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Identifying efficient ensemble perturbations for initializing subseasonal-to-seasonal prediction

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Initialization of S2S predictions

- S2S predictions is beyond atmosphere predictability limit
  → Coupled Earth system models must be used
- Usually done with ensemble:
  → How to initialize them consistently to obtain reliable results?
- Already tested, use of local properties:
  - Bred vectors (Peña & Kalnay, 2004; Yang et al., 2008; O'Kane et al., 2019)
  - Backward Lyapunov Vectors (BLVs, related to singular vectors) (Vannitsem & Duan, 2020)

  to tune the initialization of the models.

In the present work, we study the projections of the initial conditions on mainly:

- Covariant Lyapunov vectors (CLVs) and their adjoint
- Dynamical Mode Decomposition adjoint modes
- Perron-Frobenius Mode Decomposition adjoint modes

for predictions experiments with a low-order coupled ocean-atmosphere model (MAOOAM).

Dynamical Modes Decomposition (DMD)

- Considering 2 collections of states of a dynamical system $X=[\mathbf{x}_0 \dots \mathbf{x}_{K-1}]$ and $Y=[\mathbf{x}_1 \dots \mathbf{x}_K]$, then one define

$$\mathrm{M}^{\mathrm{DMD}} = \mathrm{Y}\,\mathrm{X}^{+}$$ where X^+ is the pseudoinvere

as the DMD decomposition of the observable $\mathbf{g}(\mathbf{x})=\mathbf{x}$ the system. Related to Linear Inverse Modeling

Penland (1989)

- The left eigenvectors $\mathbf{w}_i$ of M^{DMD} provides approximation of the system's Koopman operator eigenfunctions and are called *adjoint* DMD modes. (Tu et al., 2014)

- The Koopman K operator is an ∞-dimensional linear operator propagating the observable of the system. For an observable *g* :

$$\mathcal{K}^{\tau}\, g(\boldsymbol{x}) = g\left(\boldsymbol{\Phi}^{\tau}(\boldsymbol{x})\right)$$ where ϕ is the flow of the system $\dot{\boldsymbol{x}} = \boldsymbol{f}(\boldsymbol{x})$

- The action of the Koopman operator can then be approximated with the DMD as

$$\sum_{i=1}^{P} \boldsymbol{c}_i^{\mathrm{DMD}}\, \lambda_i^{\mathrm{DMD}}\, \boldsymbol{w}_i^* \quad \boldsymbol{x}$$

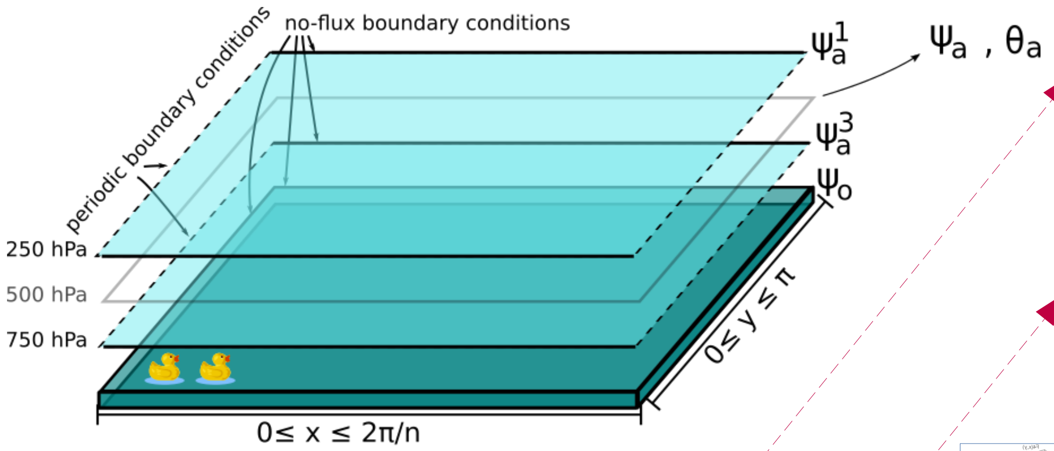
where the $\mathbf{c}_i^{DMD}$ are modes depending on the observable *g*

