

A Wavenumber–Frequency Spectrum Model for Sheared Convective Atmospheric Boundary Layer Flows

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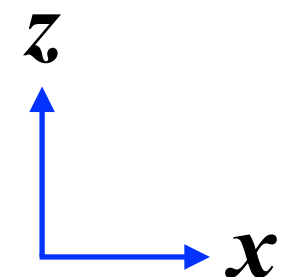
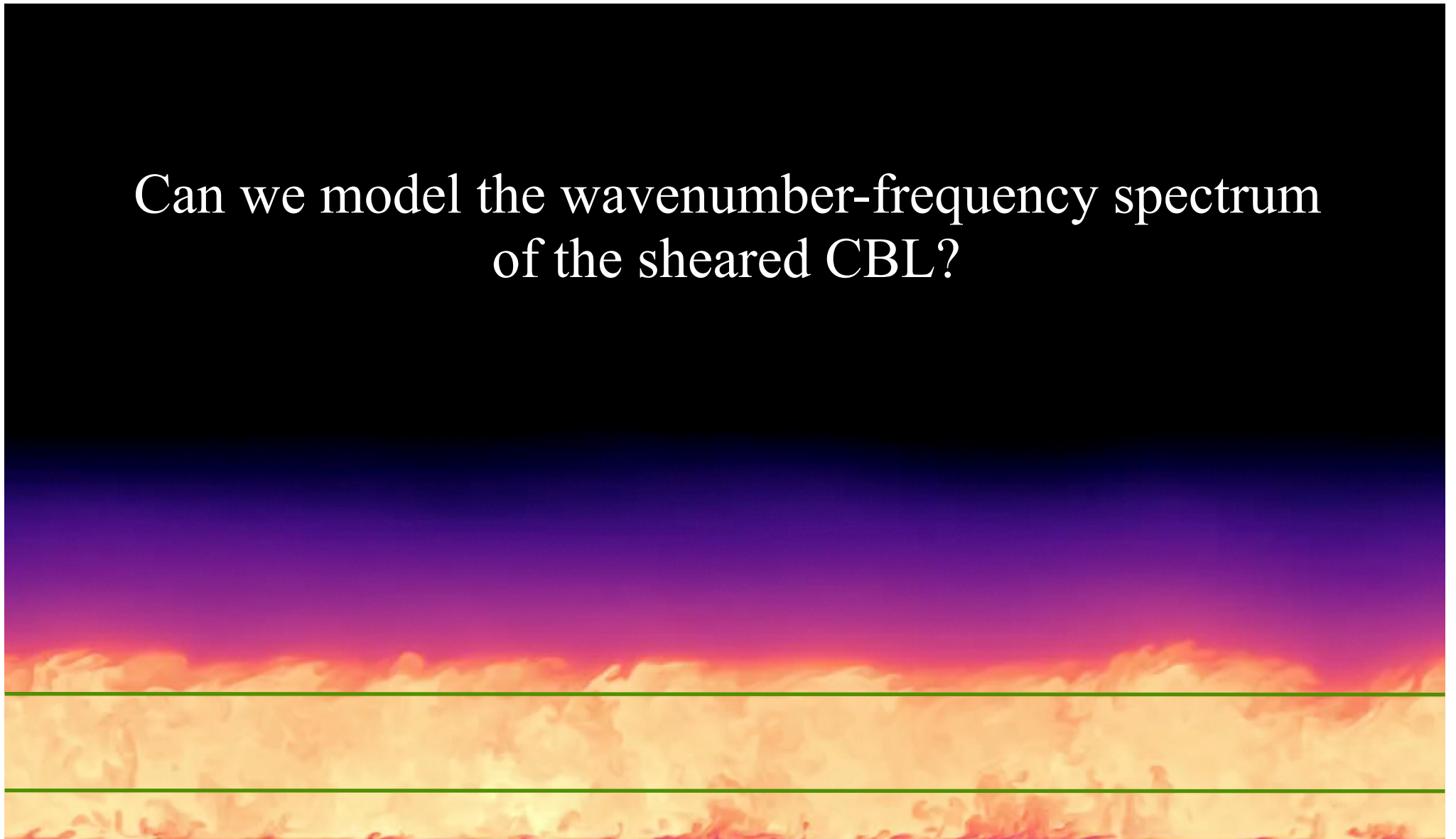
Convective Boundary Layer (CBL)

Vertical Cross-Section of the Buoyancy: Direct Numerical Simulation (DNS)

Can we model the wavenumber-frequency spectrum of the sheared CBL?

