Wave equilibration process of a non-equilibrium sea-state in shallow water after strong depth variation

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Introduction

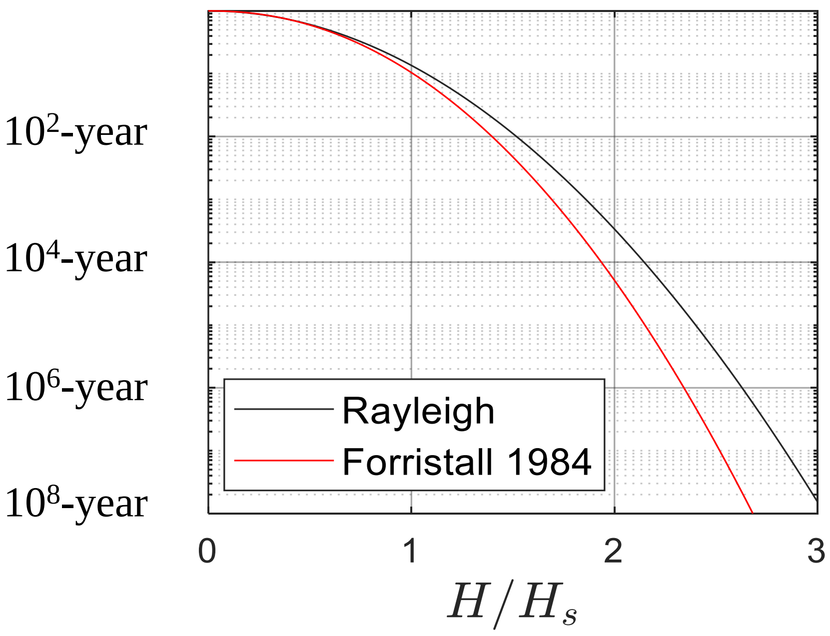
Definition / criterion

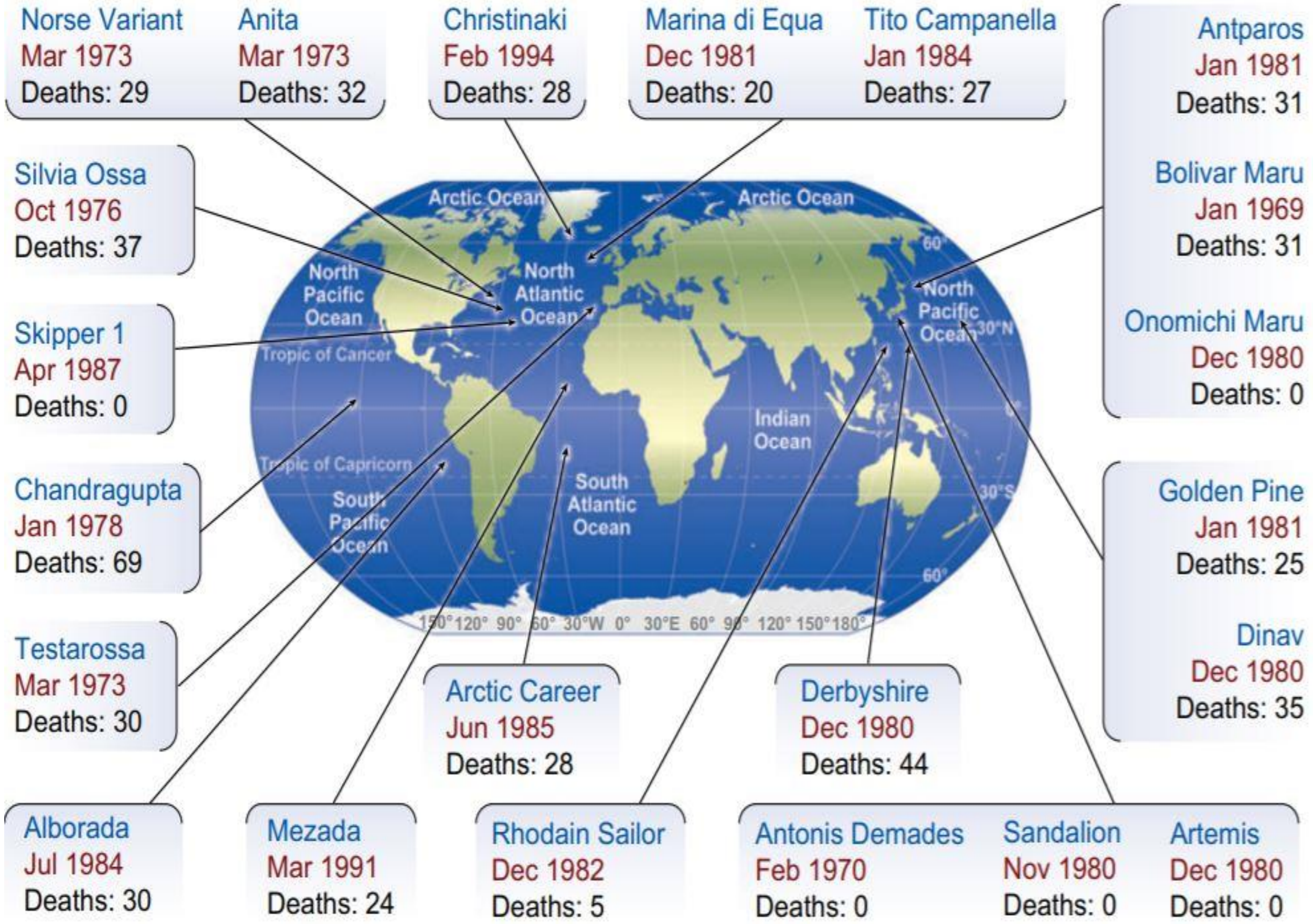
$$\frac{\boldsymbol{H}}{\boldsymbol{H_s}} > \mathbf{2} \ \text{ or } \ \frac{\eta_{crest}}{H_s} > 1.25$$