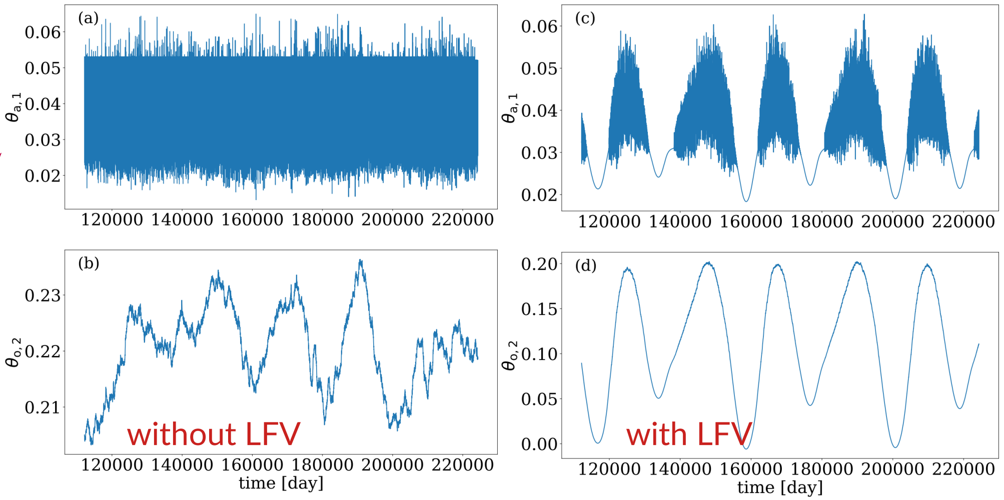
The $\mathbf{w}_i$ define approximate invariant manifolds for the Koopman operator

Same decomposition exists for the Perron-Frobenius (PF) operator propagating the probability distributions in the system.

MAOOAM: an ocean-atmosphere coupled model

QG atmosphere coupled to a shallow-water ocean



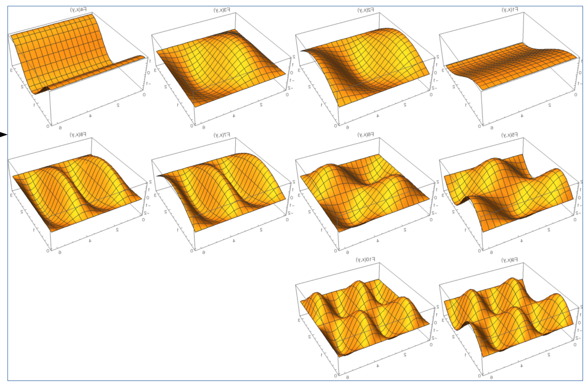


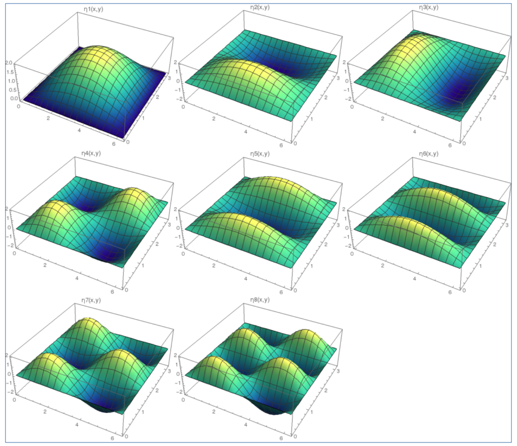
Atm. streamfunction $\psi_a(x,y) = \sum_{i=1}^{n_a} \psi_{a,i} F_i(x,y)$

