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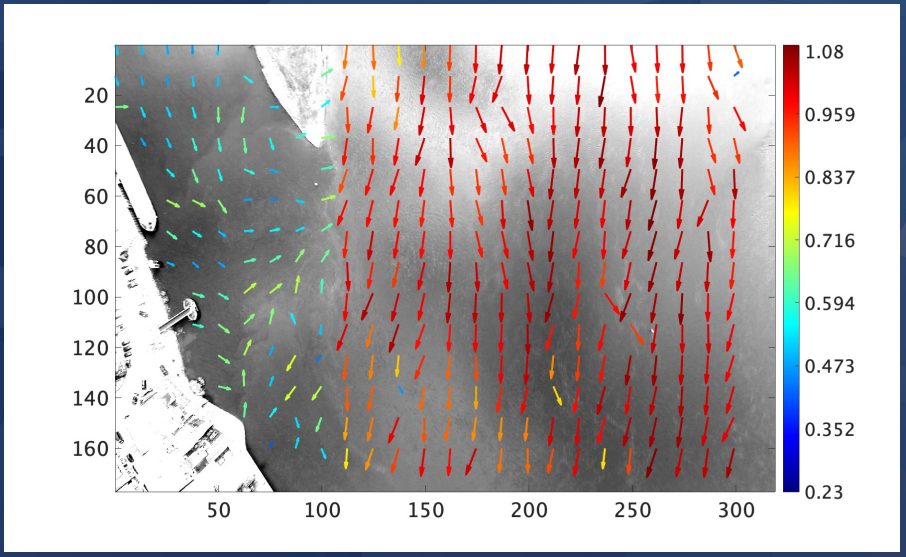
Measuring Instantaneous Velocity Fields Remotely using a Two- Dimensional Power Spectral Density Technique



Erika D. Johnson

EGU General Assembly Meeting
26 May 2022

3D PSD computational procedure calculates the FFT on a stack of images.

Computational Algorithm

Image pre-processing & rectification

↓

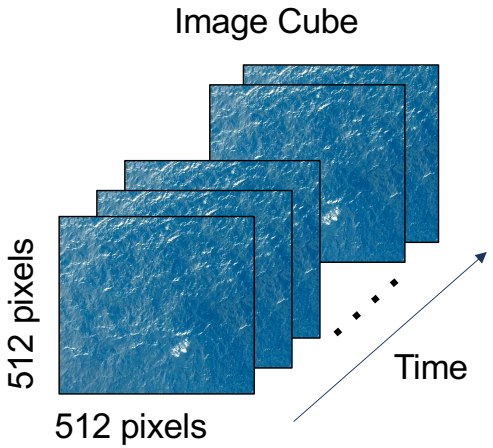
Create 3D cube of images

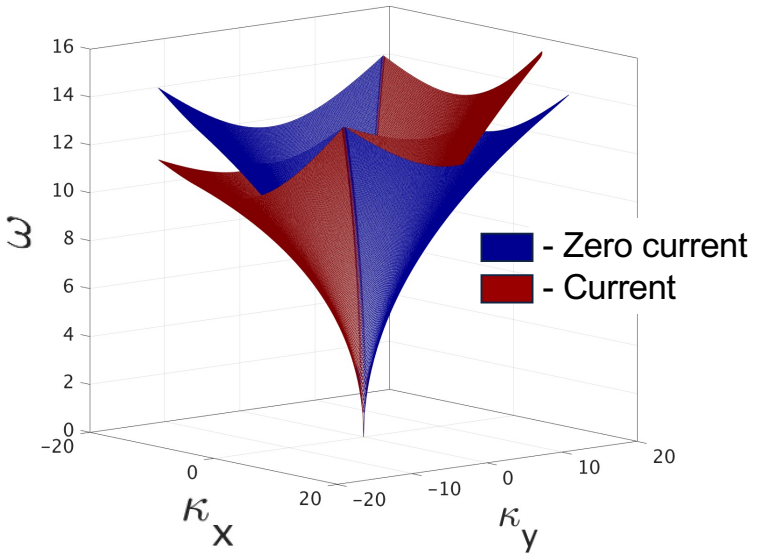
↓

Fourier Transform of Image cube

↓

Doppler-shifted Dispersion Surface





Dispersion relation with current:

$$\omega_e = \sqrt{gk\tanh(kh)} + \vec{U}\cdot\vec{k}$$

Deep water approximation:

$$\omega_e = \sqrt{gk} + \vec{U}\cdot\vec{k}$$

$$\omega_e - \sqrt{gk} = \vec{U}\cdot\vec{k}$$

Ref: Dugan, et. al. (2001), Piotrowski & Dugan (2002).

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2D PSD computational procedure calculates the FFT on a PAIR of images.