(H_s: significant wave height)

Probability distribution

Return Period – Wave height





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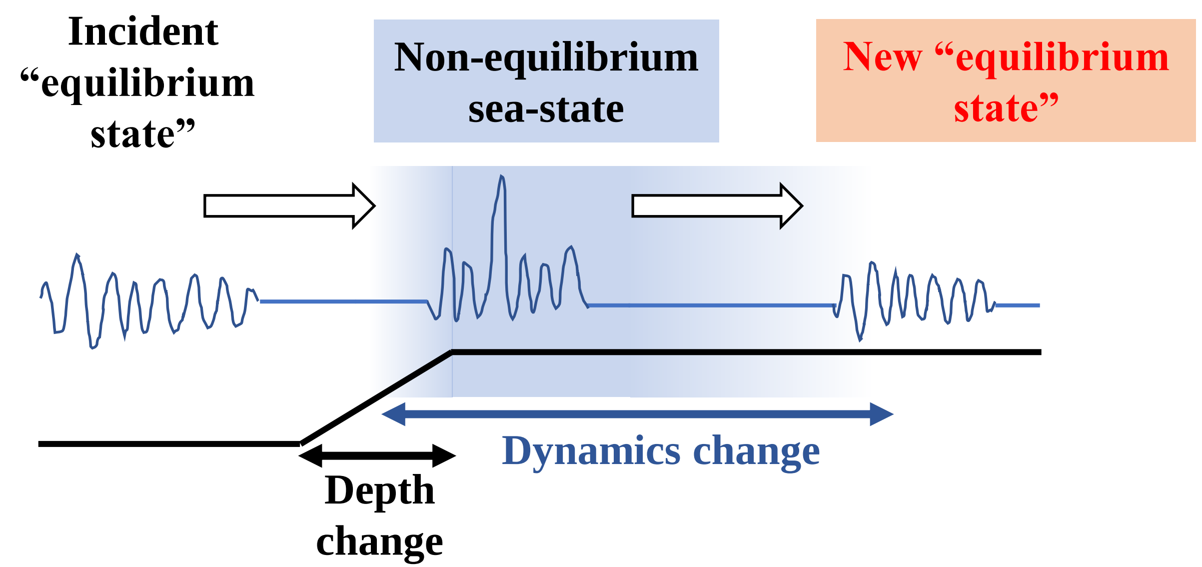
What causes freak wave formation ?

1. **Geometrical focusing** (current / bathymetry);
2. **Space-time focusing** (dispersion);
3. **Modulational instability** (unstable sidebands);
4. **Non-equilibrium dynamics** (uneven bottom).

What is Non-equilibrium dynamics (NED) ?



What do we know about NED ?

1. **Key features of non-equilibrium sea-state**
   - Non-Gaussian, Non-linear
   - Higher freak wave probability
   - Rapid spectral evolution
2. **Parametric investigation of NED**
   - Relative water depth
   - Steepness
   - Wave direction
   - Bottom profile
3. **Theoretical development**
   - Deterministic theory (Li *et al.* 2021)
   - Stochastic theory (Mendes *et al.* 2021)

The new equilibrium state and the equilibration process have not yet been studied.

Whispers3D: a fully nonlinear and dispersive potential model

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Assumptions

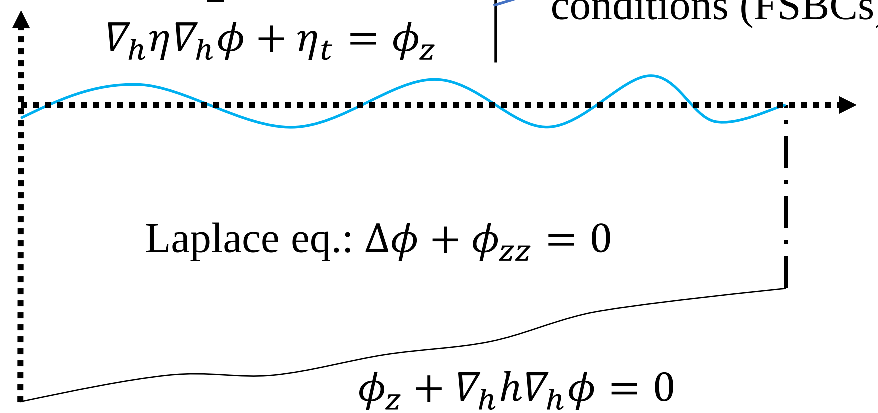
1. Incompressible fluid;
2. Inviscid and irrotational flow;
3. Continuous water column;
4. Impermeable bottom.

Governing equations of potential theory

$$\phi_t + \frac{(\nabla_h\phi)^2 + {\phi_z}^2}{2} = -g\eta$$

$$\nabla_h\eta\nabla_h\phi + \eta_t = \phi_z$$

Free surface boundary conditions (FSBCs)



Laplace eq.: $\Delta\phi + \phi_{zz} = 0$

$$\phi_z + \nabla_h h\nabla_h\phi = 0$$

Bottom boundary condition (BBC)

Formulation of Zakharov

FSBCs reformulated using $\eta(x, t)$ and $\tilde{\phi}(x, t) \equiv \phi(x, z = \eta(x, t), t)$

$$\eta_t + \nabla_h\eta\nabla_h\tilde{\phi} - \tilde{\boldsymbol{w}}\left(1 + (\nabla_h\eta)^2\right) = 0,$$

$$\tilde{\phi}_t + g\eta + \frac{1}{2}\left(\nabla_h\tilde{\phi}\right)^2 - \frac{1}{2}\tilde{\boldsymbol{w}}^2\left(1 + (\nabla_h\eta)^2\right) = 0,$$

