Modelling magneto-hydro-static equilibria in quiet Sun regions

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Magneto-Hydro-Statics (MHS)

$$\mathbf{j} \times \mathbf{B} = \nabla P + \rho \nabla \Psi$$

$$\nabla \times \mathbf{B} = \mu_0 \mathbf{j}, \ \nabla \cdot \mathbf{B} = 0$$

- ✓ MHS is a special class of MHD in equilibrium where the Lorentz-force is compensated by the pressure gradient force and gravity force.
- ✓ In the generic form the equations are nonlinear and even numerically difficult to solve. Several algorithms have been developed.
- ✓ High resolution vector magnetograms are required as boundary condition, which are available only for a few cases in active regions.
- ✓ Special classes of solutions allow a linearization of the MHS equations => Loss of generality, but easier to solve.

Linearized MHS-equations (Low 1991)

$$\nabla \times \mathbf{B} = \alpha \mathbf{B} + a \exp(-\kappa z) \nabla B_z \times \mathbf{e_z}$$

Linear force-free part

Nonmagnetic forces decrease with height

- ✓Linear MHS-equation can be solved with Fast Fourier Transform.
- ✓Require only line-of-sight magnetic field as boundary condition.
- ✓The solution has three free parameters: alpha, a, kappa
- ✓In active regions we compute the free parameters from vector magnetograms.
- ✓As vector magnetograms are not available In the quiet Sun => search for the optimum set of parameters by comparing MHS-solutions with observations.

How to obtain free parameters in Active regions?
=> use vector magnetograms

$$\nabla \times \mathbf{B} = \alpha \mathbf{B} + a \exp(-\kappa z) \nabla B_z \times \mathbf{e_z}$$

$$\alpha = \frac{\sum \left(\frac{\partial B_y}{\partial x} - \frac{\partial B_x}{\partial y} \right) \mathrm{sign}(B_z)}{\sum |B_z|}$$

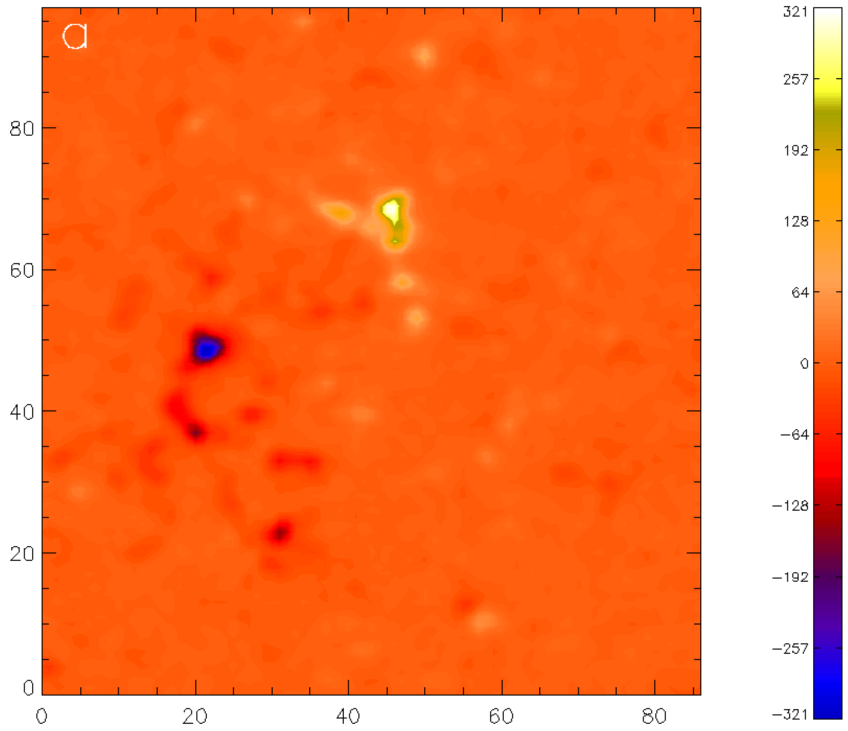
Formula for linear force-free fields
(Hagino & Sakurai, 2004)

