Spatially adaptive Bayesian estimation for Probabilistic Temperature Forecasts

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Contents

1 Introduction

NWP Forecasts and Postprocessing

- Weather forecasting today is performed with **numerical weather prediction (NWP) models**.
- **Deterministic simulation models**, differential equations representing physics of the atmosphere.
- Common approach: use of **forecast ensembles** to represent forecast uncertainty.
- **Ensemble member**: single NWP forecast, obtained by running NWP model with different initial conditions and/or model formulations.

State-of-the-art Univariate Postprocessing

- **Ensembles of NWP forecasts address forecast uncertainty**, cannot capture uncertainty to full extent, thus often suffer from **biases and dispersion errors** (underdispersion).
- Postprocessing models correct for these shortcomings **in coherence with recently observed forecast errors**, yield **full predictive probability distributions**.
- Popular approach: Ensemble model output statistics (EMOS, Gneiting et al., 2005), **combines ensemble members in regression approach**.

Ensemble Model Output Statistics (EMOS)

- Y **univariate weather quantity which can be assumed to follow a normal distribution** (e.g. temperature), $x_1, \ldots, x_k$ ensemble of forecasts.

$$\begin{aligned} Y &= a + b_1\, x_1 + \ldots + b_k\, x_k + \varepsilon, \\ \varepsilon &\sim N(0, c + dS^2), \end{aligned}$$

  a, $b_1, \ldots, b_k$ bias-correction parameters, $c, d \geq 0$, variance parameters, and S^2 (empirical) ensemble variance.

- **EMOS predictive distribution**

$$Y | x_1, \ldots, x_k \sim N(a + \sum_{\ell=1}^{k} b_\ell\, x_\ell, c + dS^2).$$

- For exchangeable forecasts: $b_1 = \ldots = b_k = b$.

2 Markovian EMOS

Extension of Univariate Methods

- Many standard methods **designed for univariate weather quantity**, at **fixed locations**, for **fixed forecast horizons**.
- However, performing “univariate” (individual) postprocessing is **not utilizing multivariate information** of raw ensemble.
- Need for models that explicitly account for **multivariate dependence structures**, such as inter-variable, spatial, temporal,... to obtain physically coherent forecasts.

Proposal of this work

Postprocessing procedure based on basic EMOS model that **combines univariate and multivariate postprocessing**

2 Stage Procedure for Postprocessing

- **Basic approach**: Spatially adaptive Markovian extension of standard EMOS (MEMOS), performs **individual postprocessing at each location** (using information from neighbouring stations)
- **Add-on approach**: Provide basic MEMOS with **additional spatial dependence structure** to obtain **multivariate postprocessing procedure**

Markovian Ensemble Model Output Statistics (MEMOS)

For Y_s surface temperature and $x_{1,s}, \dots, x_{k,s}$ exchangeable ensemble at location s, the MEMOS model is given as

$$\begin{aligned} Y_s &= \eta_s + \varepsilon_s \\ \eta_s &= \gamma + a_s + b_s \, \bar{x}_s. \end{aligned}$$

- $\varepsilon_s \sim N(0, \sigma^2)$ for all s,
- γ overall fixed effect intercept,
- $\bar{x}_s = \sum_{\ell=1}^{k} x_{\ell,s}$,
- random effects $a_s \sim \mathbf{a}(s)$, $b_s \sim \mathbf{b}(s)$, where $\mathbf{a}(s)$, $\mathbf{b}(s)$ are assumed to be **latent Gaussian random fields (GRFs)**.

MEMOS predictive distribution:

$$Y_s | \eta_s, \sigma^2, x_{1,s}, \dots, x_{k,s} \sim N(\eta_s, \sigma^2).$$

The SPDE Approach, Lindgren et al. (2011)

- A Gaussian Random Field (GRF) $x(s)$ with **Matern covariance function** is stationary solution to the **linear fractional stochastic partial differential equation (SPDE)**:

$$(\kappa^2 - \triangle)^{\alpha/2}\,(\tau x(s)) = W(s),$$

  $s \in \mathbb{R}^d$, $\alpha = \nu + d/2$, $\kappa > 0$, $\nu > 0$, $\tau > 0$, $\triangle = \sum_{i=1}^{d} \frac{\partial^2}{\partial s_i^2}$, W spatial white noise.