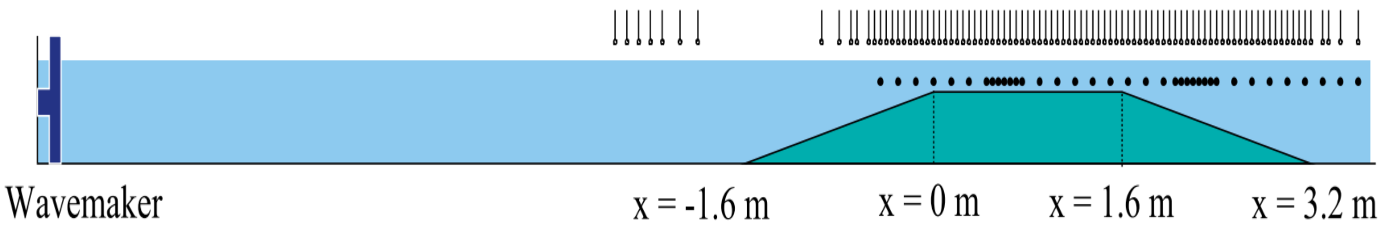
Dirichlet-to-Neumann (DtN) problem

with $\tilde{w}$: vertical velocity at the free surface $\phi_z(x, z = \eta(x, t), t)$;

Validation with experiments in Trulsen *et al.* 2020, JFM

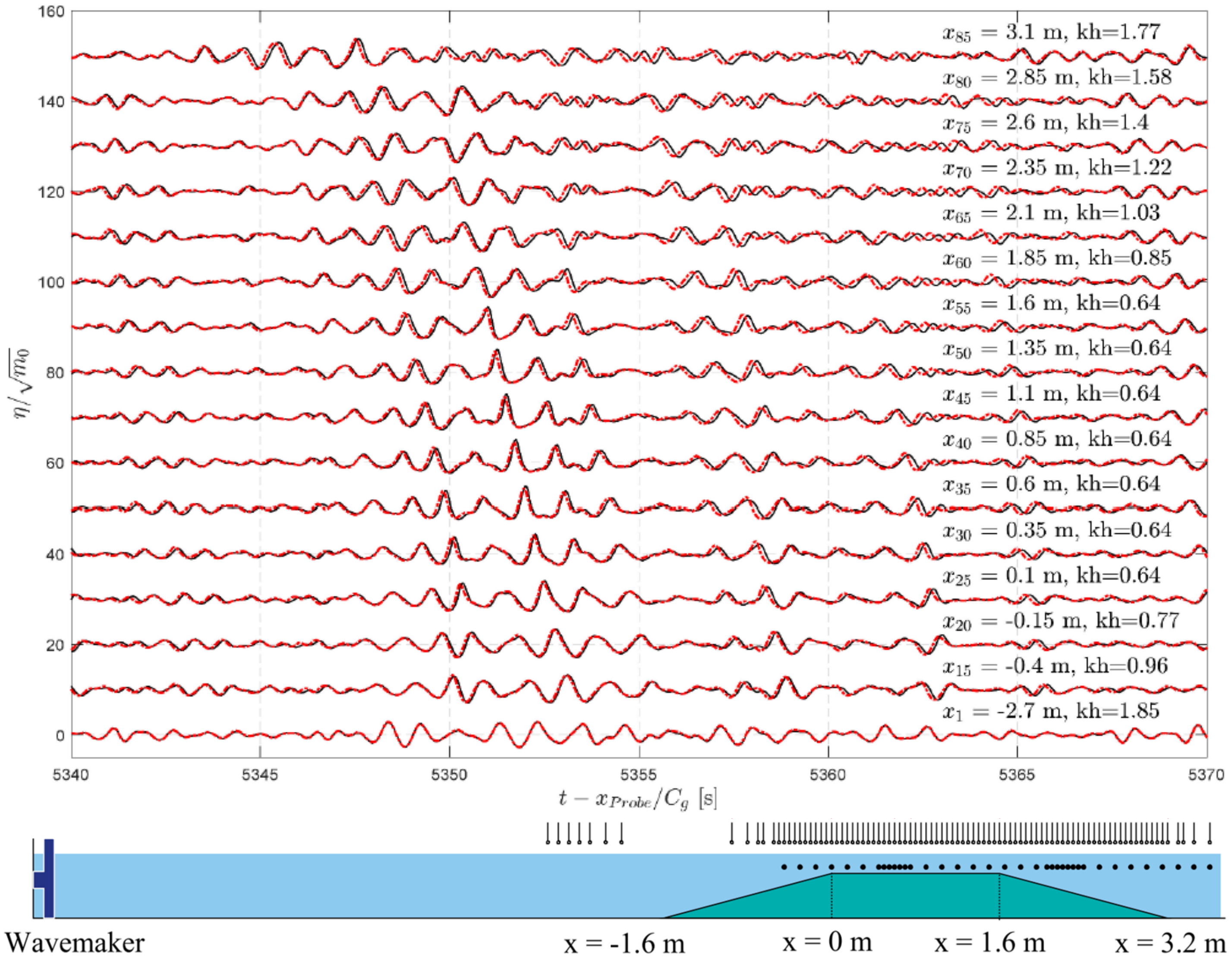
Run	T_p (s)	γ	Deeper region h_1 (m)	μ_1	H_{m_0} (m)	ϵ_1	Ur_1	Shallower region h_2 (m)	μ_2	H_{m_0} (m)	ϵ_2	Ur_2
3	1.1	3.3	0.53	1.85	0.025	0.031	0.0049	0.11	0.64	0.025	0.052	0.1918

TABLE 1. Key parameters of the experimental case reported as Run 3 in Trulsen *et al.* (2020).