Atm. temperature $\theta_a(x,y) = \sum_{i=1}^{n_a} \theta_{a,i} F_i(x,y)$

Oc. streamfunction $\psi_o(x,y) = \sum_{i=1}^{n_o} \psi_{o,i} \phi_i(x,y)$

Oc. temperature $\theta_o(x,y) = \sum_{i=1}^{n_o} \theta_{o,i} \phi_i(x,y)$





Experiments design

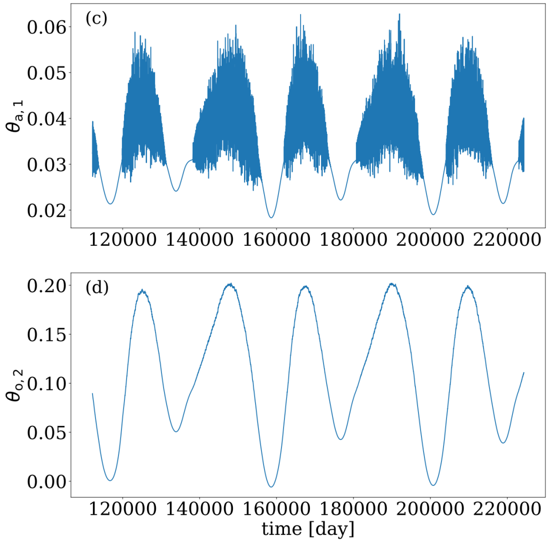
N ensemble forecasts along a reference trajectory:

Control run IC: $\boldsymbol{x}_n^{\text{ctrl}}(0) = \boldsymbol{x}_n(0) + \boldsymbol{\delta x}_0^{\text{ctrl}}$ n = 1,…,N

