Treatment of Noise in GRACE Gravity Field Recovery

A Comparison between Empirical Parameterization and Stochastic Modelling

Yufeng Nie[1], Yunzhong Shen[1], Roland Pail[2], and Qiujie Chen[1]

1. College of Surveying and Geo-Informatics, Tongji University, Shanghai, China
2. Institute of Astronomical and Physical Geodesy, Technical University of Munich, Munich, Germany

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Noise-reduction Methods Revisited

Dominant noise sources in the GRACE solution

- K-band range-rate measurement noise
- Kinematic orbit noise
- Accelerometer measurement noise
- Ocean tide model errors
- Imperfection in AOD product

⇨ Measurement noise

⇨ Dynamic force errors

Common methods for noise reduction in gravity recovery

Empirical parameterization

KBR method: range-rate *kinematic* empirical parameters

ACC method: high-frequency (constrained) *dynamic* empirical accelerations

COV method: fully-populated covariance matrix

FILT method: time-series model-based filtering

Stochastic modelling

Noise-reduction Methods Revisited

Theoretical connections among four methods

KBR method

$$\delta\dot{\rho} = A + Bt + (E + Ft)\cos n\,t + (G + Ht)\sin n\,t$$

LOS projection

$$\delta\dot{\rho} = \left(\delta\dot{S}_2 - \delta\dot{S}_1\right)\cos\theta/2 + \left(\delta\dot{R}_2 + \delta\dot{R}_1\right)\sin\theta/2$$

Linear perturbation

perturbed acc.

$$\mathbf{p} = a_R\mathbf{i}_R + a_S\mathbf{i}_S + a_N\mathbf{i}_N$$

ACC method

$$\mathbf{y} = \mathbf{Ax} + \mathbf{Bp} + \mathbf{e}$$
$$\mathbf{0} = \mathbf{p} + \mathbf{e}_p$$
$$E\left(\mathbf{e}_p\right) = \mathbf{0} \quad D\left(\mathbf{e}_p\right) = \sigma_p^2\mathbf{I}$$

⇐ $\#\mathbf{p} \ll \#\mathbf{s}$

Least-squares collocation

$$\mathbf{y} = \mathbf{Ax} + \mathbf{Bs} + \mathbf{e}$$
$$E(\mathbf{e}) = \mathbf{0} \quad D(\mathbf{e}) = \sigma_e^2\mathbf{Q}_e$$
$$E(\mathbf{s}) = \mathbf{0} \quad D(\mathbf{s}) = \sigma_s^2\mathbf{Q}_s$$

⇒ $\mathbf{e}^* = \mathbf{Bs} + \mathbf{e}$

FILT method

$$\mathbf{v}_i = \phi_1\mathbf{v}_{i-1} + \phi_2\mathbf{v}_{i-2} + \cdots + \phi_p\mathbf{v}_{i-p} + \boldsymbol{\epsilon}_i$$
$$\mathbf{Fy} = \mathbf{FAx} + \mathbf{Fe}$$

AR psd estimate

$$\hat{P}(f) = \frac{\sigma_\epsilon^2\Delta t}{1 - \sum_{k=1}^{p}\phi_k\exp(-j2\pi fk\Delta t)}$$

Spectral analysis

periodogram

$$\hat{P}(f) = \Delta t\sum_{\tau=-(N-1)}^{N-1}\hat{c}_\tau\exp(-j2\pi f\tau\Delta t)$$

COV method

$$\mathbf{y} = \mathbf{Ax} + \mathbf{e}^*$$
$$E(\mathbf{e}^*) = \mathbf{0}$$
$$D(\mathbf{e}^*) = \sigma_s^2\mathbf{BQ}_s\mathbf{B}^T + \sigma_e^2\mathbf{Q}_e$$