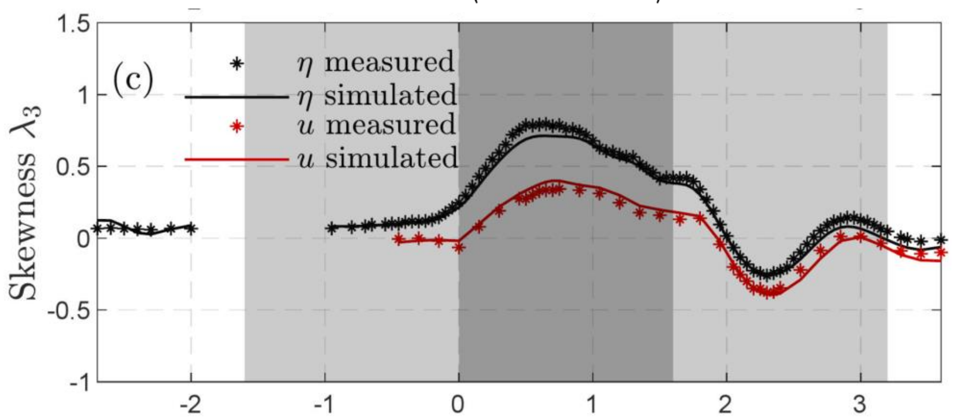


Whispers3D: Validation in simulating NED

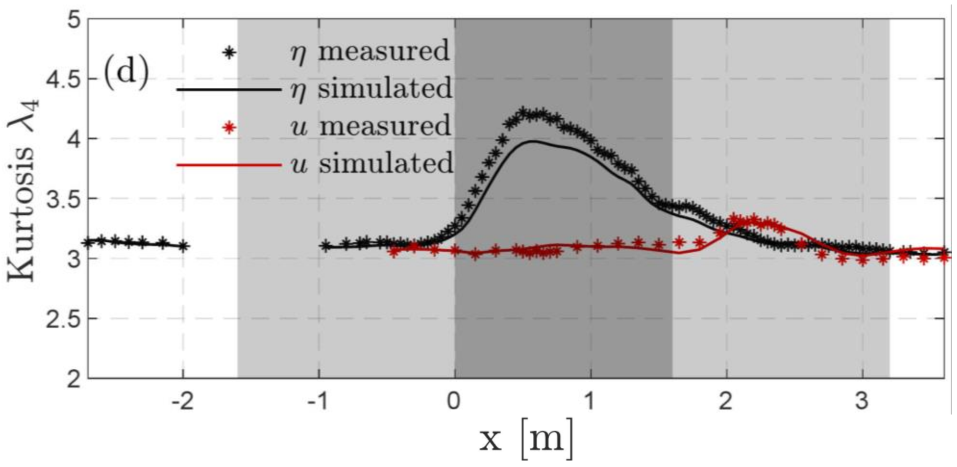
Black lines: measurements; **Red lines: simulations**



$$\lambda_3(\eta) = \left\langle (\eta - \langle\eta\rangle)^3 \right\rangle / \sigma^3$$