Computational Algorithm

Image pre-processing & rectification

↓

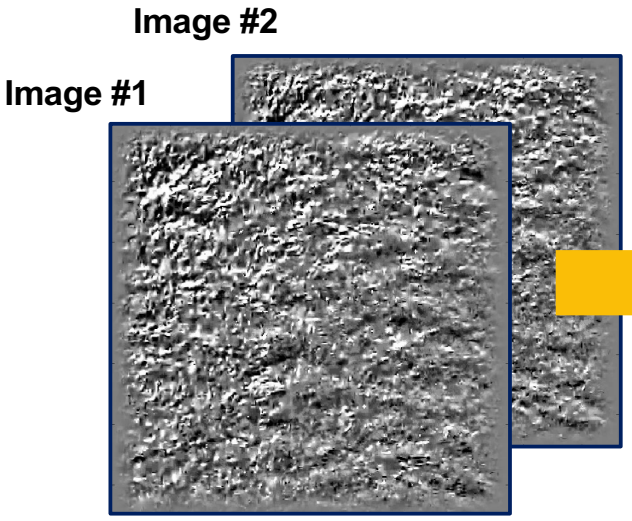
Calculate co-spectrum of image pair

↓

Subtract cross-spectral phase from theoretical phase

↓

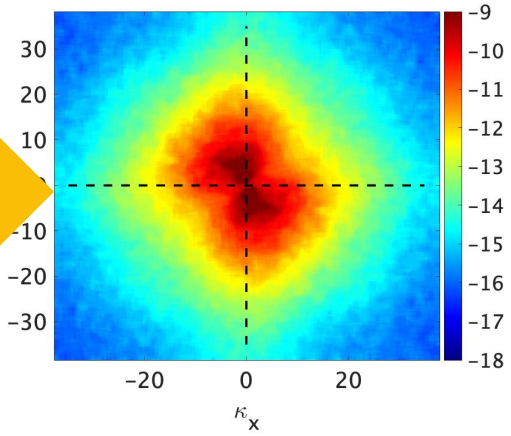
Least-squares to determine velocity components



Δt=0.4 s

2D FFT

Co-spectrum of image pair



Ref: Abileah (2013), Abileah & Trizna (2010), Yurovskaya, et al. (2019).

2D PSD computational procedure calculates the FFT on a PAIR of images.

Cross-spectral phase: $\phi(\vec{\kappa}) = \arg[S_{ac}(\vec{\kappa})]$

Computational Algorithm

Image pre-processing & rectification

↓

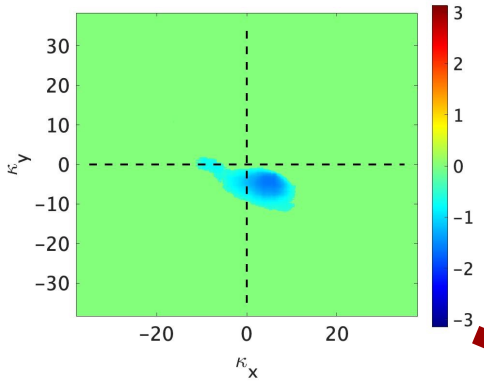
Calculate co-spectrum of image pair

↓

Subtract cross-spectral phase from theoretical phase

↓

Least-squares to determine velocity components



$$\omega = \sqrt{g\kappa \tanh(\kappa h)} + \vec{\kappa}\vec{U}$$

$$c = \frac{\omega}{\kappa} = \sqrt{\frac{g}{\kappa}\tanh(\kappa h)} + \frac{\vec{\kappa}}{\kappa}\vec{U}$$

$$c = \frac{\phi(\kappa, \varphi)}{\kappa \Delta t}$$

$$\frac{\phi(\vec{\kappa})}{\kappa \Delta t} - \sqrt{\frac{g}{\kappa}} = \vec{U}$$

Ref: Abileah (2013), Abileah & Trizna (2010), Yurovskaya, et al. (2019).

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2D PSD computational procedure calculates the FFT on a PAIR of images.

Cross-spectral phase: $\phi(\vec{\kappa}) = \arg[S_{ac}(\vec{\kappa})]$

Theoretical phase $\sqrt{\frac{g}{\kappa}}$

Computational Algorithm

Image pre-processing & rectification

↓

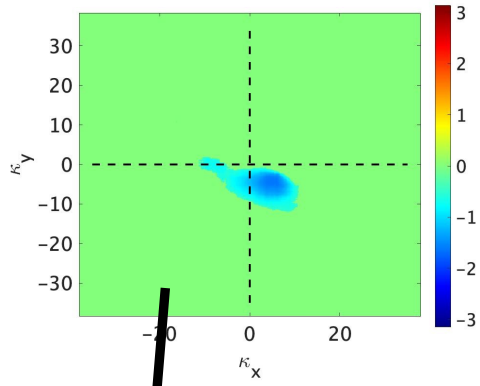
Calculate co-spectrum of image pair

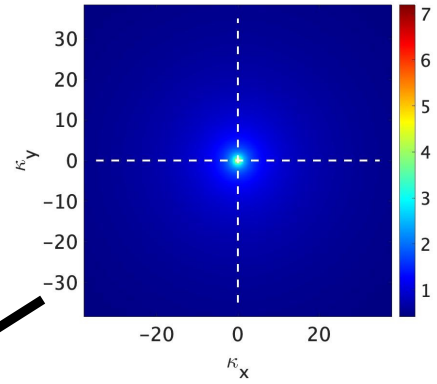
↓

Subtract cross-spectral phase from theoretical phase

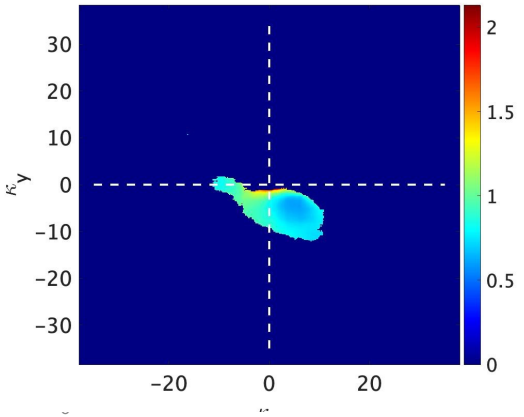
↓

Least-squares to determine velocity components





$$\frac{\phi(\vec{\kappa})}{\kappa \Delta t} - \sqrt{\frac{g}{\kappa}} = \vec{U}$$



Ref: Abileah (2013), Abileah & Trizna (2010), Yurovskaya, et al. (2019).

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Drone images from Streßer, et. al. (2017) taken over the Elbe river in Lauenburg, Germany were used in this analysis.

