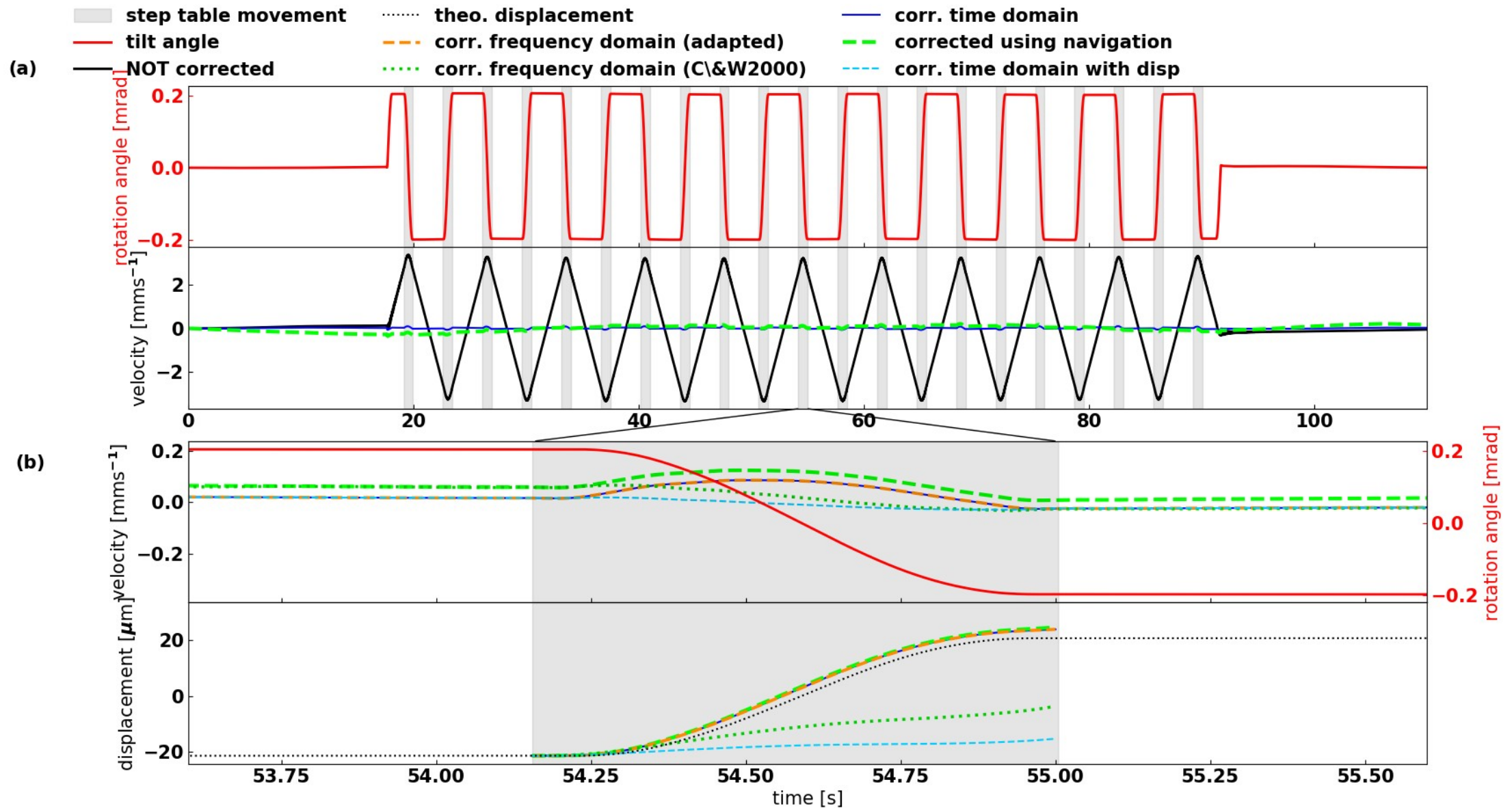
*Tests conducted by Bernauer et al, published in 2020*

Bernauer, F., Wassermann, J., & Igel, H. (2020). Dynamic tilt correction using direct rotational motion measurements. *Seismological Research Letters*, 91(5), 2872-2880.

# Comparison with lab tests

- Tilt table with BlueSeis3A and Trillium Compact, rotated around East axis
- 3 methods in Bernauer et al to remove tilt
  - Frequency method (Crawford & Webb 2000)
  - Adapted frequency method
  - Time domain
- « Navigation » method with rotation matrix added

# Comparison with lab tests



# Comparison with lab tests

- Same results with new method
- Advantages : real time method
- But main advantages not seen in this experiment :
  - Easy rotation when data on several FOG axes
  - Gravity projection ! Here only rotation on East axis...