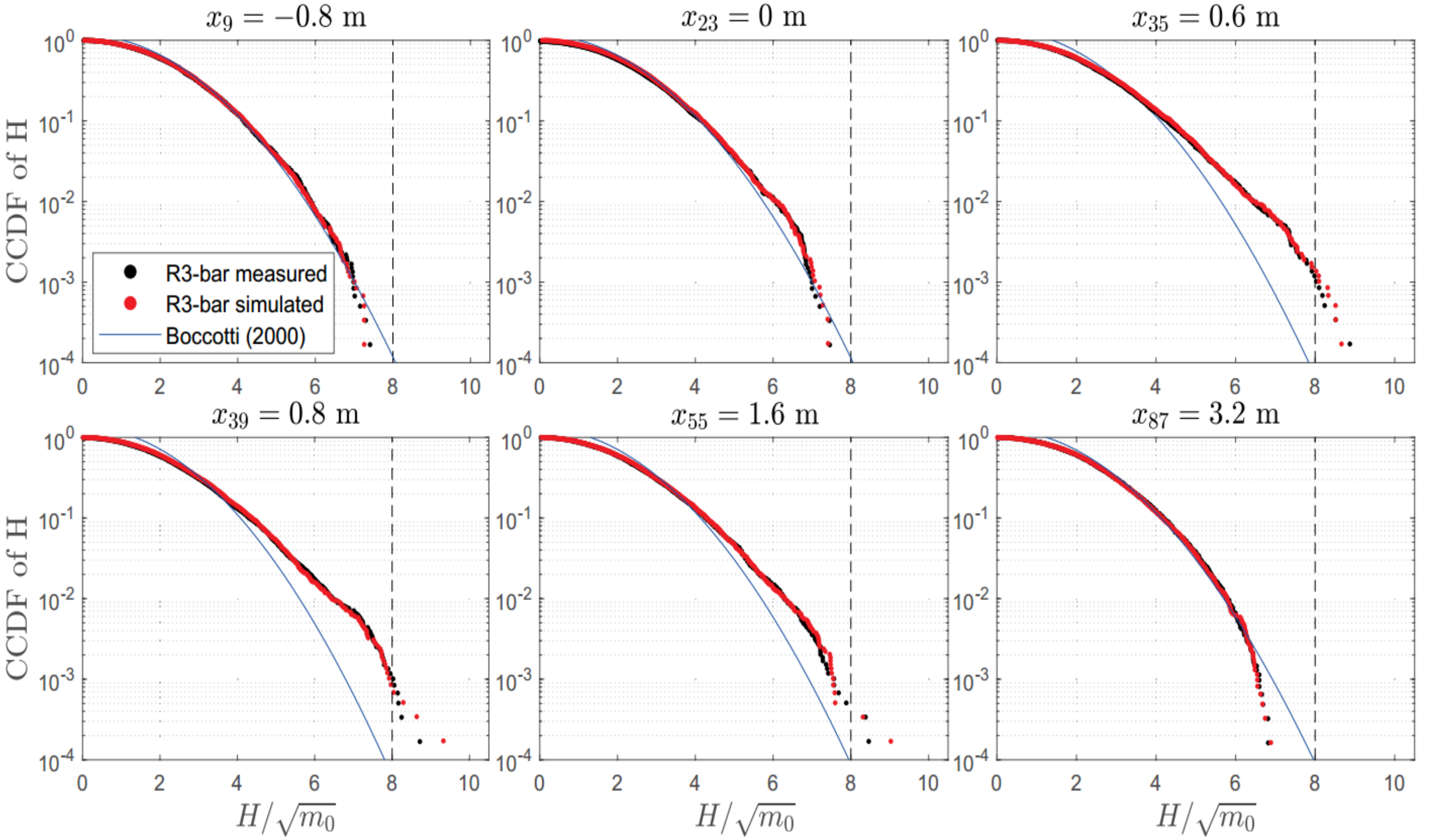


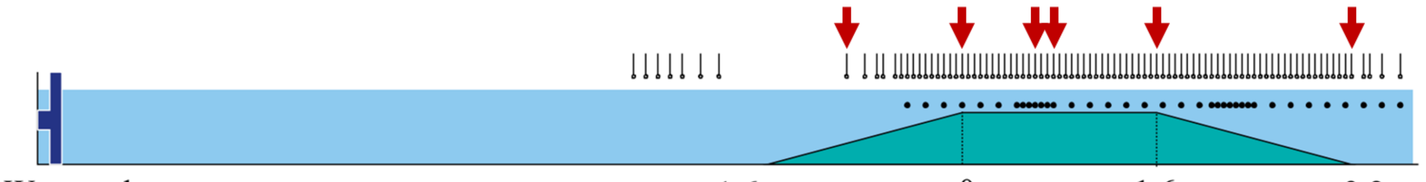
$$\lambda_4(\eta) = \left\langle (\eta - \langle\eta\rangle)^4 \right\rangle / \sigma^4$$



Results are published in **Zhang and Benoit 2021, JFM**

Whispers3D: Validation in simulating NED





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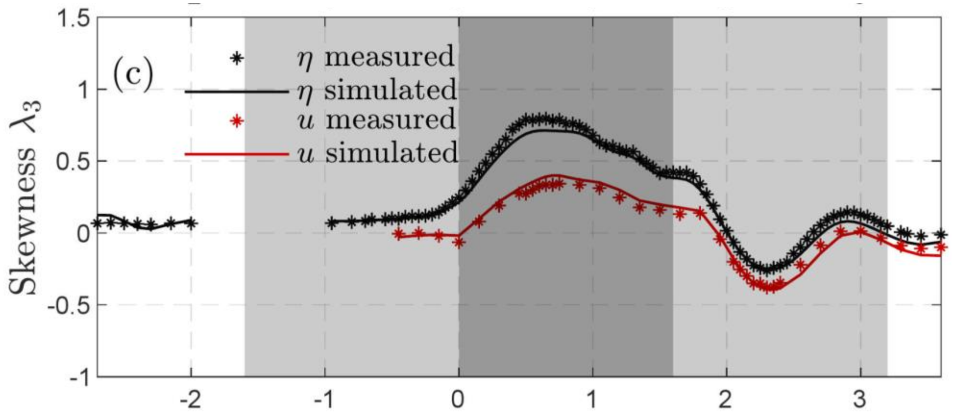

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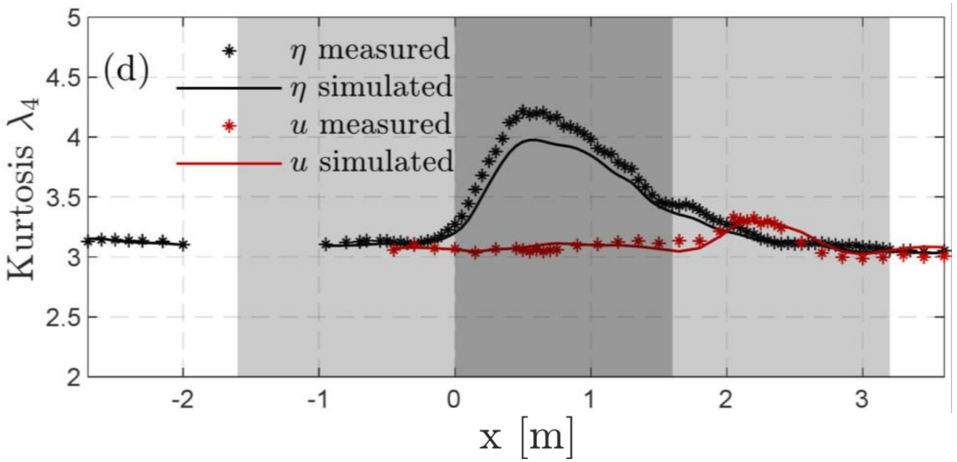


Results are published in **Zhang and Benoit 2021, JFM**

Set-up of the simulations: Long shallower region

Incident irregular waves (JONSWAP spectrum)