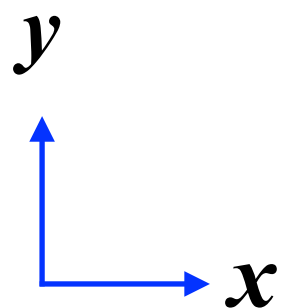
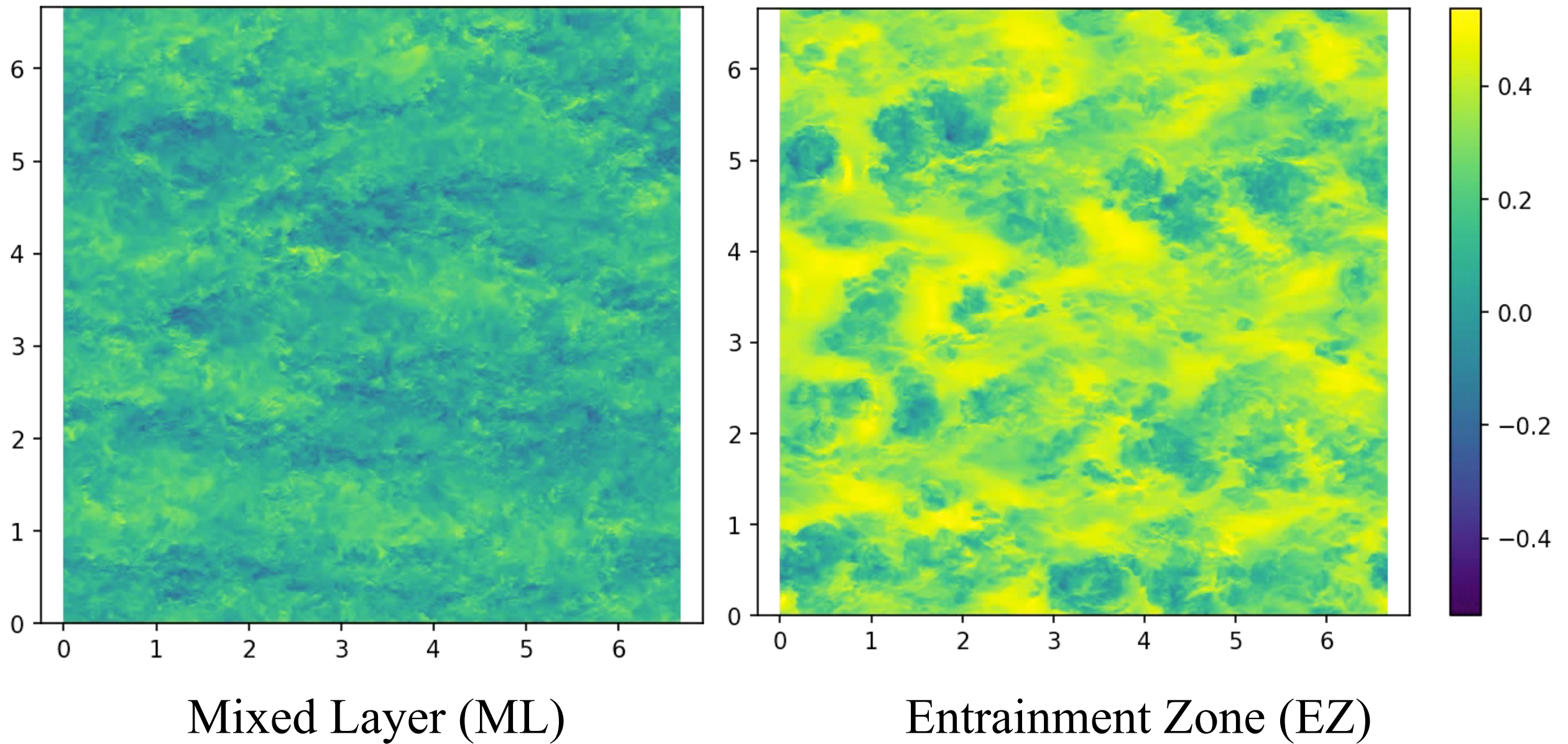
Entrainment Zone