Numerical Simulations

Background force models

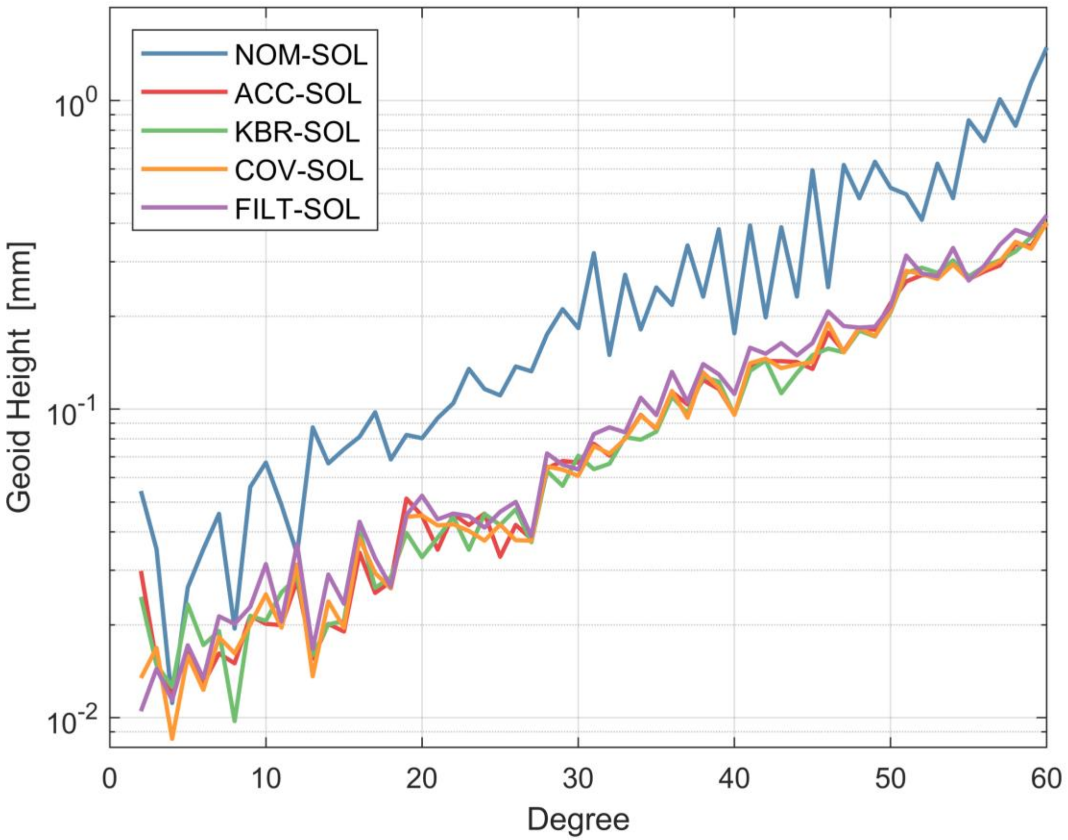
Model	*True*	*Reference Case-1*	*Reference Case-2*
Static field	GOCO05s	GOCO05s	EIGEN-6C4
Ocean tide	EOT11a	EOT11a	GOT4.7
Non-tidal signal	ESM AOHIS	ESM AOHIS	ESM DEAL & AOerr