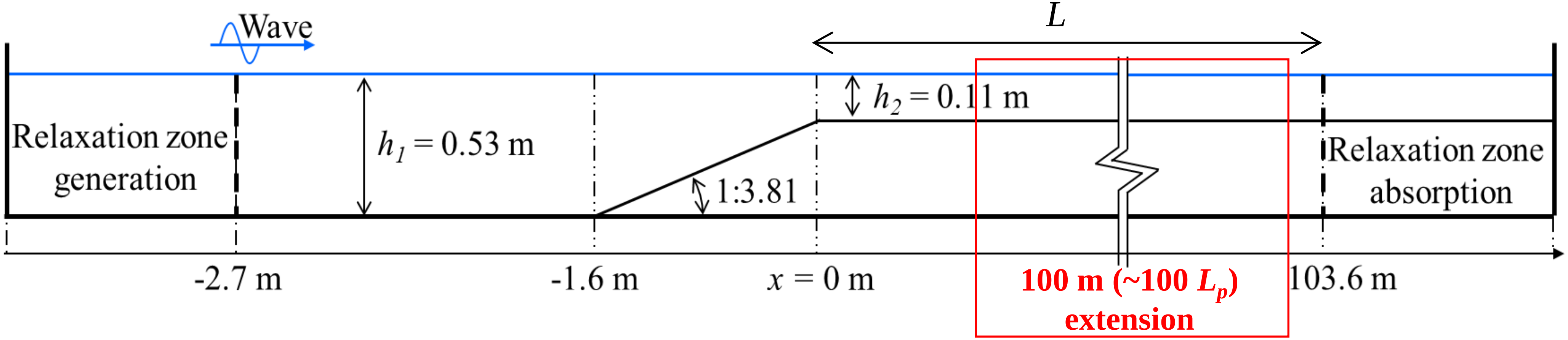
Summary of wave field parameters, in SI units.

Case	H_s	L	Deeper/shallower area		
			μ	ε	Ur
1	0.030	103.6	1.85/0.64	0.037/0.062	0.0058/0.232

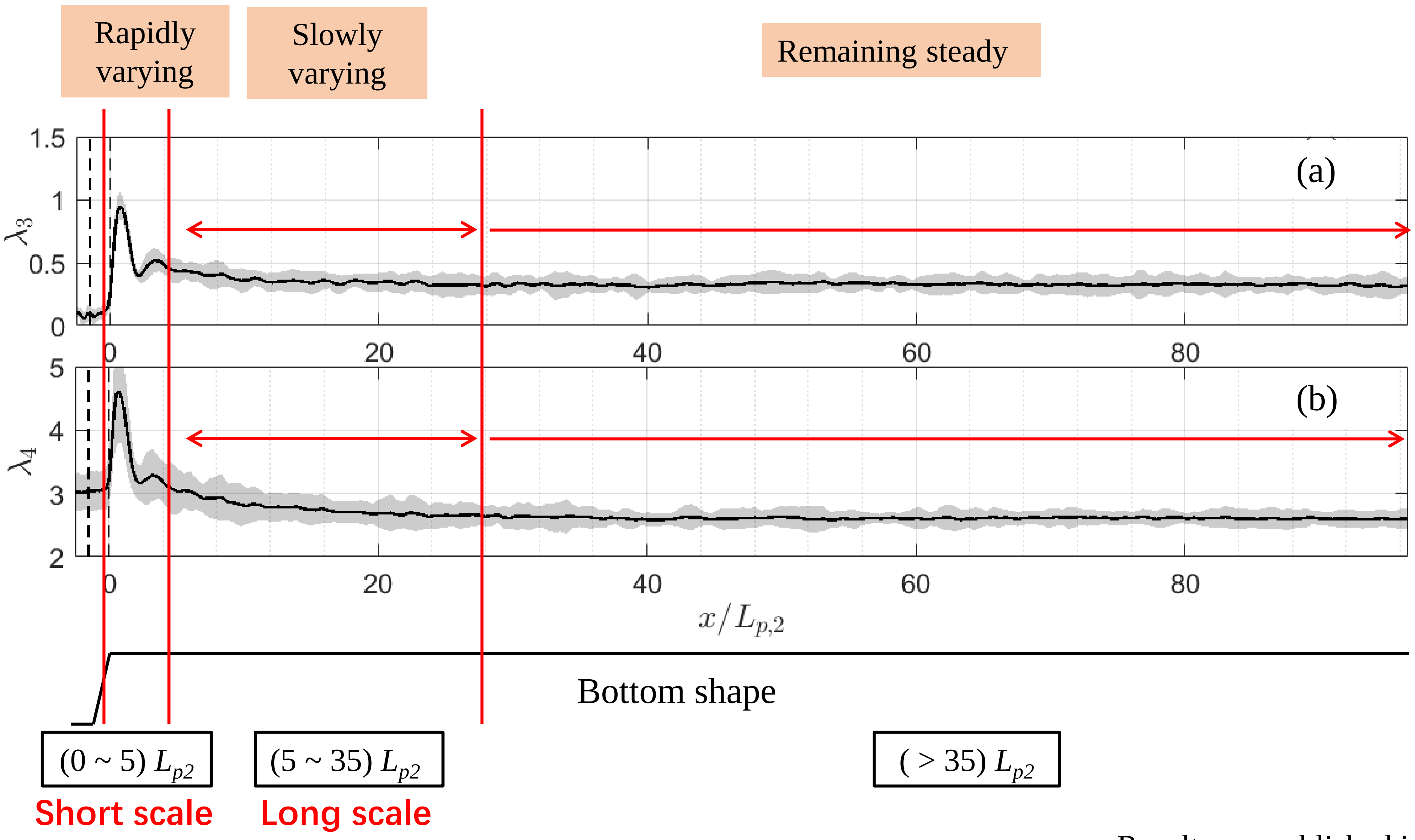
[a] For all cases, $T_p = 1.1$ s, $\gamma = 3.3$, deeper region depth $h_1 = 0.53$ m, shallower region depth $h_2 = 0.11$ m.

relative water depth $\mu = k_p h$
steepness $\epsilon = k_p a_c$
Ursell number $U_r = \epsilon / \mu^3$
$a_c = \sqrt{2}\sigma$
$\sigma^2 = \langle (\eta - \langle \eta \rangle)^2 \rangle = m_0$