Mixed Layer

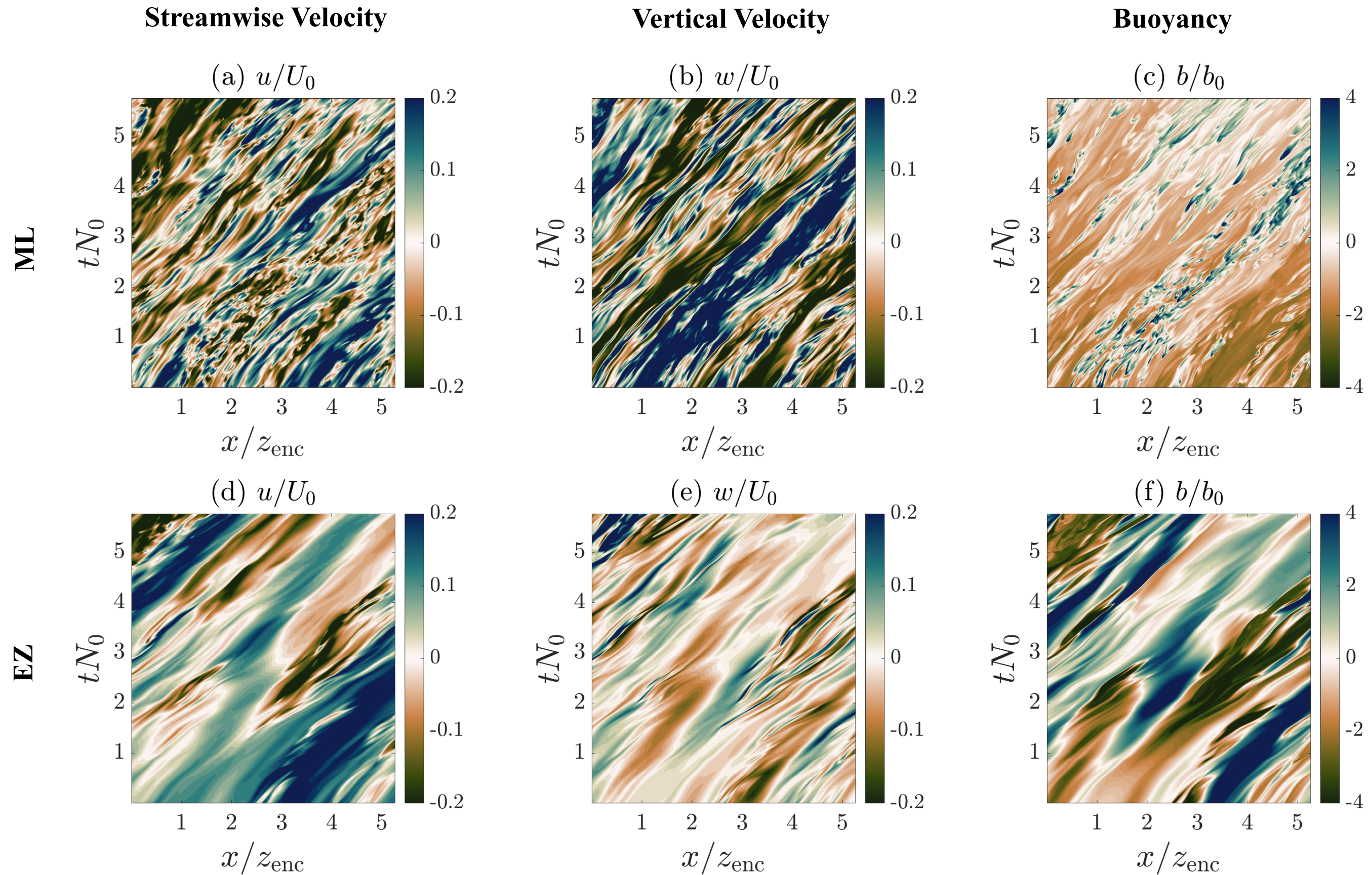


Convective Boundary Layer: Mixed Layer and Entrainment Zone

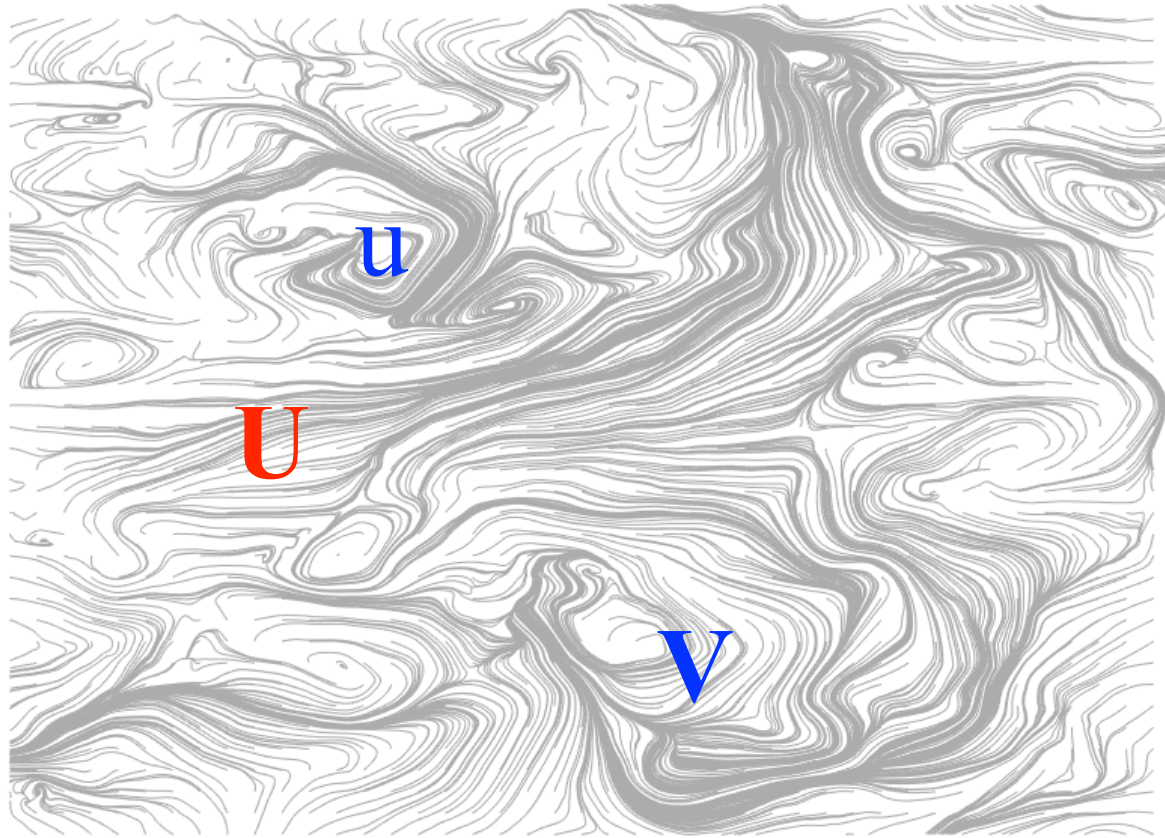
Horizontal Cross-Section of the Buoyancy



Numerical Simulations: space-time structure



Linear Random Advection Model (LRAM)



$$\frac{\partial}{\partial t} u + (\mathbf{U} + \mathbf{V}) \cdot \nabla u = 0$$