Perfect ensemble IC: $\boldsymbol{y}_{m,n}(0) = \boldsymbol{x}_n^{\text{ctrl}}(0) + \boldsymbol{\delta x}_0^m \quad , \quad m = 1, \ldots, M-1$

Same distribution U[-ε/2,ε/2]

$\boldsymbol{\delta x}_0^M = 0$



<u>Experiments</u>

Projection onto subspaces spanned by selected vectors:

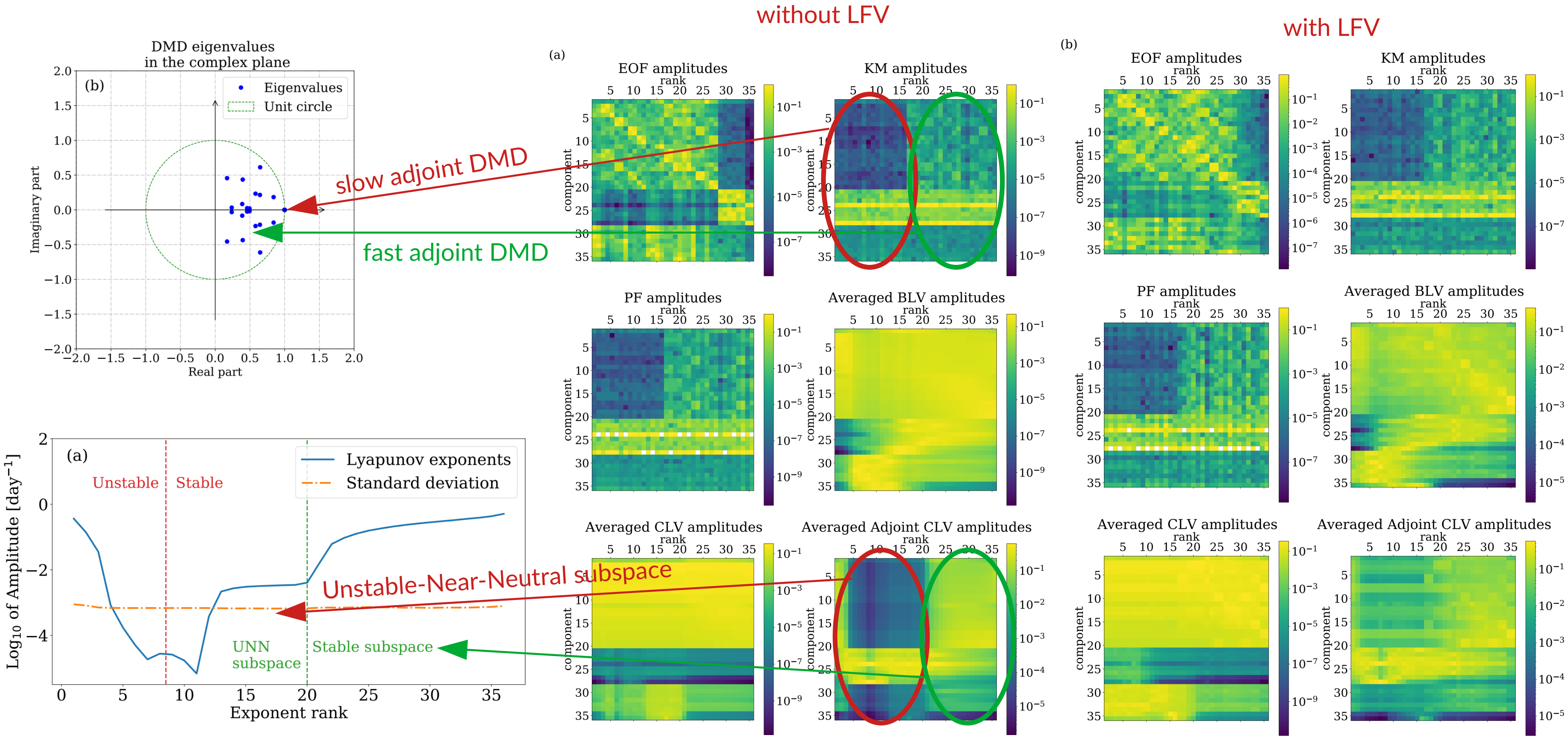
Projector: $\Pi = \mathrm{B}(\mathrm{B}^* \mathrm{B})^{-1} \mathrm{B}^*$ where B is the column matrix of the selected vectors