New bathymetry with extended shallower region



Results and interpretation: Statistical parameter evolution



1. Strong modulation of **statistical moments** happens in the **short scale**;

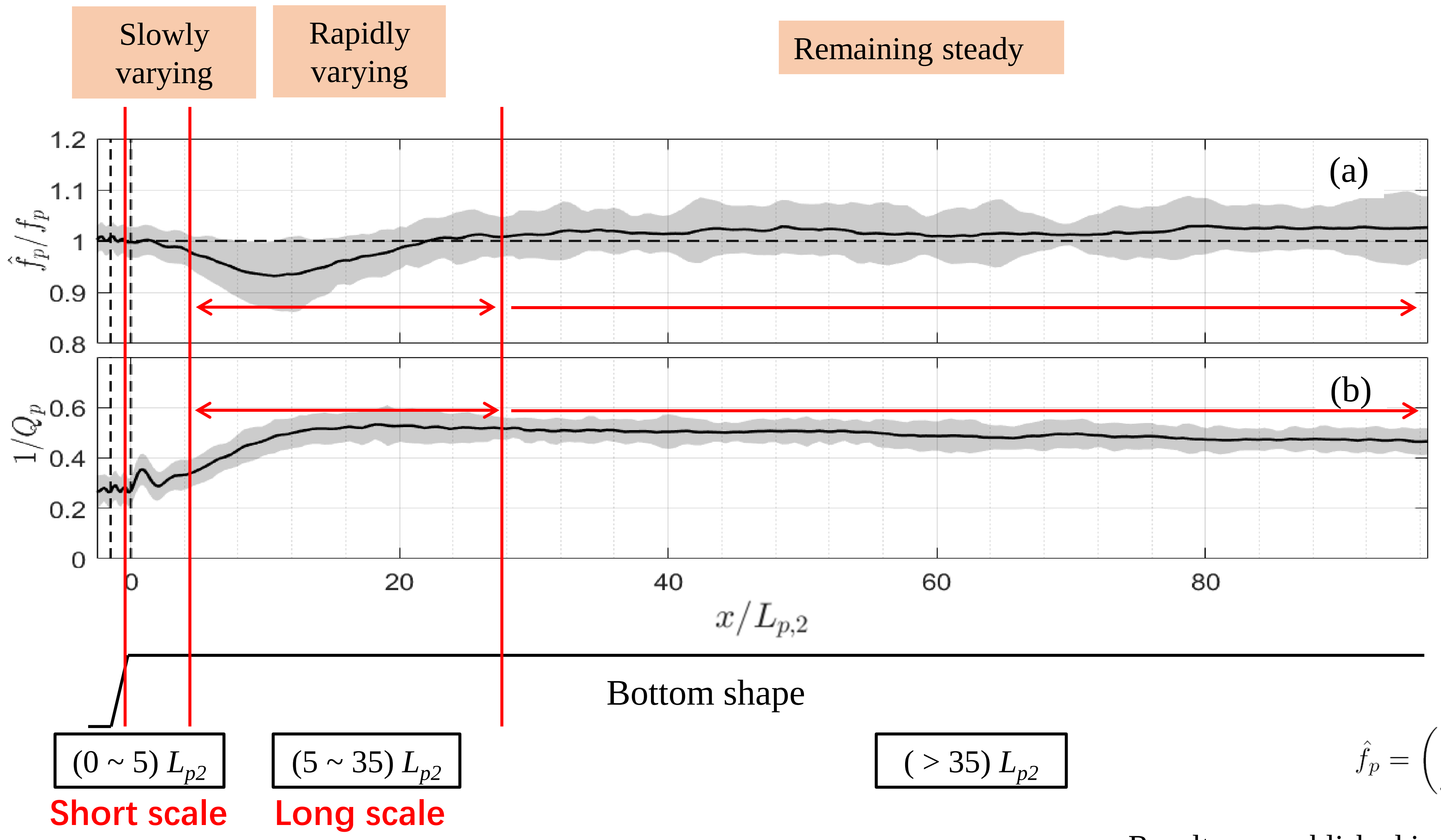
2. Steady **Kurtosis < 3** in the new equilibrium state.

$$\lambda_3(\eta) = \left\langle (\eta - \langle \eta \rangle)^3 \right\rangle / \sigma^3$$

$$\lambda_4(\eta) = \left\langle (\eta - \langle \eta \rangle)^4 \right\rangle / \sigma^4$$

Results are published in **Zhang et al. 2022, Coastal. Eng.**

Results and interpretation: Spectral parameter evolution



1. Strong modulation of **spectral parameters** happens in the **long scale**;
2. **Significant spectral broadening** happens in the new equilibrium state.

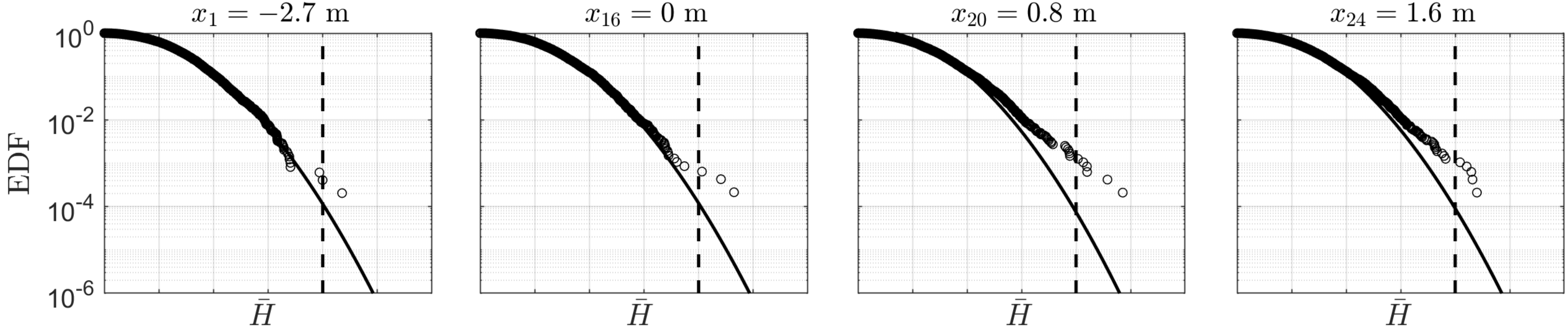
$$Q_p = \frac{2}{m_0^2}\int_0^{\infty} f S^2(f)\,\mathrm{d}f$$

$$\hat{f}_p = \left(\int_0^{\infty} f S^4(f)\,\mathrm{d}f\right) / \left(\int_0^{\infty} S^4(f)\,\mathrm{d}f\right)$$