$$E_u(\mathbf{k}, \omega; z) = \frac{E_u(\mathbf{k}; z)}{[2\pi \langle (\mathbf{V} \cdot \mathbf{k})^2 \rangle]^{1/2}} \exp \left[- \frac{(\omega - \mathbf{k} \cdot \mathbf{U})^2}{2 \langle (\mathbf{V} \cdot \mathbf{k})^2 \rangle} \right]$$

Wavenumber Spectrum

Doppler Shift

Doppler Broadening

Fourier Transform
Gaussian Large Scale Advection

$E_w(\mathbf{k}, \omega; z)$ ✓

$E_b(\mathbf{k}, \omega; z)$ ✓

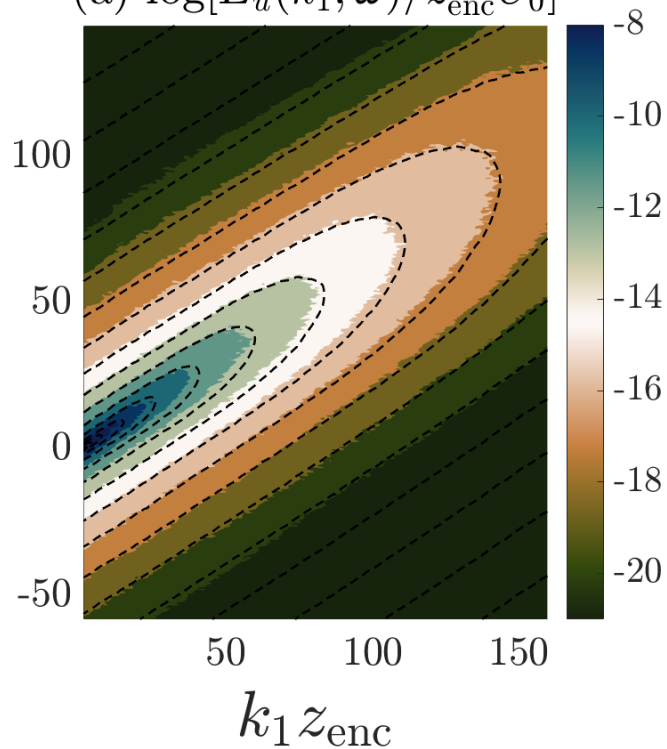
Wavenumber-Frequency Spectra: LRAM vs. DNS