$$\oint T \mathbf{ds}$$

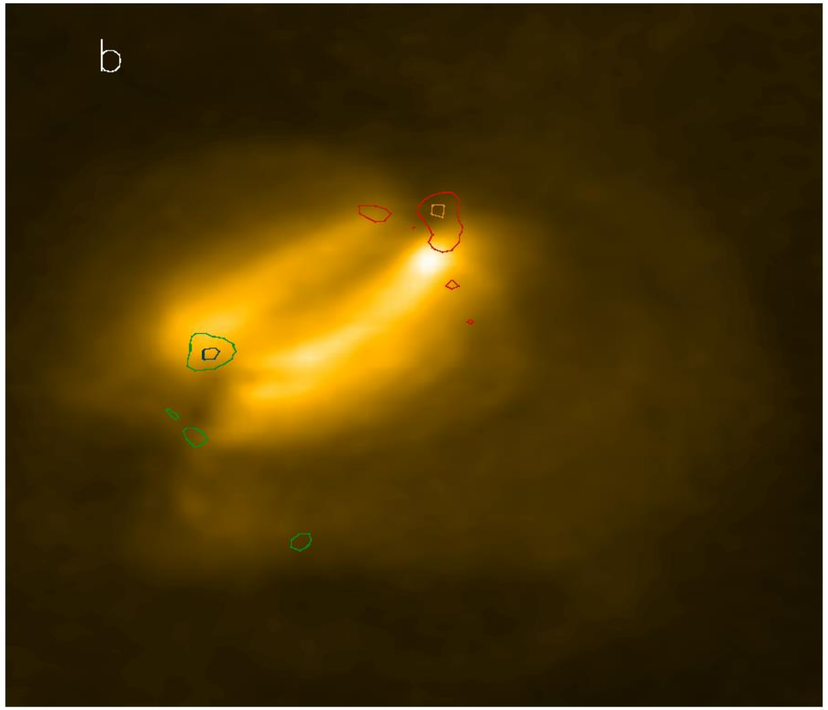
Write Maxwell Stress Tensor
(Forces) in components
Wiegelmann et al. 2017)

$$a = \frac{\left|\sum B_x B_z\right| + \left|\sum B_y B_z\right| + \left|\sum (B_x^2 + B_y^2) - B_z^2\right|}{\frac{1}{2} \sum (B_x^2 + B_y^2 + B_z^2)}$$

How to obtain free parameters in the quiet Sun?



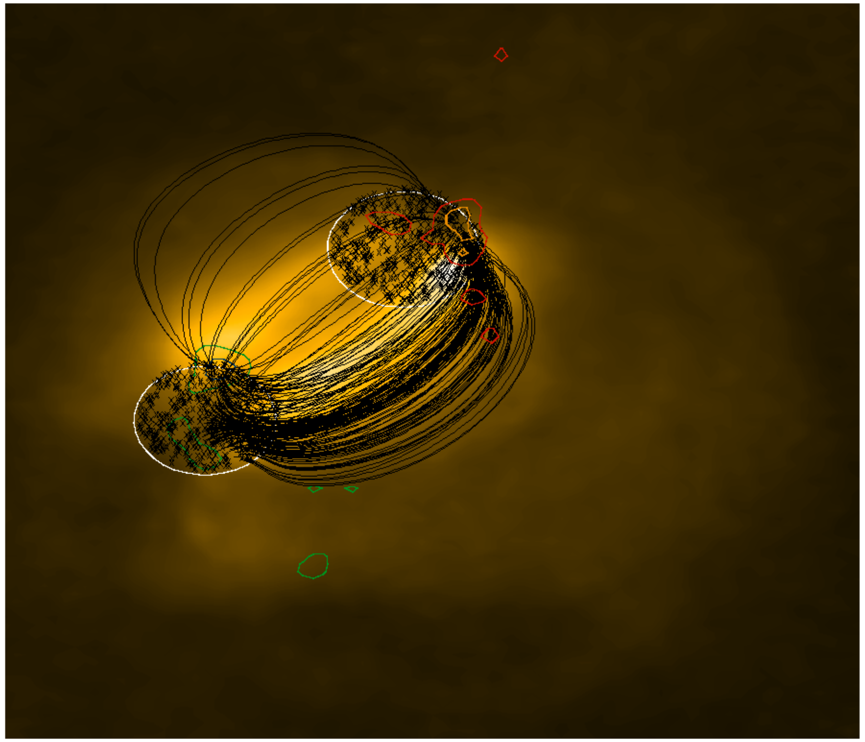
Use a line-of-sight magnetogram from SDO/HMI as boundary condition for linear MHS-model