Projected perturbations: $\boldsymbol{\delta x}_0'^m = \Pi\, \boldsymbol{\delta x}_0^m$

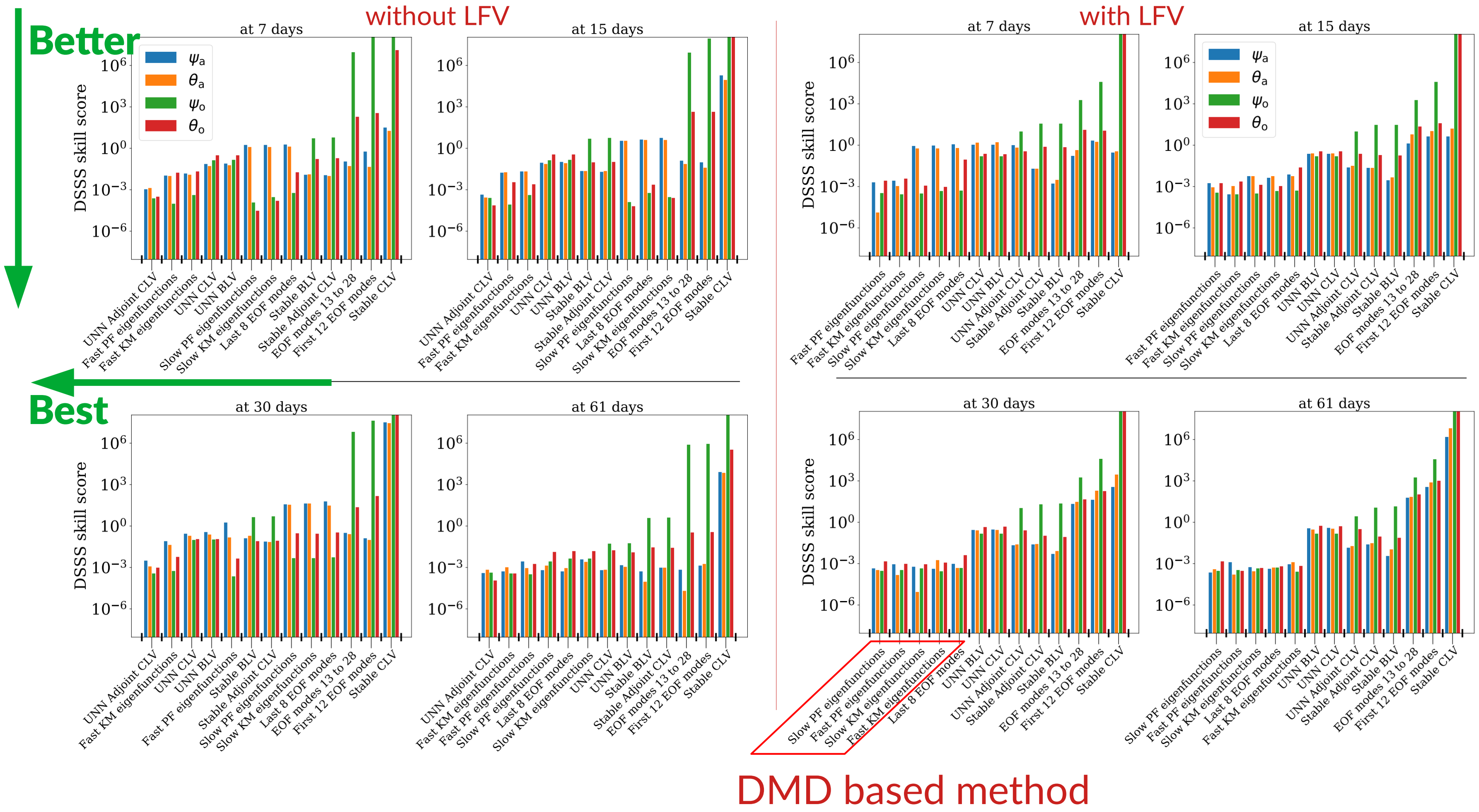
New ensembles: $\boldsymbol{y}'_{m,n}(0) = \boldsymbol{x}_n^{\text{ctrl}}(0) + \boldsymbol{\delta x}_0'^m \quad , \quad m = 1, \ldots, M$

<u>Goal:</u>
→ Obtain forecasts as reliable as the ones provided by the perfect ensembles.

Selected bases: analysis of the reference trajectory



Results: DSSS skill scores at different lead times



DSSS:

Skill score based on the Dawid-Sebastiani score. Worth 0 if same reliability as the perfect ensemble. Characterize the moments.

Dawid & Sebastiani (1999)

Leutbecher (2019)

Conclusion

Key results:

- Approximated KM and PF eigenfunctions obtained using DMD provide reliable ensemble forecasts, and are “easy” to compute.
- Adjoint CLVs also provide reliable forecasts (sometimes the most reliable ones), but are notably hard to compute.
- A consistent link exists between the two frameworks, which explains the results.
- Results seem to not depend on the regime (with or withour LFV)
- Results of Vannitsem & Duan (2020) with the BLVs can also be explained with the DMDs.

Forthcoming developments:

- Many, but most notably, replication of the study with a higher-dimensional system, with a non-trivial dimensionality reduction to make DMD tractable.
- Ultimately, development of the approach in a realistic S2S framework.

Notice finally that the first forward Liapunov vector is not the direction of the fastest growing perturbation for large future time, since all perturbations contained within $F_1^+(t_1)\backslash F_2^+(t_1)$ eventually grow at the same average rate λ_1. In other words, perturbations starting from, say, the subspace spanned by $\mathrm{f}_1^+(t_1)$ and $\mathrm{f}_2^+(t_1)$ are, in practice, not growing faster than almost any random perturbation. This indicates that, for long-range forecasts, the use of singular vectors for initial perturbations looses its relevance.

Legras & Vautard (1996)
A guide to Liapunov vectors. ECMWF seminar on predictability.