Altitude: 204 m
Sample rate: 24 fps
Record length: 60 sec
Spatial Res: 8 cm/pix
Data cube:150 x 150 pixels

Ref: Streßer, et. al. (2017)

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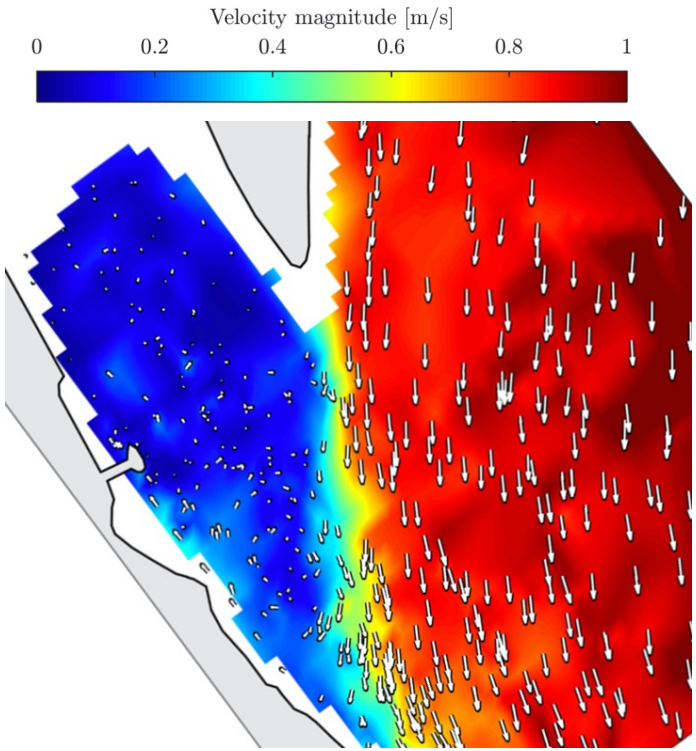
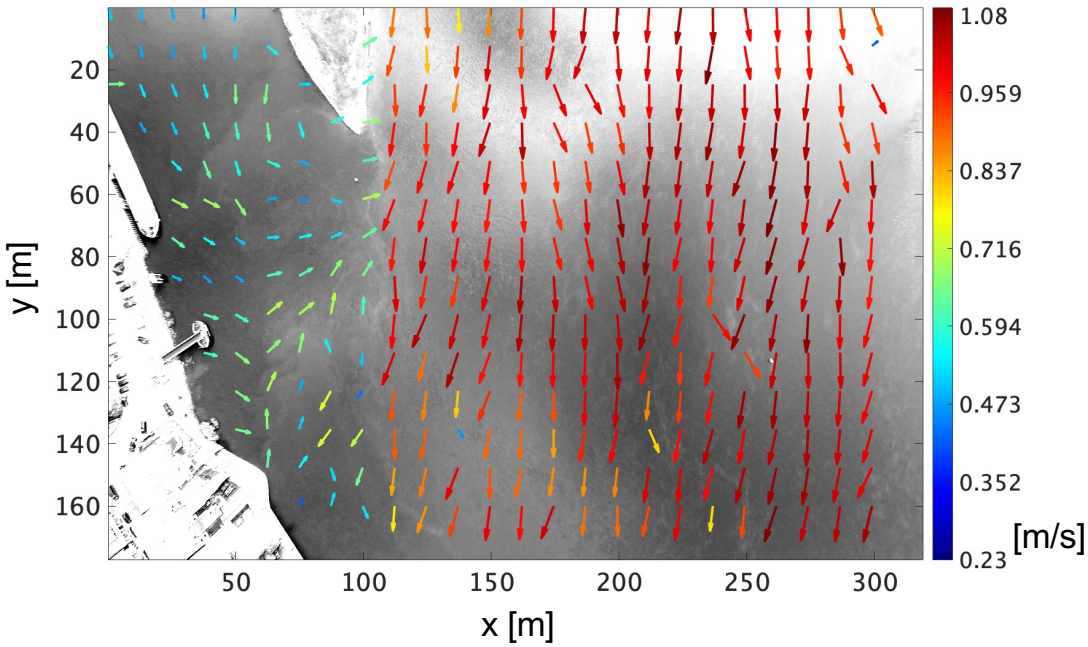
Our results compare favorably with the results from Streßer, et. al. (2017).

Mean velocity measured by ADCP results from Streßer, et. al. (2017).