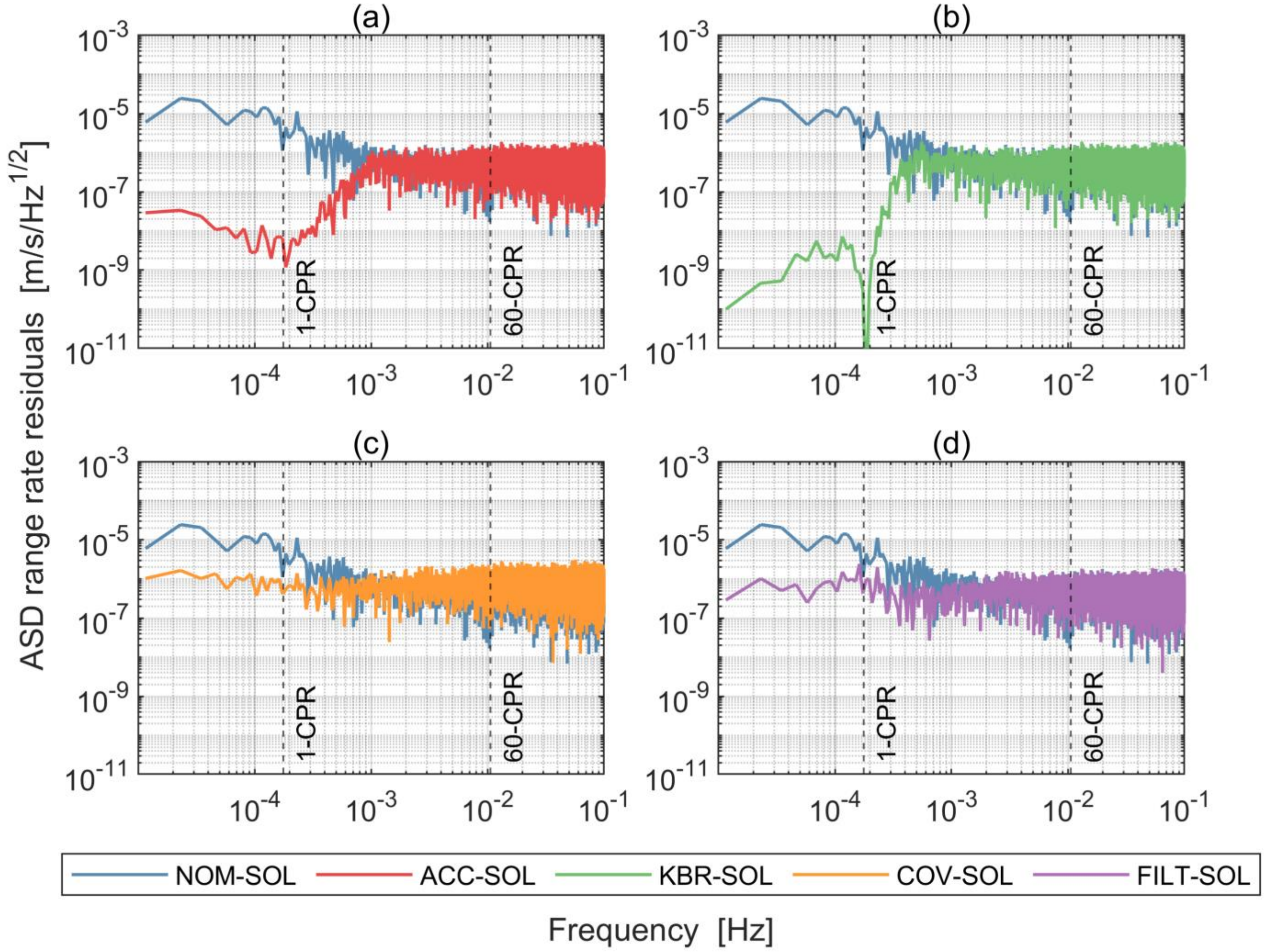
Noise models

Observation	*Case-1*	*Case-2*
Orbit positions	2 cm white noise	2 cm white noise
KBR range-rates	0.2 μm/s white noise	$1.8 \times 2\pi f$ μm/s/$\sqrt{\text{Hz}}$
Accelerometer	along & radial: 1×10^{-10} m/s^2 white noise cross: 1×10^{-9} m/s^2 white noise	along & radial: $(1+0.005/f)^{1/2} \times 10^{-10}$ m/s$^2/\sqrt{\text{Hz}}$ cross: $(1+0.1/f)^{1/2} \times 10^{-9}$ m/s$^2/\sqrt{\text{Hz}}$