Streamwise Velocity

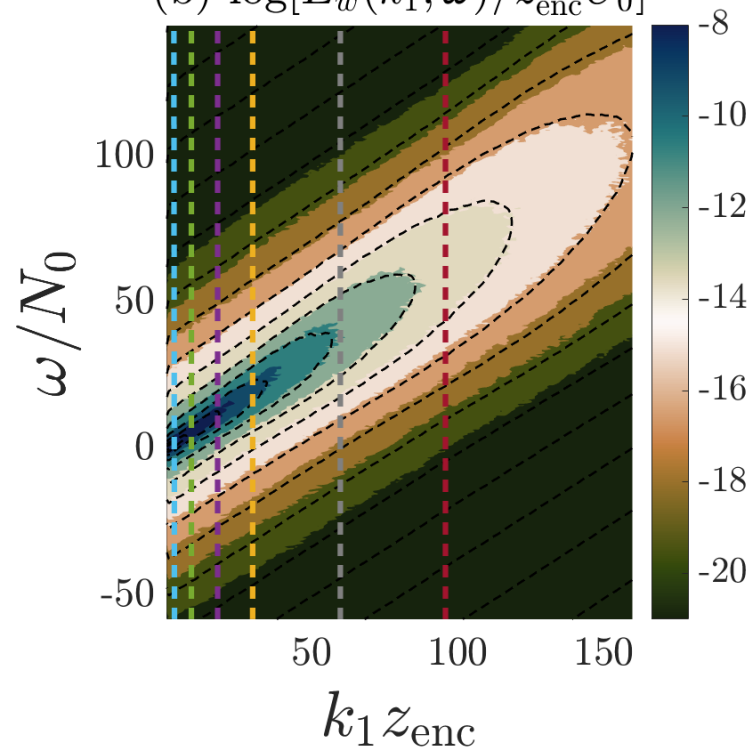
Vertical Velocity

Buoyancy

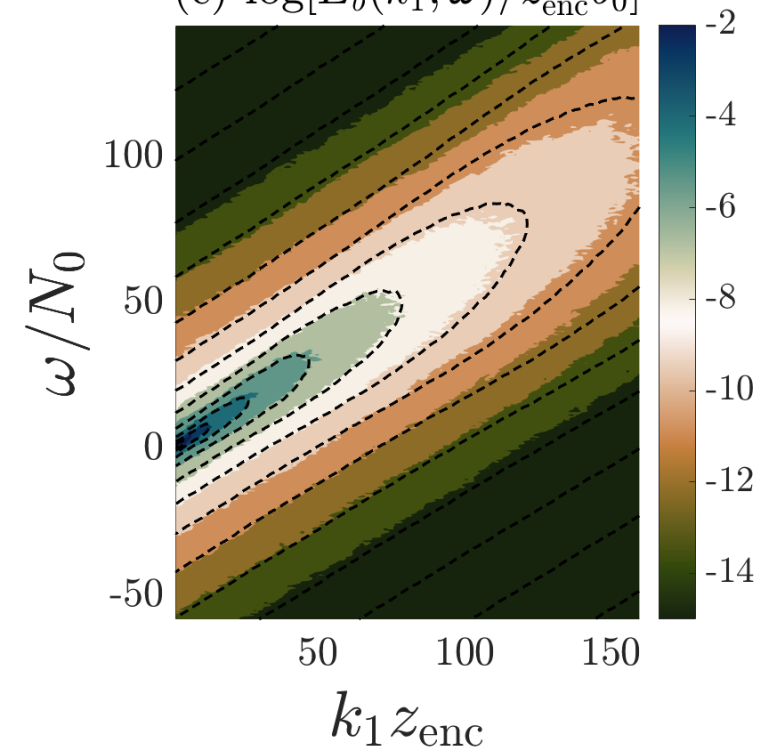
(a) $\log[E_u(k_1, \omega)/z_{\text{enc}}^2 U_0]$



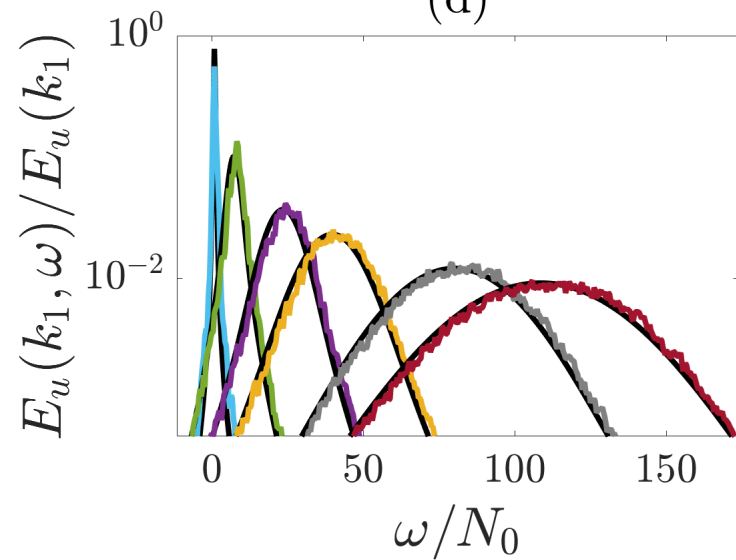
(b) $\log[E_w(k_1, \omega)/z_{\text{enc}}^2 U_0]$