- Provides **Gaussian Markov Random Field (GMRF) representation of GF**.
- Value of α influences smoothness of underlying field.
- Model fitting in a Bayesian framework with **Integrated Nested Laplace Approximation (INLA)**.
- Approximation of **posterior marginal distributions** of $a_s, b_s, \eta_s, \sigma^2$ with **R-SPDE-INLA**.

Basis Function Representation

- GMRF representation of GRF obtained via **linear combination of basis functions** defined on triangulated domain (mesh) with n vertices

$$x_n(s) = \sum_{k=1}^{n} \psi_k(s)\, w_k,$$

- $\psi_k\colon \mathbb{R}^2 \to [0, 1]$, continuous **piecewise linear basis functions**, such that $\psi_k(r) = 1$ for $r = k$ and $\psi_k(r) = 0$ for $r \neq k$, $r, k = 1, \ldots, n$.
- Vector of weights $\mathbf{w} = (w_1, \ldots, w_n)'$ is GMRF with precision matrix $\mathbf{Q}(\boldsymbol{\theta})$, depending on a set of hyperparameters $\boldsymbol{\theta}$, here $\boldsymbol{\theta} = (\tau, \kappa)'$.

Link between GRF and GMRF

- Continuous field $x(s)$ **fully determined by finite dimensional distribution** of **w**.
- Weights w_k determine value of $x(s_n)$ at locations s_n that are vertices of mesh, values s_{int} at arbitrary locations in **interior of triangles** determined by simple **linear interpolation** of ψ_k to s_{int}
- Establishes **explicit link between GRF parameters** κ^2, τ and **elements of the precision matrix Q** of GMRF representation via the distribution of weights vector **w**.

Adding Spatial Dependence Structure

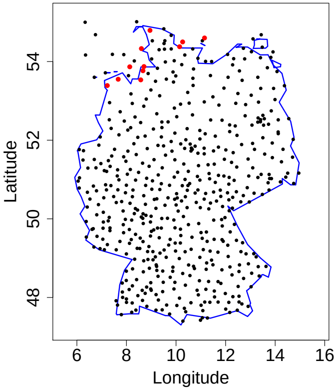
1. Combine univariate models with **Ensemble copula coupling (ECC)**: Reorder samples from univariate postprocessed margins according to rank structure of the raw ensemble (Schefzik et al., 2013).
2. $\textbf{ECC}^2$ for MEMOS: Obtain k ECC ensembles each of size m by **repeatedly drawing from MEMOS posterior**, yields ECC ensemble of size $k \cdot m$.
3. Benchmark approach: **Assume independence** of the margins of the multivariate distribution.

3 Results

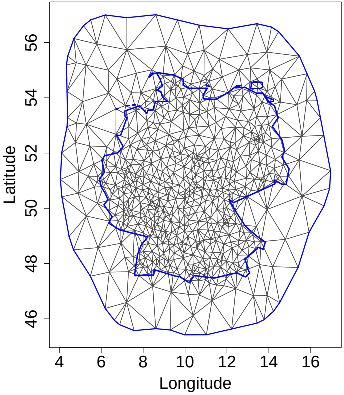
Case study: ECMWF Temperature Forecasts

- **Temperature data for Germany**, time period February 2010 - April 2011
- 50-member (exchangeable) **European Centre for Medium-Range Weather Forecasts (ECMWF) ensemble** (Buizza, 2006)
- Daily **24-h ahead forecasts**, initialized 00 UTC
- Observations provided by German Weather Service (DWD), **518 stations for temperature**

Observation Stations and Mesh



Observation stations

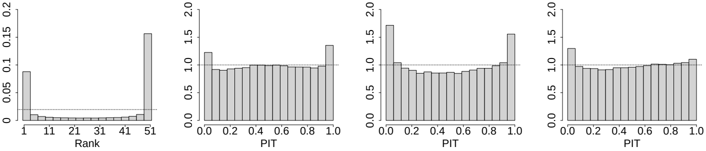


Corresponding Mesh

Univariate Overall Performance

Results aggregated over all stations and dates in March 24, 2010 - April 30, 2011.