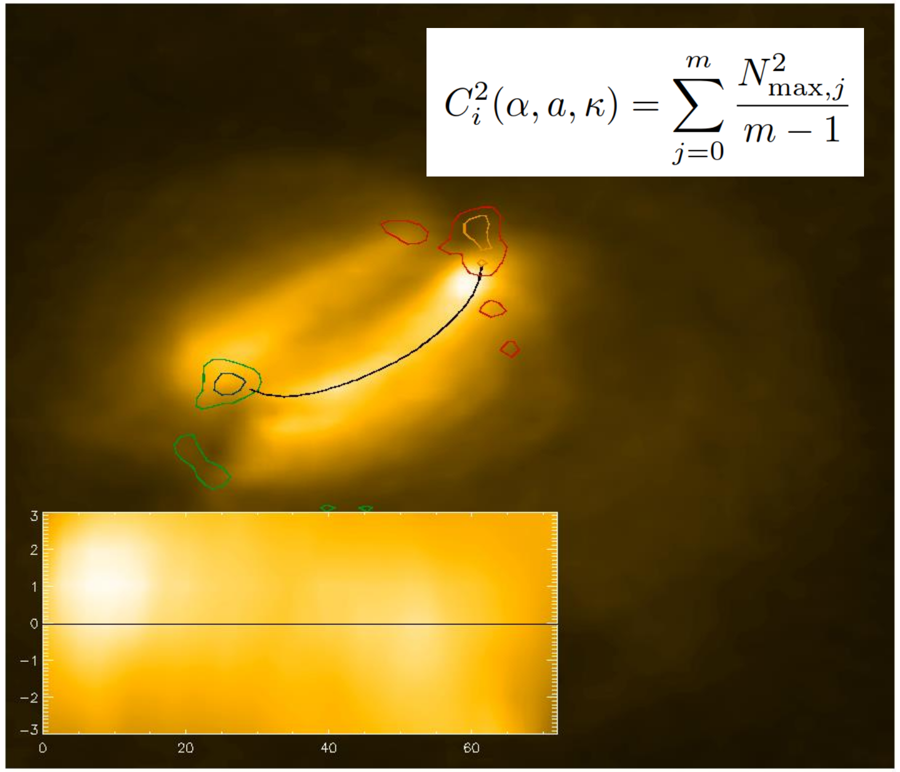


- ✓ SDO/AIA in the Fe XII 193 Å channel (T ∽ 1MK)
- ✓ Compare our model with the image to constrain alpha, a, kappa

How to compare magnetic field model and images?

Modified version from Carcedo et. al. 2003





- ✓ Compute many loops from footpoint areas (white circles) of coronal loops.
- ✓ Consider only closed loops connecting the footpoints.
- ✓ Uncurl the loops and use Gaussfit to quantify how well a field line and a coronal loop agree.
- ✓ Find minimum with Simplex-Downhill iteration.

Conclusions

- ✓ Special classes of magneto-hydro-static equilibria are useful when vector magnetograms are not available (e.g. quiet Sun).
- ✓ We developed a method to automatically find footpoint areas of magnetic loops and by an uncurling method we can compare the magnetic model and the coronal image.
- ✓ A Simplex Downhill method finds the optimum set of the 3 free parameters (alpha, a, kappa).
- ✓ If the image shows several loops, the optimum parameter set is usually different for each structure.