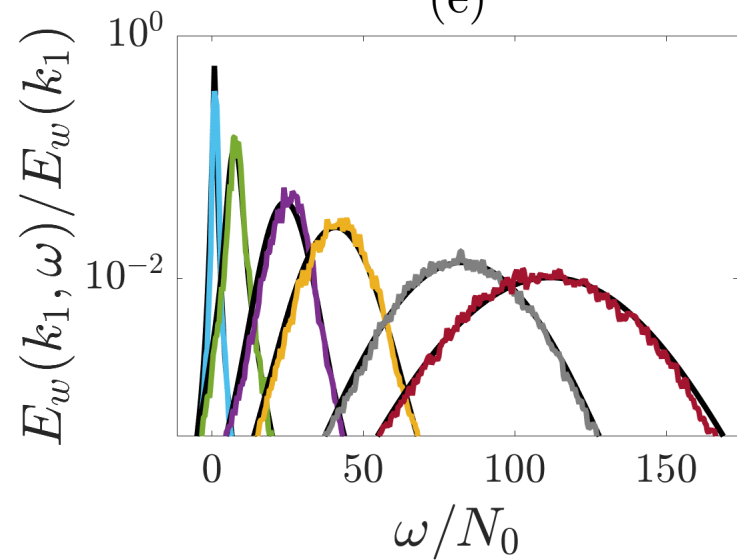
(c) $\log[E_b(k_1, \omega)/z_{\text{enc}}^2 b_0]$



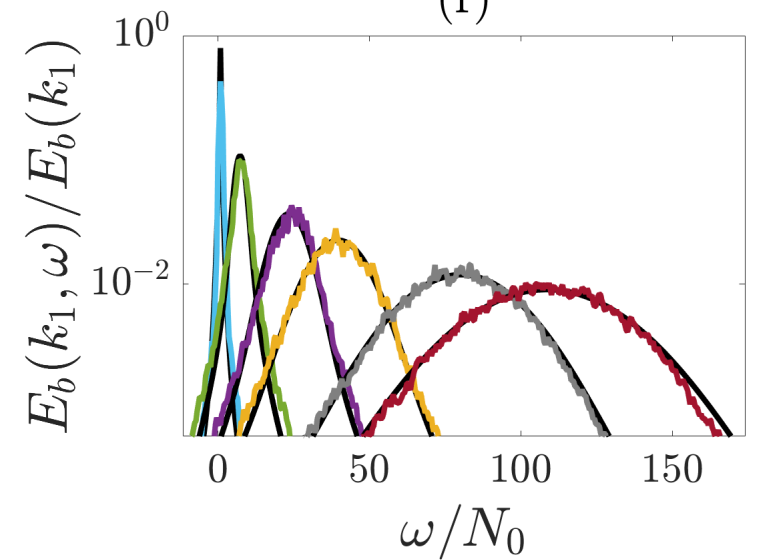
(d)



(e)



(f)



Wavenumber Spectrum Model: von Kármán approach vs. DNS

$$E(k) = \frac{A_1 (k/k_e)^4}{[1 + (k/k_e)^2]^{17/6}},$$



$$\phi_{ij}(\mathbf{k}) = \frac{E(k)}{4\pi k^2} \left[\delta_{ij} - \frac{k_i k_j}{k^2} \right],$$