Results and Analysis

- Simulation Case 1: white sensor noise only

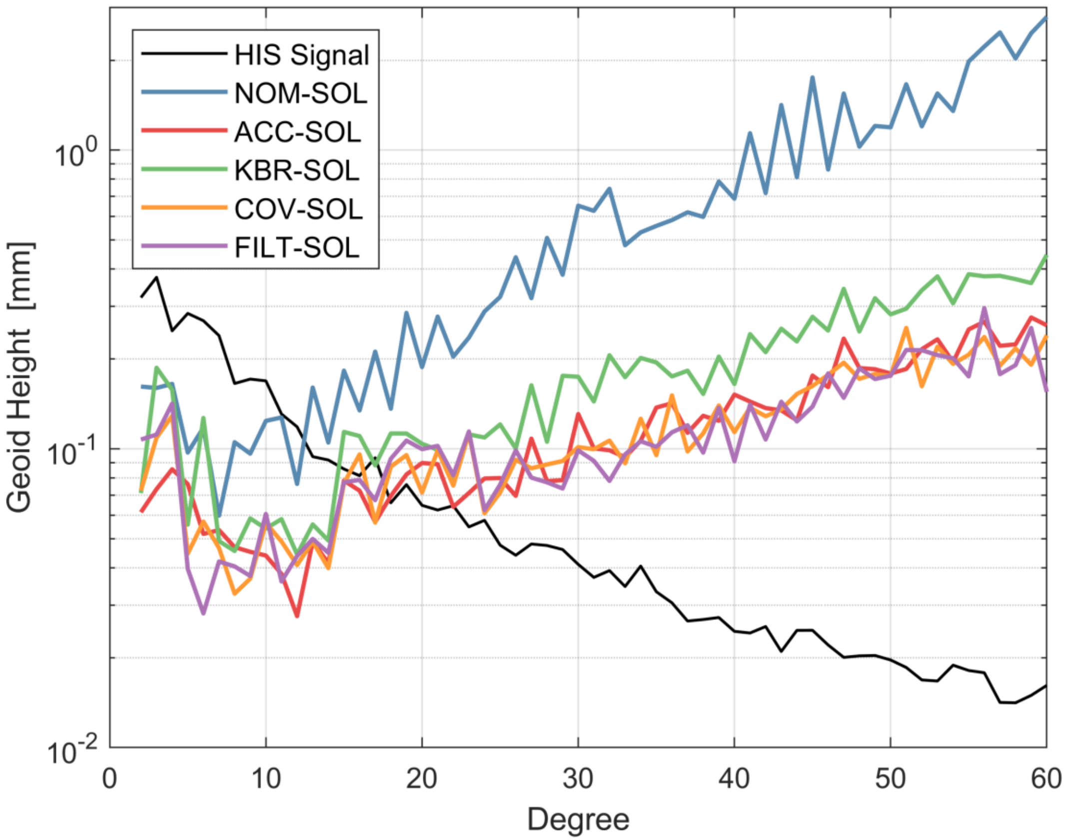


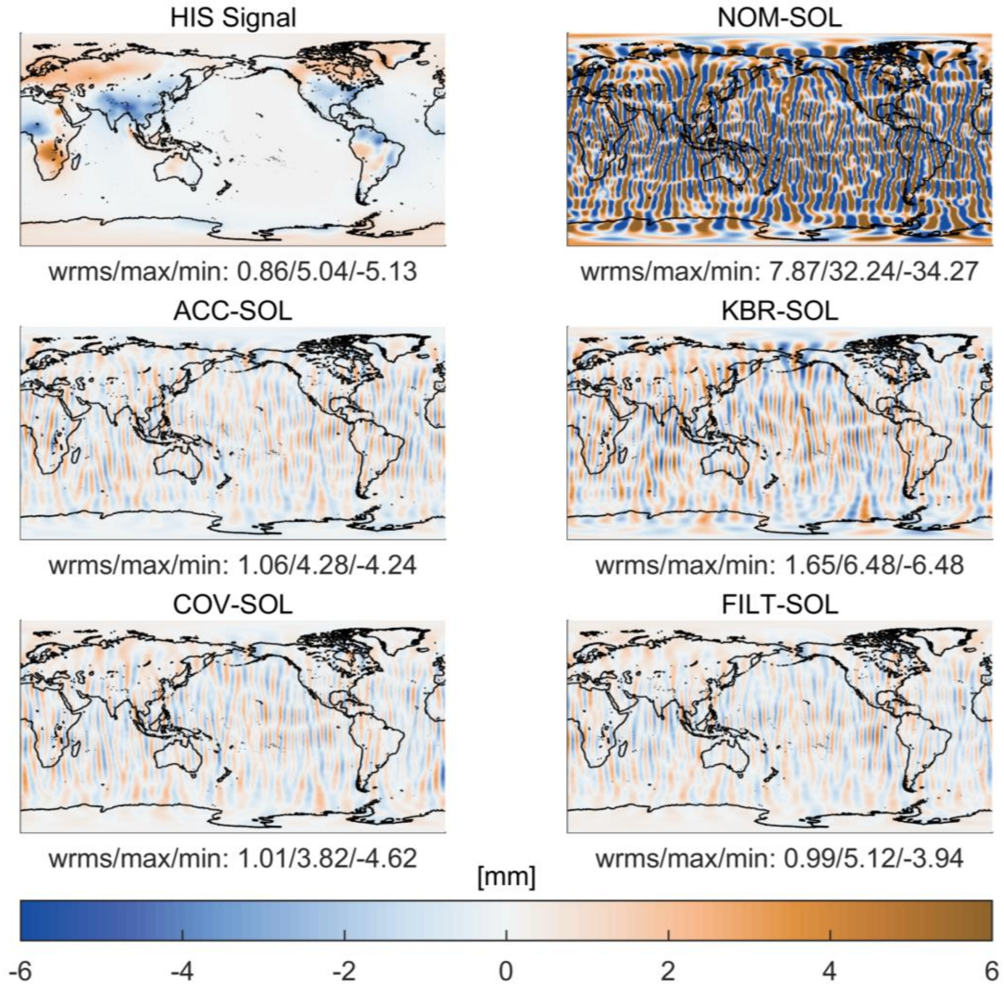


Consistent performances among the four methods

Results and Analysis

- Simulation Case 2: colored sensor noise & temporal aliasing

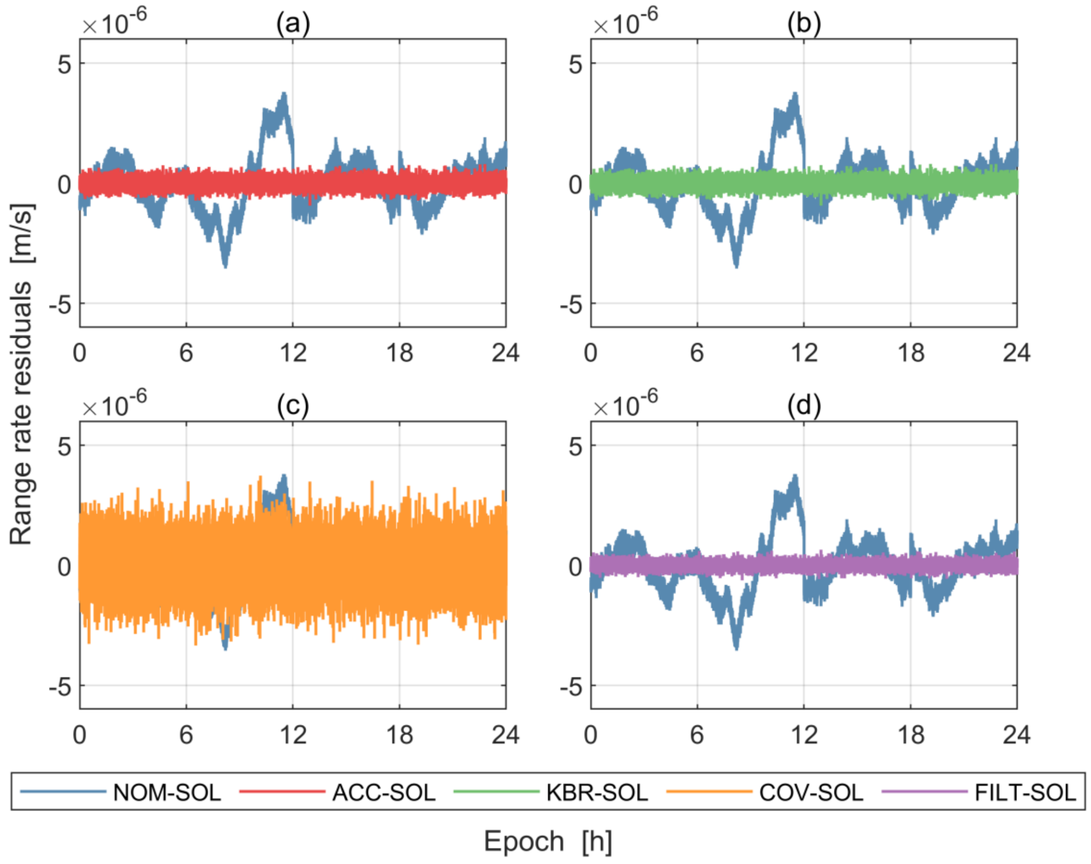


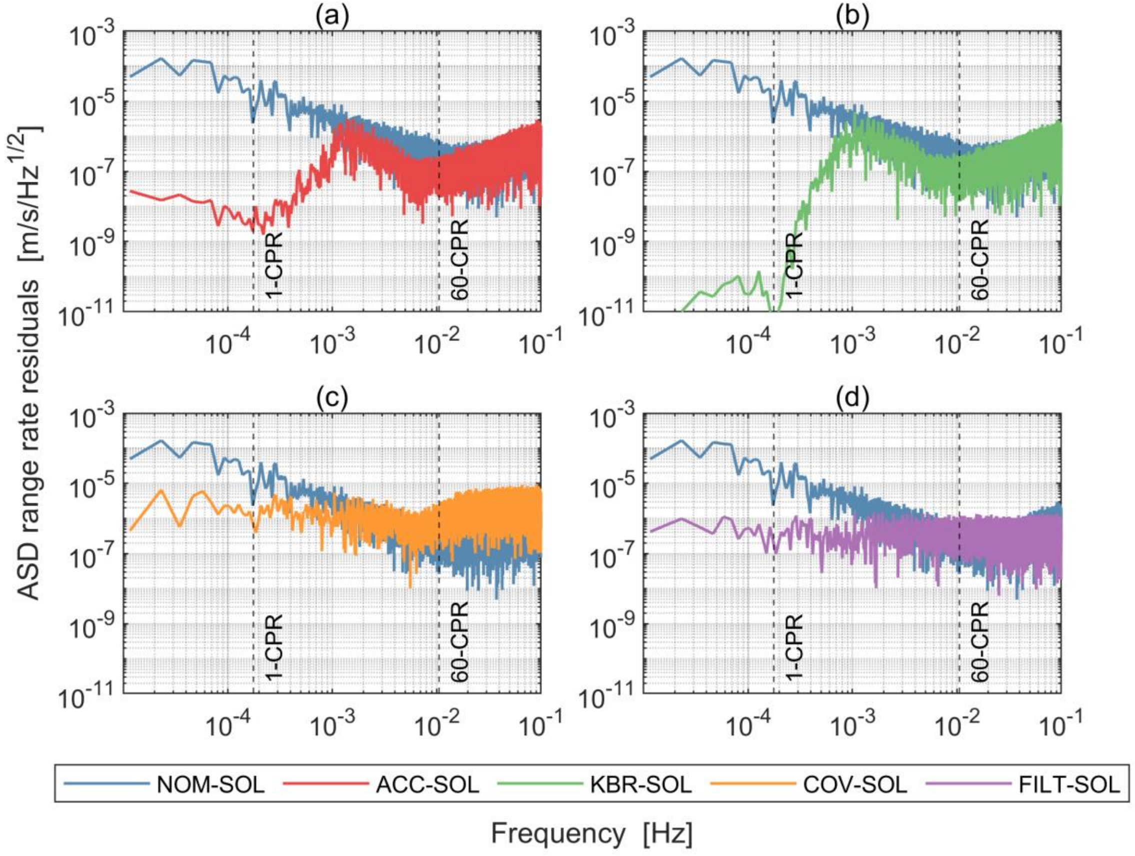


ACC, COV and FILT solutions have less noise

Results and Analysis

- Simulation Case 2: colored sensor noise & temporal aliasing





Low-frequency patterns are mitigated

Colored noise behavior beyond 10^{-3} Hz is not properly handled in ACC and KBR-SOL

Conclusions

- The ACC and COV can be regarded as special cases of LSC
- KBR and ACC are linked by linear perturbation theory, FILT and COV represent parametric and non-parametric spectral estimation techniques
- Four methods are of consistent performances when only white sensor noise is considered
- ACC, COV and FILT perform better when both colored sensor noise and temporal aliasing are considered
- Empirical parameterization (ACC and KBR) works as high-pass filter
- Stochastic modelling (COV and FILT) can deal with colored noise for the whole frequency band

Thank you for your attention !

yufeng.nie@tongji.edu.cn

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