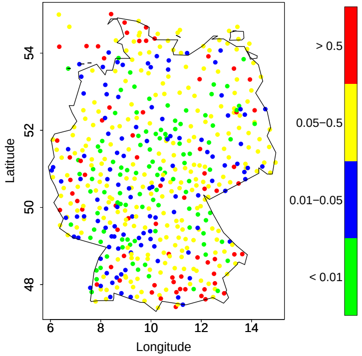
	CRPS	**MAE**	**RMSE**
Raw ECMWF	2.498	2.807	3.762
Global EMOS	1.792	2.489	3.242
Local EMOS	1.415	1.964	2.551
MEMOS	1.402	1.966	2.545



Raw ECMWF Global EMOS Local EMOS MEMOS

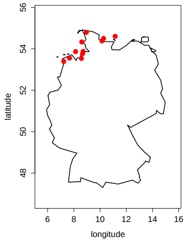
Station-wise (one-sided) Diebold-Mariano tests of CRPS

p-value	< 0.01	$0.01 - 0.05$	$0.05 - 0.50$	> 0.50
♯ stations	102	109	238	69



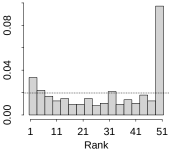
Multivariate Performance, North Sea Coast

11 stations along the North Sea coastline, aggregated over March 24, 2010 - April 30, 2011.

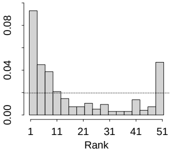


ES	Independence	ECC
Raw ECMWF	6.330	6.372
Global EMOS	5.406	5.239
Local EMOS	4.797	4.742
MEMOS	4.798	4.659

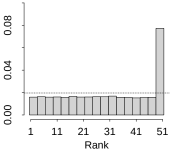
Multivariate Rank Histograms, North Sea Coast



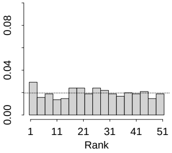
Raw ECMWF Ind



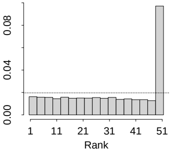
Raw ECMWF ECC



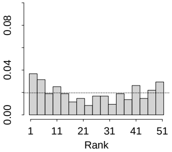
Global EMOS Ind



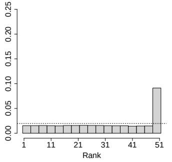
Global EMOS ECC



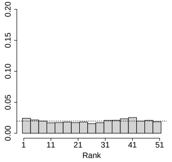
Local EMOS Ind



Local EMOS ECC



MEMOS Ind



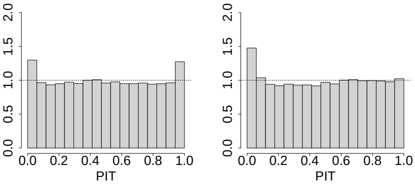
MEMOS ECC

Performance at out-of-sample Stations

- Randomly **sample 10 out of the 518 stations**, remove them completely from training data.
- Fit model on training data (containing 508 stations), **predict at out-of-sample stations.**
- **Repeat process 10 times**, aggregate results over all out-of-sample stations in the 10 runs, and all days in the period March 24, 2010 - April 30, 2011.

Overall out-of-sample Predictive Performance

	CRPS	MAE	RMSE
Global EMOS	1.77	2.47	3.17
MEMOS	1.50	2.09	2.70



Global Emos Memos

4 Outlook

Summary & Outlook

- MEMOS **highly competitive** to state-of-the-art (local) EMOS.
- Advantage over local EMOS: **spatially out-of-sample prediction** possible at arbitrary locations.
- Combining postprocessing models with **ECC yields improvement over independence** approach.
- **Future Work**:
  - Investigate **choice of mesh, tuning of mesh parameters** to improve MEMOS properties.
  - Further investigations show **assumption of constant variance** not too reasonable, incorporate/implement spatially varying variance within SPDE-INLA framework.
  - Implement procedure for other non-Gaussian variables (wind speed, precipitation).
  - Test models on a data set containing longer (and more recent) time period.

THANK YOU FOR YOUR ATTENTION!

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