Mean velocity magnitude results from 2D PSD algorithm



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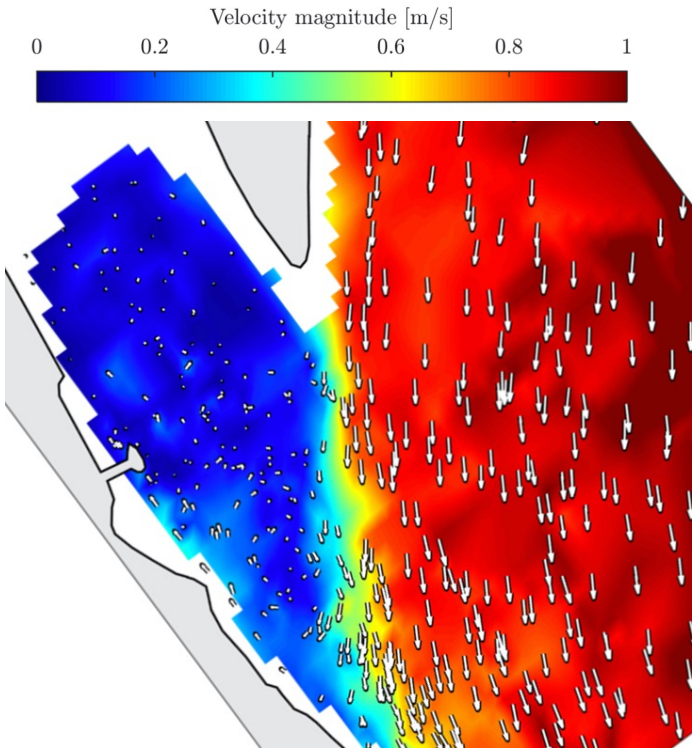
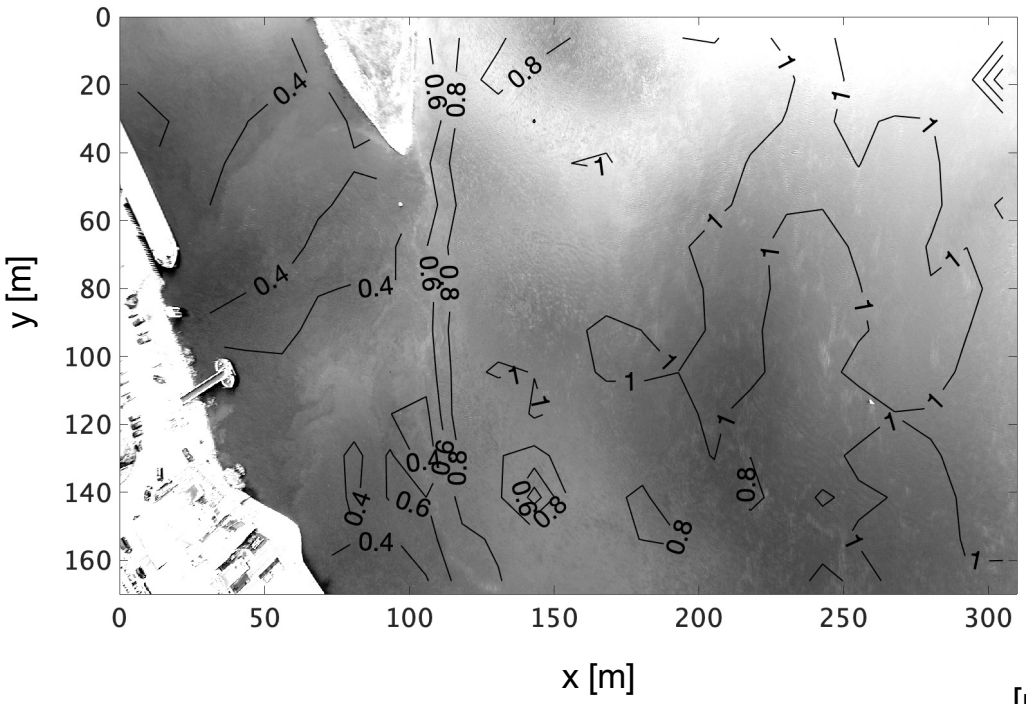


Velocity magnitude results from 2D PSD algorithm



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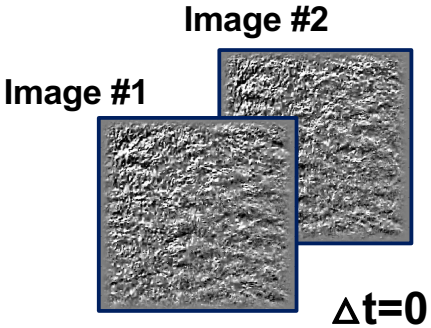
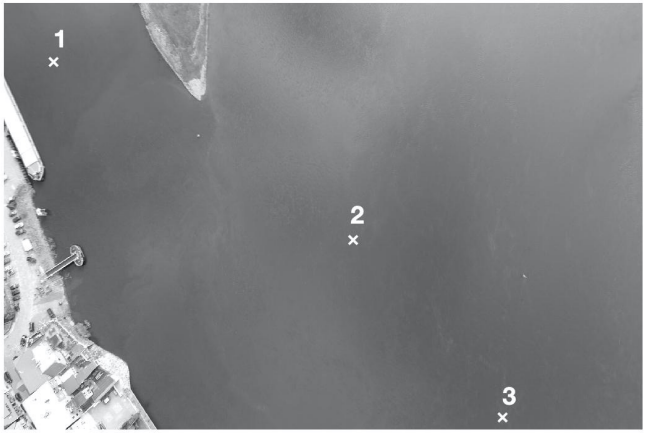
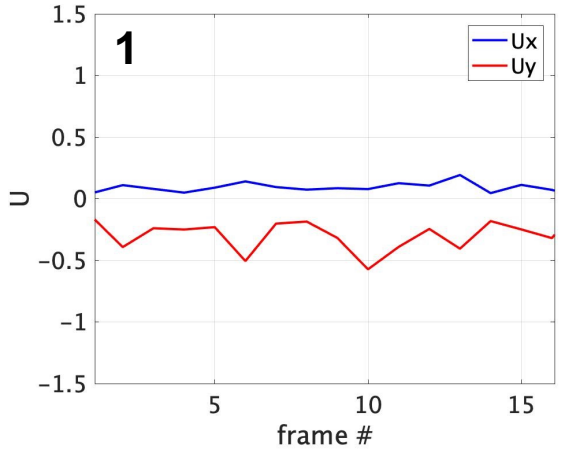
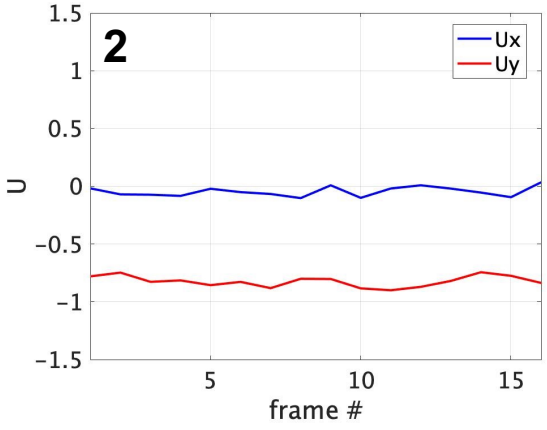
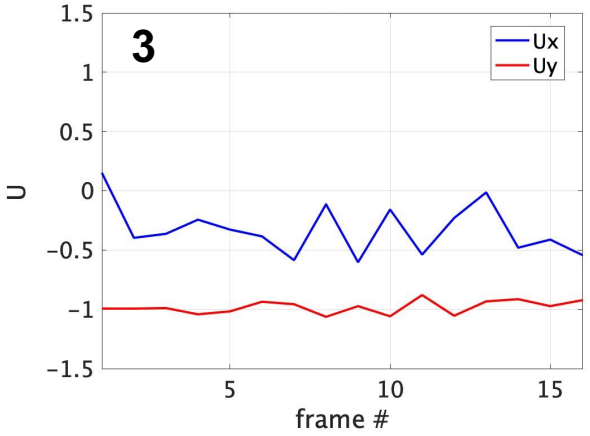
The 2D PSD technique can be used to measure velocity spectra, time series and sub surface velocity profiles.