# **Algorithm and simulations**

# Algorithm and simulations

$$\dot{C}_B^N = C_B^N [\omega_{IB}^B \times] - [\omega_{IE}^N \times] C_B^N$$

This equation is good analytically but is flawed numerically because of the integration. Numerical methods have been developed to mitigate numerical errors and non-orthogonality. They use the rotation vector formalism.

## Simulation:

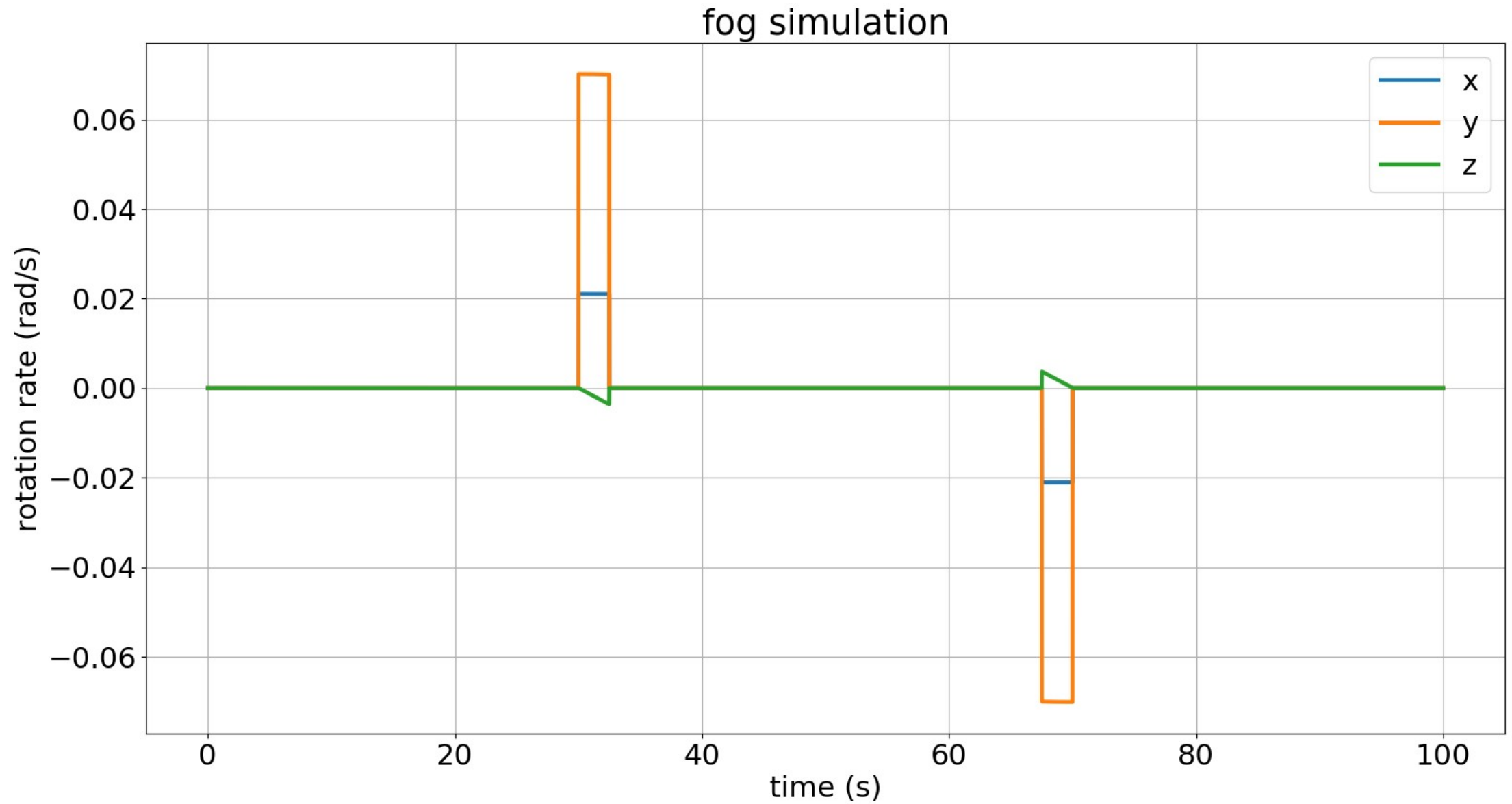
- roll and pitch angles applied
- Simulation of FOGs
- Retrieval of input angles from FOGs

3 methods are tested to retrieve the angles:

- “direct” method
- “Without gravity”
- navigation

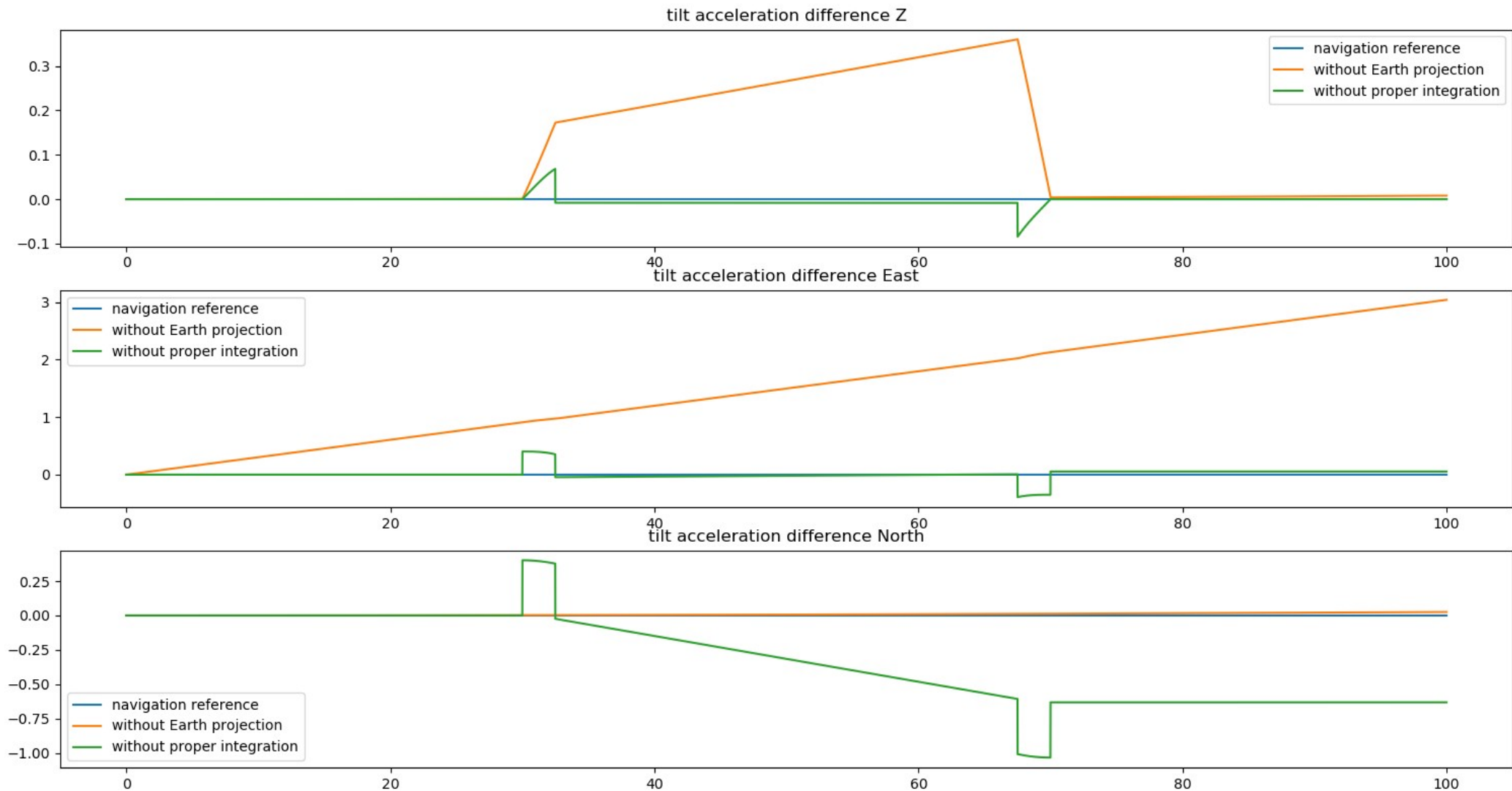


# Algorithm and simulations



# Algorithm and simulations

Difference (Percent of maximum tilt)



# Conclusion

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## INPUTS

- Rotation sensors measurements
- Position of station
- Initial “verticality” of rotation sensors



## OUTPUTS

- Heading
  - Pitch
  - Roll at every time
- Rotation vector at every time



Attitude of  
seismometer



Tilt

- Always reliable, fewer hypothesis:
  - Can work for several axis rotation and complex rotations
  - No need to correlate to the seismometer output
  - Calculates Earth projection for large tilts
- Can be directly implemented as a FOG output
- Real time, no additional lowpass filtering needed for Earth projection

**Thanks for your attention !**