$$\int_{-\infty}^{\infty} \phi_{ij} dk_3.$$

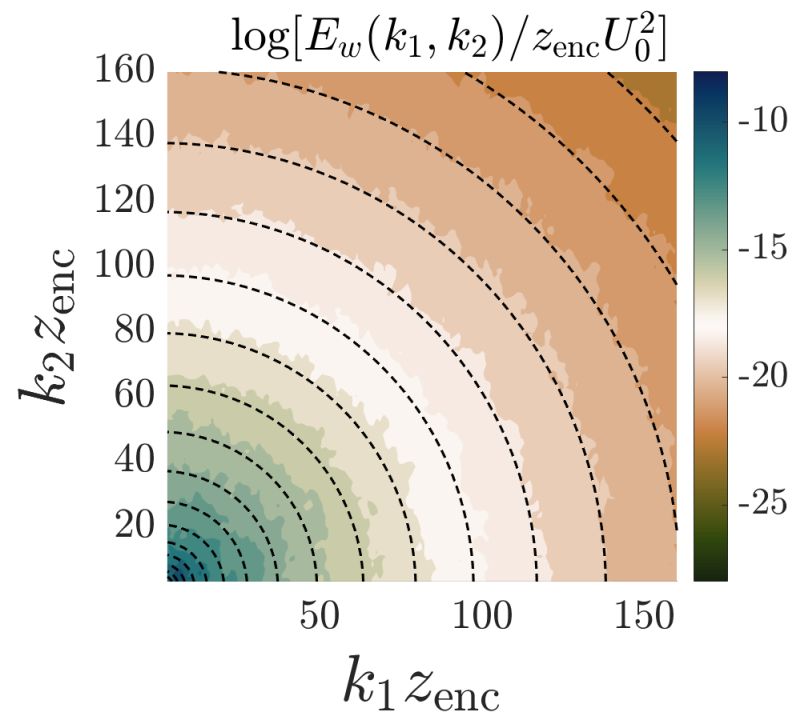


$$E_w(k_1, k_2) = \frac{4}{9\pi} \frac{C_w w_*^2}{k_e^2} \frac{\left[\left(\frac{k_1}{k_e} \right)^2 + \left(\frac{k_2}{k_e} \right)^2 \right]}{\left[1 + \left(\frac{k_1}{k_e} \right)^2 + \left(\frac{k_2}{k_e} \right)^2 \right]^{7/3}},$$