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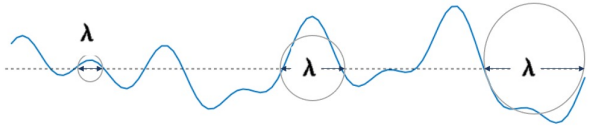
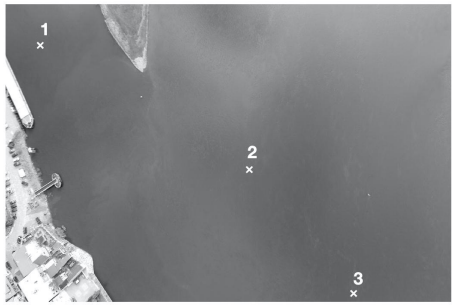
The 2D PSD technique can be used to measure velocity spectra, time series and sub surface velocity profiles.

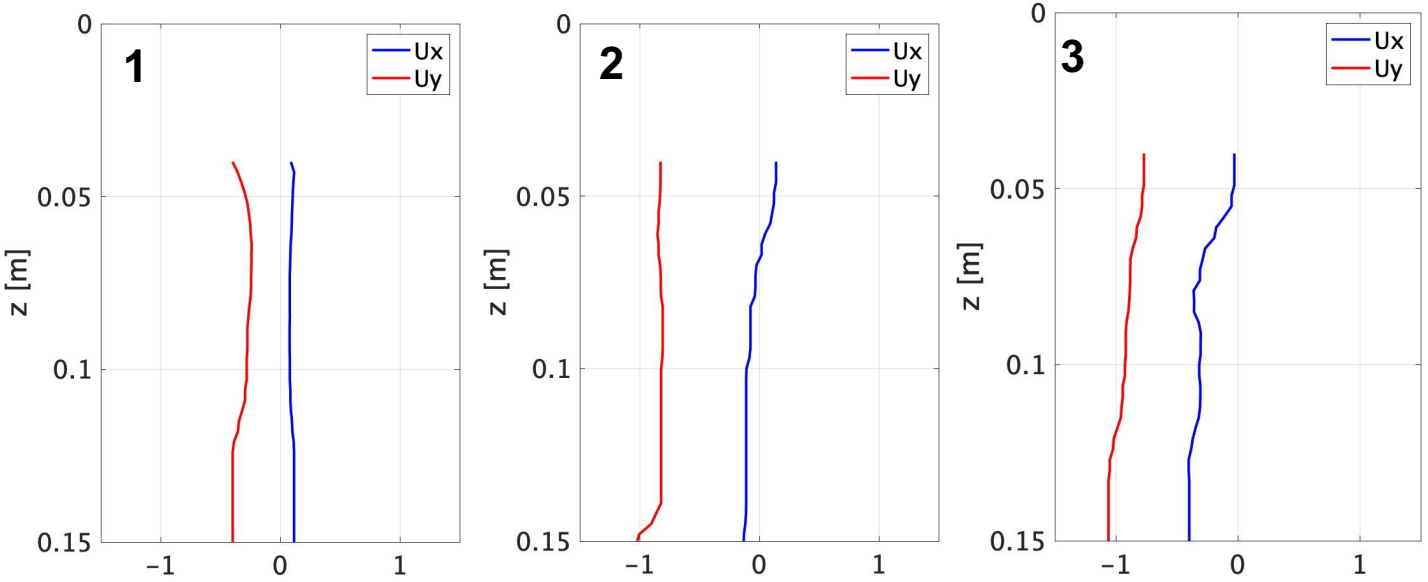




$$U(k) = 2k \int_{-d}^{o} U(z) e^{2kz}\, dz,$$

$$d_m \simeq \frac{1}{2k} = \frac{\lambda_o}{4\pi}.$$

Ref: Stewart & Joy(1974)