Results are published in **Zhang et al. 2022, Coastal. Eng.**

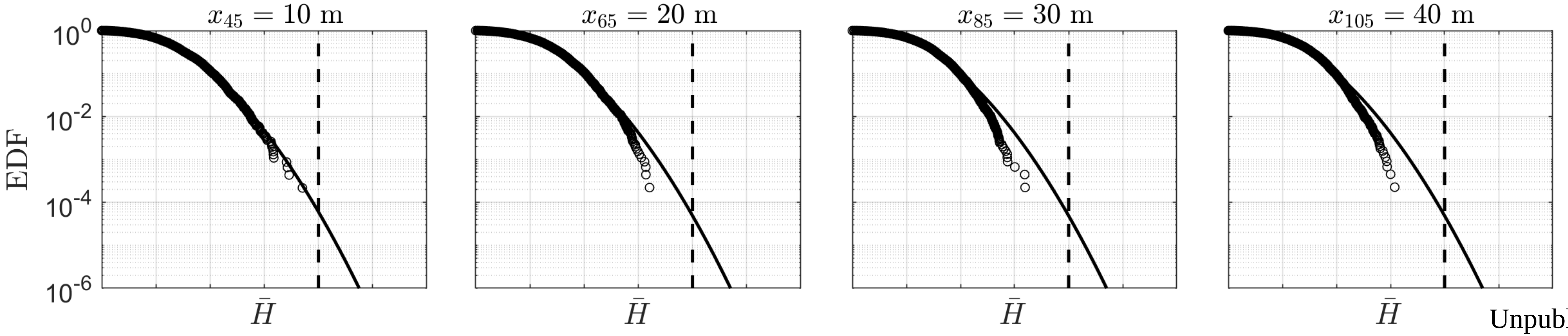
Results and interpretation: Wave height distribution

Short scale results: NED results in enhanced freak wave probability



○ **Empirical distri.** —— **Boccotti (2000)**

Long scale results: NED results in reduced freak wave probability



Unpublished

Conclusion

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- The NED takes effects in **two spatial scales**, a short scale O(L_p) and a relatively long scale O(10~10^2 L_p).
- In the short scale, strong modulation of statistical parameters, freak wave probability is **intensified**;
- In the long scale, the spectral shape undergoes significant broadening, freak wave probability is **suppressed**;
- The knowledge of the "scales" of the NED would help for the design of coastal structures.

Thank you for your attention !

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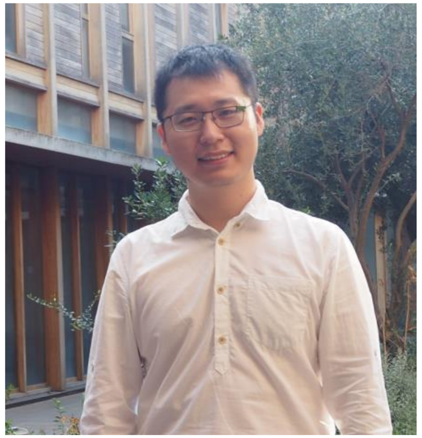

Zhang Jie

2021 - now	Post-doc	Dalian Univ. Tech.	SLCOE
2016 - 2020	Doctor	Centrale Marseille	IRPHE
2014 - 2016	Master	Hohai Univ.	
2010 - 2014	Bachelor	Haerbin Univ.	

Research interests

Extreme waves; Freak waves;

Bragg and Fabry-Perot resonances;

Modulation instability; breathers;

Non-equilibrium dynamics;

References:

[1] Haver, S. 2001 Evidences of the existence of freak waves. *In Rogue waves*, pp. 129–140.

[2] Gemmrich, J., Cicon, L. 2022 Generation mechanism and prediction of an observed extreme rogue wave. *Sci. Rep.* **12**, 1718.

[3] Li, Y., Draycott, S., Zheng, Y., Lin, Z., Adcock, T. A. A., van den Bremer, T. S. 2021 Why rogue waves occur atop abrupt depth transitions. *J. Fluid Mech.* **919**, R5.

[4] Mendes, S., Scotti, A., Brunetti, M., Kasparian, J. 2022 Non-homogeneous analysis of rogue wave probability evolution over a shoal. *J. Fluid Mech.* **939**, A25.

[5] Trulsen, K., Raustøl, A., Jorde, S., Rye, L. B. 2020 Extreme wave statistics of long-crested irregular waves over a shoal. *J. Fluid Mech.* **882**, R2.

[6] Zhang, J., Benoit, M., Kimmoun, O., Chabchoub, A., Hsu, H.-C., 2019. Statistics of extreme waves in coastal waters: large scale experiments and advanced numerical simulations. *Fluids* **4**, 99.

[7] Zhang, J., Benoit, M., 2021. Wave–bottom interaction and extreme wave statistics due to shoaling and de-shoaling of irregular long-crested wave trains over steep seabed changes. *J. Fluid Mech.* **912**, A28.

[8] Zhang, J., Benoit, M., Ma, Y. 2022 Equilibration process of out-of-equilibrium sea-states induced by strong depth variation: Evolution of coastal wave spectrum and representative parameters. *Coastal Eng.* **174**, 104099.