Takeaway message

To initialize ensemble forecast for S2S:

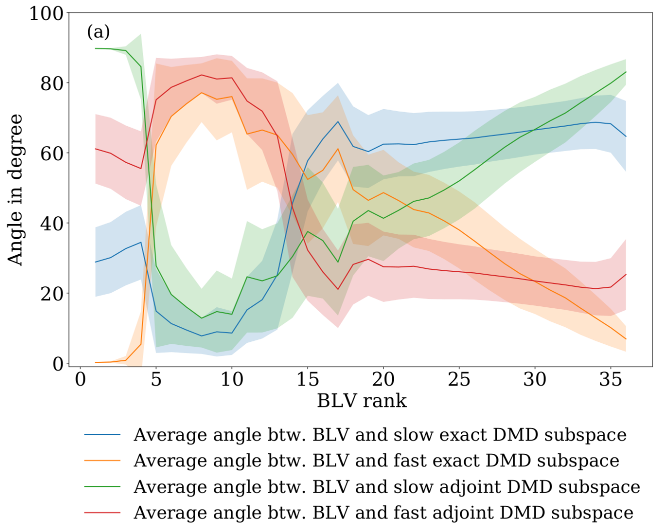
- Local stability properties are less important
- However, ensembles initial conditions can be constructed in more relevant subspaces

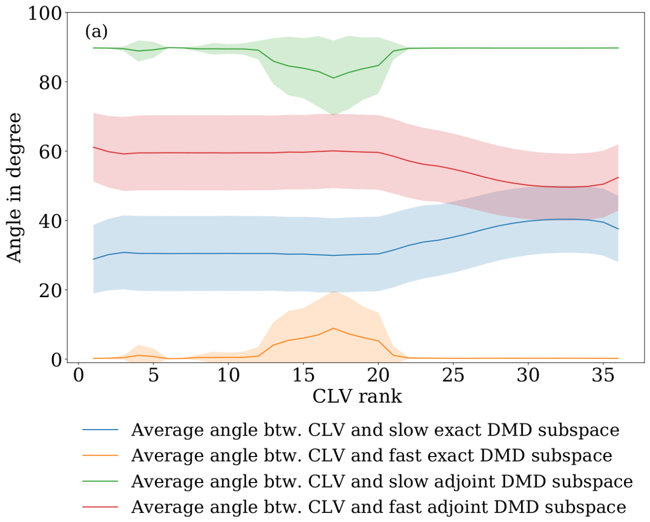
  → constructed with *measures* of the system valid at these timescales (through DMD)

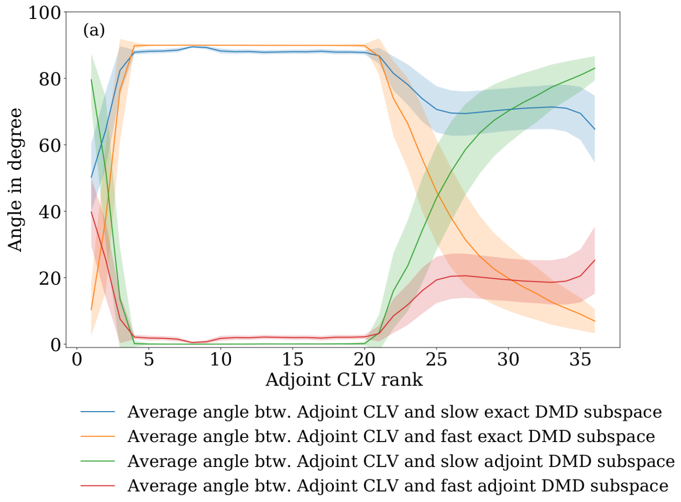
Reference

Jonathan Demaeyer, Stephen G. Penny, Stéphane Vannitsem. Identifying efficient ensemble perturbations for initializing subseasonal-to-seasonal prediction. JAMES, in press, doi: 10.1029/2021MS002828

Backup: Correspondence between bases







Fast BLVs close to adjoint DMDs (KM eigenfunctions) → Explains good results obtained with fast BLVs in Vannitsem & Duan (2020)

CLVs related to DMDs (KM modes)

Adjoint CLVs related to adjoint DMDs (KM eigenfunctions)

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