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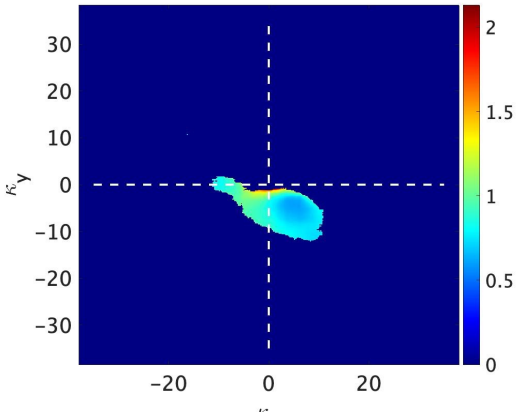
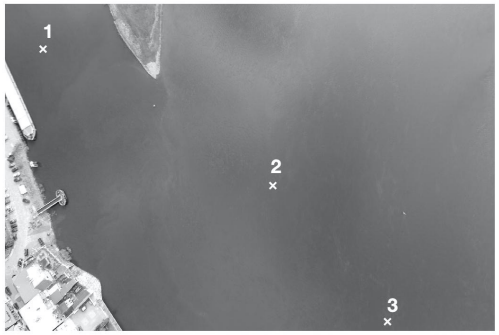
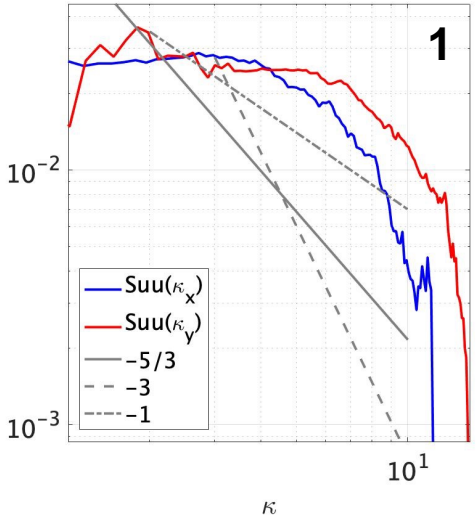
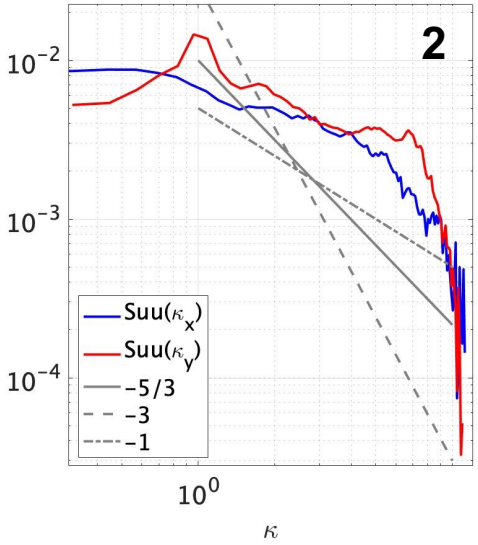
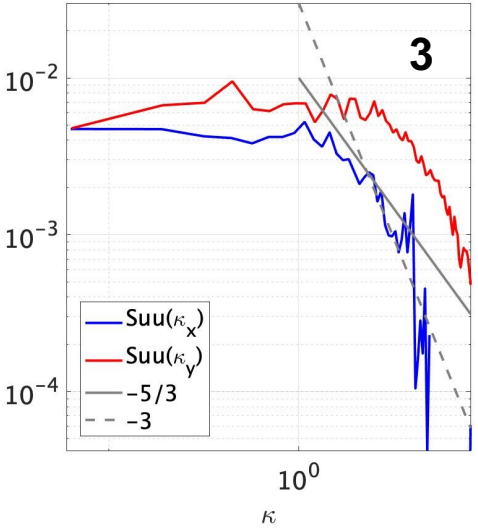
The 2D PSD technique can be used to measure velocity spectra, time series and sub surface velocity profiles.











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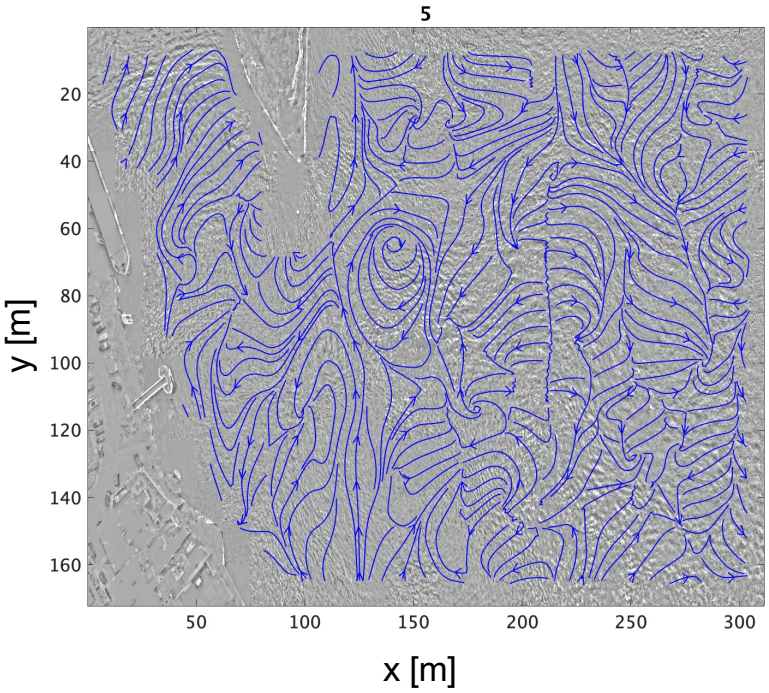
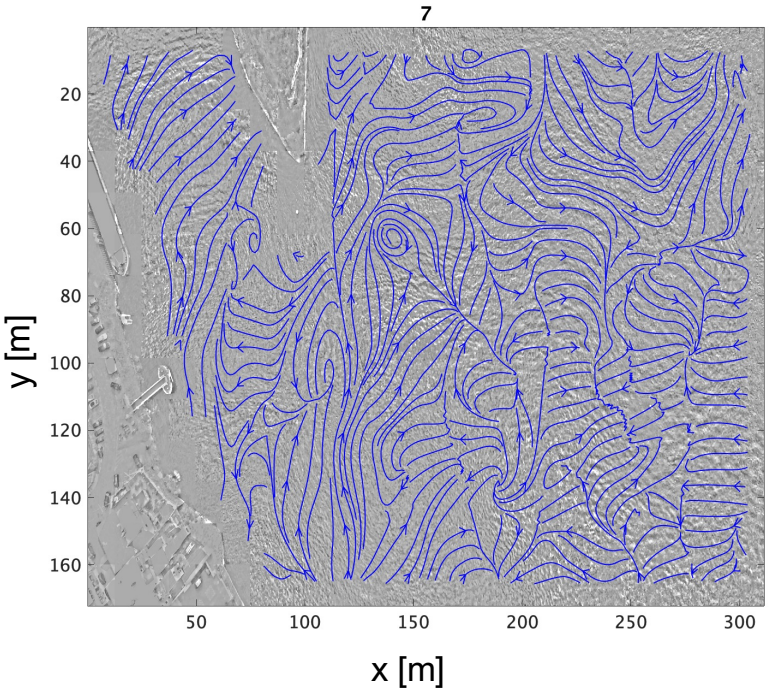
Instantaneous velocity fields reveal the presence of eddies.