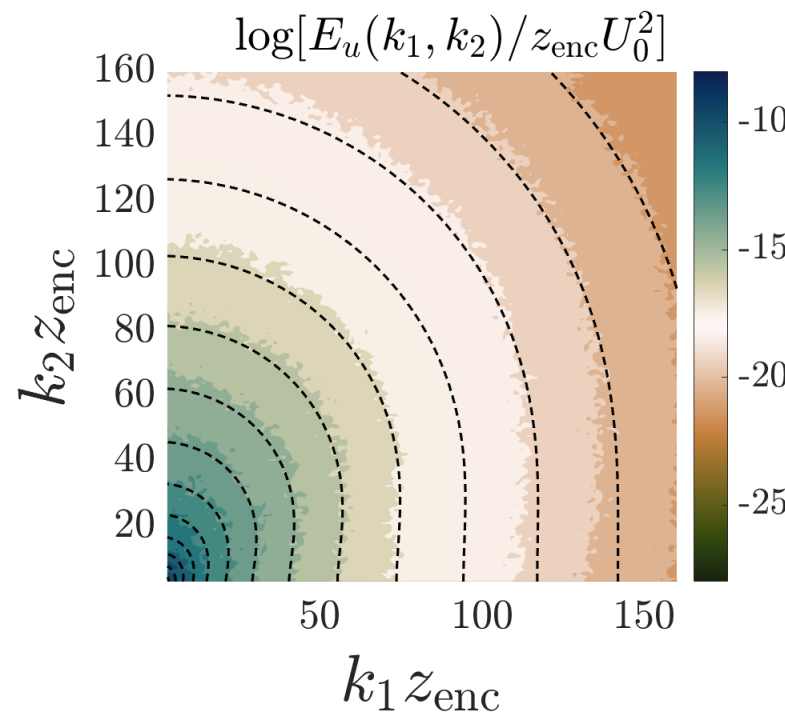
$$E_u(k_1, k_2) \quad \checkmark$$

$$E_b(k_1, k_2) \quad \checkmark$$

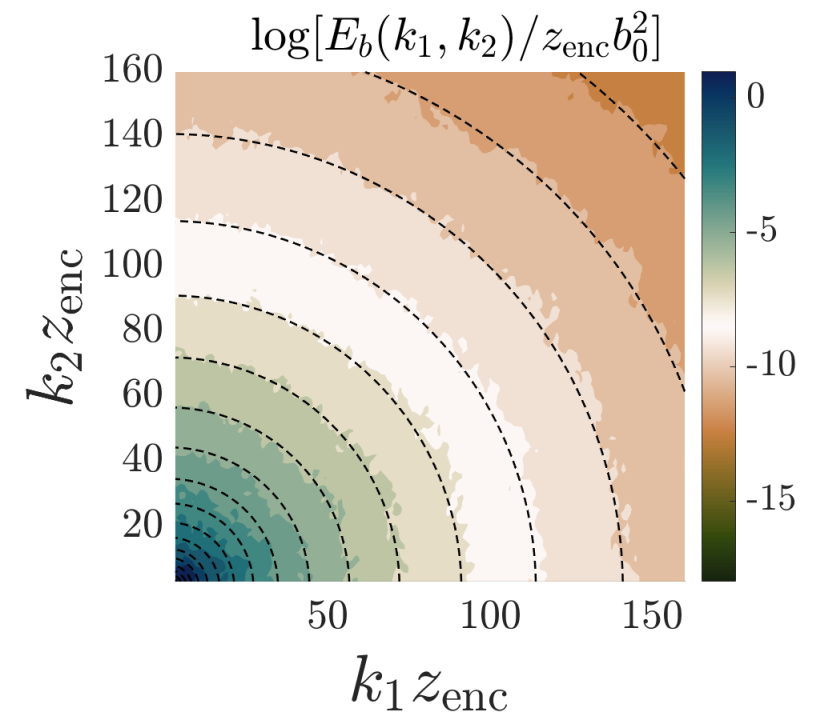
Vertical Velocity



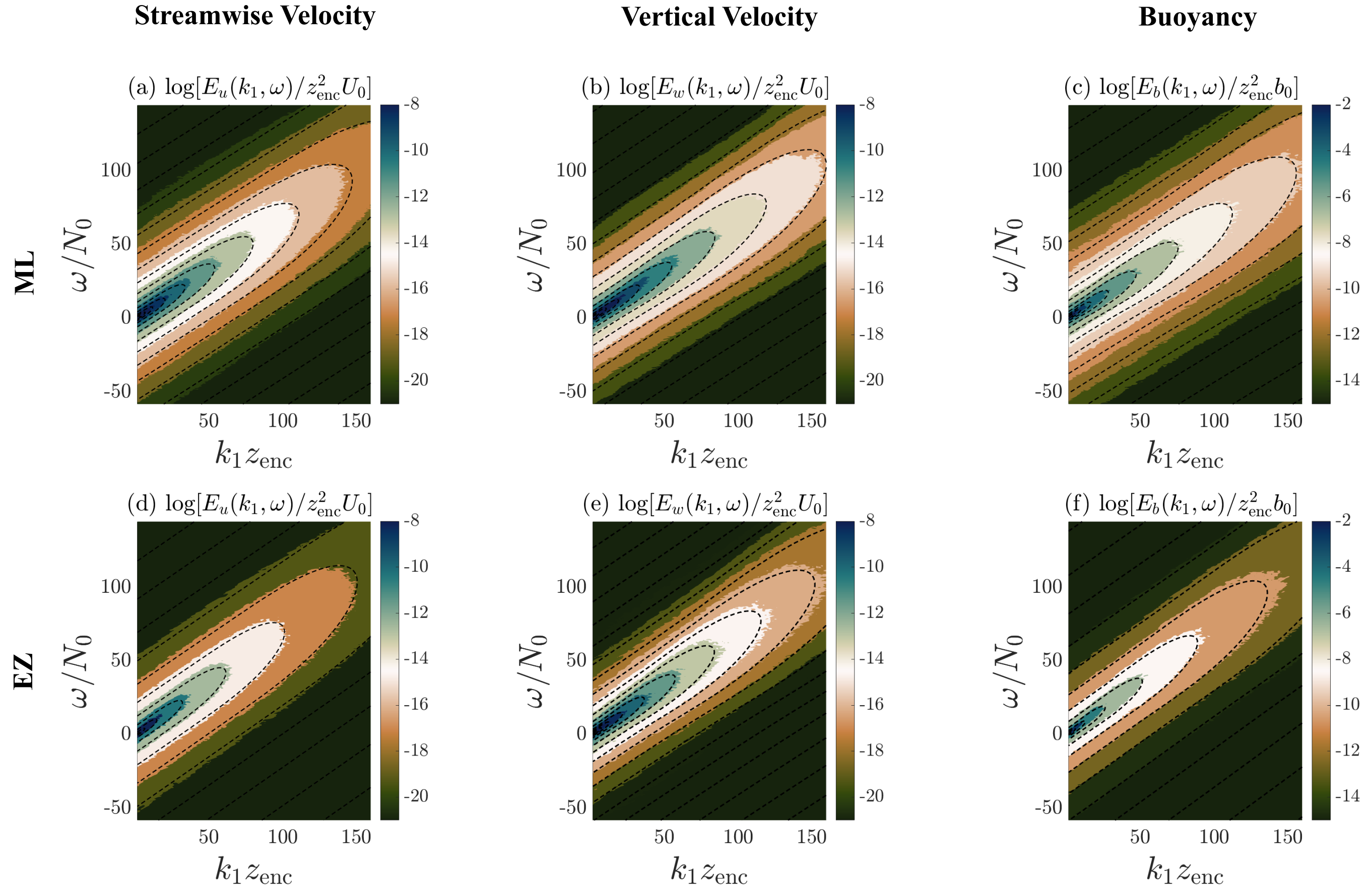
Streamwise Velocity



Buoyancy



Wavenumber-Frequency Spectra: von Kármán-based LRAM vs. DNS



Conclusions

- We applied a linear random advection approach to model the wavenumber-frequency spectra for sheared CBL flows.
- We used a von Kármán spectrum to parameterize two-dimensional wavenumber spectra for velocity components and buoyancy.