The National Academies of SCIENCES ENGINEERING MEDICINE

Email: Erika.Johnson@nrl.navy.mil
Twitter: @erikaj_314159

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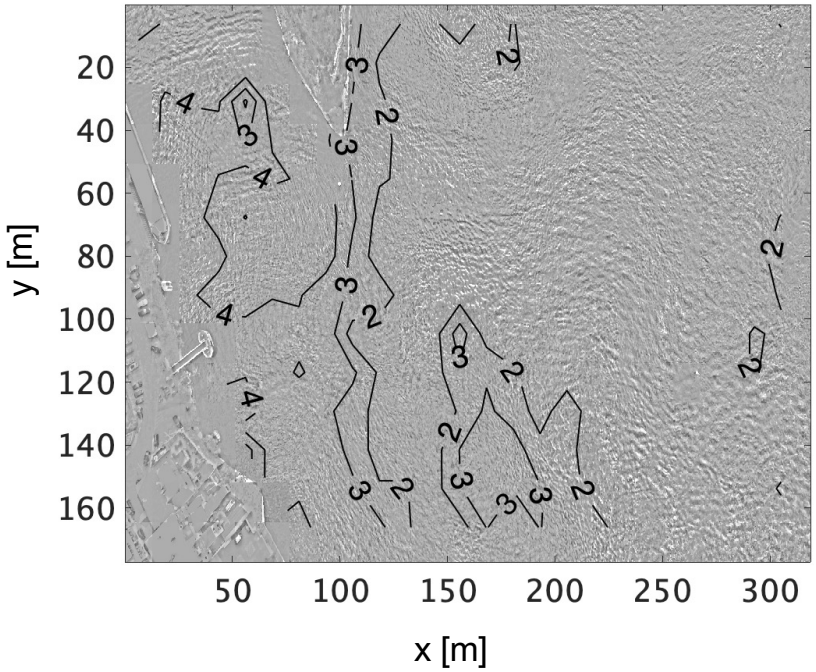
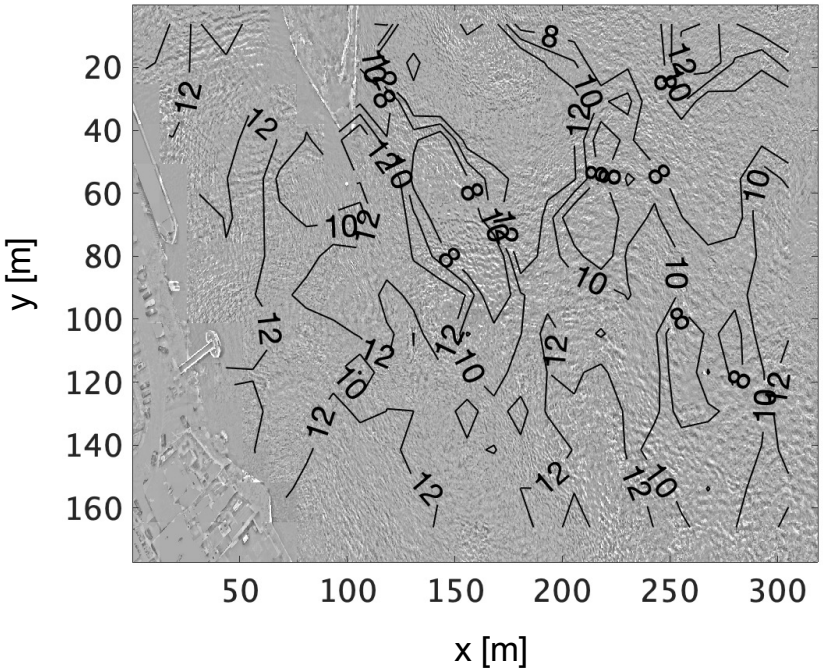
Δt was adjusted to accommodate for varying wave numbers across the river.

Lowest Wave Numbers Present



Highest Wave Numbers Present



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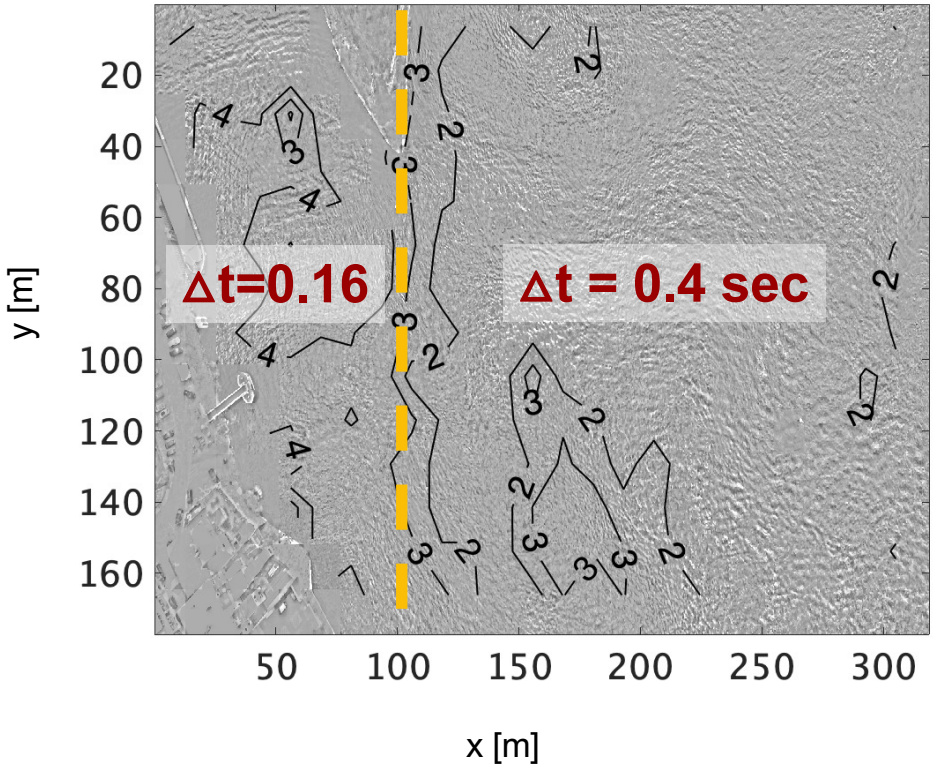
Δt was adjusted to accommodate for varying wave numbers across the river.

Lowest Wave Numbers Present



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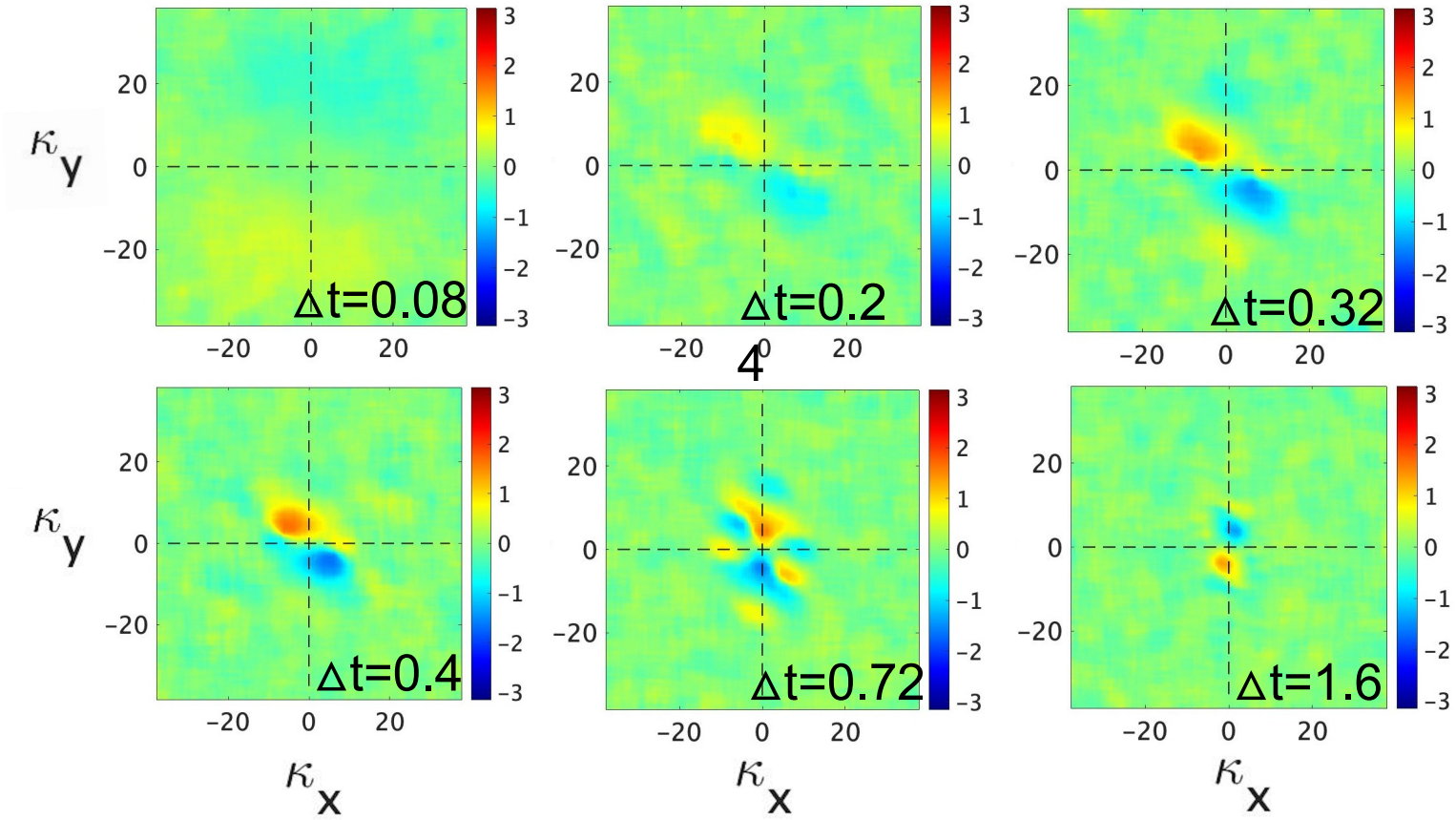
Δt was adjusted to accommodate for varying wave numbers across the river.



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Δt was adjusted to accommodate for varying wave numbers across the river.